LPV H-infinity Controller Design for a Wind Power Generator

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Abstract— There are many control methods of wind power generator set, and various control technology has successful applied cases, however, with the content augment of set, we need to improve dynamic stability. The routine traditional control technology (for example PID control) can't satisfy the requirements now. Aiming at the particularity of wind power generator set, the paper puts forward a new gain scheduling LPV robust H^{\pi} controller design method. The method places closedloop poles into the field satisfying dynamic response in order to solve the problem that the routine PID control can't solve. Take use of LPV convex decomposition method, transform wind turbine model into convex polyhedron structure LPV model, and then by using LMI method design to meet $H\infty$ performance and closed-loop pole placement feedback gain for each vertex of convex polyhedron. Synthesize each vertex feedback gain to get LPV controller with convex polyhedron structure. The result of simulation validates that the controller has well control performance.

Keywords—wind power generator, LPV(linear parameter varying), $H\infty$ (H-infinity)control, convex decomposition, LMI(linear matrix inequalities)

I. INTRODUCTION

In the control of wind power generator, the tradition PID control is a common method. But the tradition PID control can't satisfy the systems whose model parameter large range, nonlinear and multivariable. Therefore the results usually bring overshoot or fluctuation. In order to solve the problem, many scholars put forward a new gain scheduling controller synthesis method. The controllers designed depend on scheduling variables, and ensure a certain degree of robust stability and robust performance in the range considered.

Gain scheduling control is an engineering design method that is widely applied to nonlinear time-varying systems [1, 2]. The theory is to design local controllers to get a global controller by interpolation method. The essential characteristic is to use linear controller design method to design parameterdependent or nonlinear time-varying system controller. Recently, with robust control development, gain scheduling control received further study [3], especially, gain scheduling control based on LPV method has been applied to actual engineering design [4]. Dynamic characteristic of wind power generator set is a nonlinear system; however, by transform we Zhoujie Shenyang University of Technology Wind Energy Institute Shenyang, China zhoujie_19830312@163.com

can get its LPV model with wind speed, pitch, and rotate speed of wind turbine as parameters.

Aiming at MW(megawatt) grade drive system of variable speed and variable pitch wind power generator set, the paper combines gain scheduling control and H ∞ control theory, take use of LPV synthesis method [5], and put forward a new method that designs gain scheduling robust H ∞ controller. The method ascertains variety range of elected parameters firstly. Then use LPV convex decomposition technique, transform wind power generator model to LPV model with convex polyhedron structure. Design separately feedback gain to meet H ∞ performance and dynamic characteristic, and synthesize each vertex feedback gain designed to get LPV controller with convex polyhedron structure.

Our aim is to design controller with disturbance attenuation, robust stabilization, and closed-loop response satisfying certain requirements. These aims can be solved through H ∞ synthesis and pole placement. With the application of LMI method, H ∞ control problem and pole placement can be described as convex optimization problem included LMIs restriction [6, 7]. Therefore, by solving a set of LMIs design multivariable aims controller to meet H ∞ performance and also get good dynamic characteristic. The paper solves LMIs through MATLAB LMI Control Toolbox [8]. Lastly, validate control performance of controller by simulation.

II. DRIVE SYSTEM MODEL AND LPV EXPRESSION

The definition of LPV system: LPV system is a linearity time-varying system, and its state-place model A, B, C, D is a function of time-varying parameters $\theta(t)$. The important difference between it and average time-varying system is that $\theta(t)$ can be measured on-line and its varying range is known beforehand. Namely $\theta(t) \in \Theta, \forall t \ge 0, \Theta$ is the range of parameters.

Its state equation expression is:

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t),$$

$$y(t) = C(\theta)x(t) + D(\theta)u(t),$$

$$\theta(t) \in \Theta$$

Where x, u, y denote separately state, input, output. $\theta(t)$ is time-varying vector of parameter. Limit the time-varying system:

- The state matrixes $A(\theta), B(\theta), C(\theta), D(\theta)$ depend on $\theta(t)$;
- θ(t) is varying in the polyhedron field whose vertexes are w₁, w₂,..., w_r;
- $\Theta(t) \in \Theta := \{w_1, w_2, \dots, w_r\}, Co \text{ expresses convex hull.}$

So $\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} \in Co := \left\{ \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} := \begin{bmatrix} A(w_i) & B(w_i) \\ C(w_i) & D(w_i) \end{bmatrix}, i = 1, 2 \cdots r \right\}$ Three-step model of drive train:

$$\begin{bmatrix} \dot{\theta}_{s} \\ \Omega_{r} \\ \Omega_{g} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{K_{s}}{J_{r}} & -\frac{B_{s}}{J_{r}} & \frac{B_{s}}{J_{r}} \\ \frac{K_{s}}{J_{g}} & \frac{B_{s}}{J_{g}} & -\frac{B_{s}}{J_{g}} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta} \\ \Omega_{r} \\ \Omega_{g} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{T_{r}}{J_{r}} & 0 \\ 0 & -\frac{T_{g}}{J_{g}} \end{bmatrix}$$
(1)

Where: aerodynamics torque is $T_r = \frac{1}{2}\rho\pi R^3 C_Q(\lambda,\beta)V^2$, generator torque can be described as $T_g = B_g(\Omega_g - \Omega_z)$ in its linear field.

Aerodynamics torque is a nonlinear function of speed, rotated speed and pitch in the whole wind speed field. It can be linearized as:

$$\begin{split} \hat{T}_{r} &= -B_{r}(\overline{\Omega}, \overline{\beta}, \overline{V}) \cdot \hat{\Omega}_{r} + k_{r,V}(\overline{\Omega}, \overline{\beta}, \overline{V}) \cdot \hat{V} + k_{r,\beta}(\overline{\Omega}, \overline{\beta}, \overline{V}) \cdot \hat{\beta} \\ B_{r}(\overline{\Omega}, \overline{\beta}, \overline{V}) &= -\frac{\partial T_{r}}{\partial \Omega_{r}} \bigg|_{(\overline{\Omega}, \overline{\beta}, \overline{V})} = -\frac{T_{r}(\overline{\Omega}, \overline{\beta}, \overline{V})}{\overline{\Omega}} \frac{\partial C_{\varrho} / \partial \lambda}{C_{\varrho} / \lambda} \bigg|_{(\overline{\lambda}, \overline{V}, \overline{\beta})} \\ k_{r,V}(\overline{\Omega}, \overline{\beta}, \overline{V}) &= -\frac{\partial T_{r}}{\partial V} \bigg|_{(\overline{\Omega}, \overline{\beta}, \overline{V})} = \frac{T_{r}(\overline{\Omega}, \overline{\beta}, \overline{V})}{\overline{V}} \left(2 - \frac{\partial C_{\varrho} / \partial \lambda}{C_{\varrho} / \lambda} \bigg|_{(\overline{\lambda}, \overline{V}, \overline{\beta})} \right) \\ k_{r,\beta}(\overline{\Omega}, \overline{\beta}, \overline{V}) &= \frac{\partial T_{r}}{\partial \beta} \bigg|_{(\overline{\Omega}, \overline{\beta}, \overline{V})} = \frac{T_{r}(\overline{\Omega}, \overline{\beta}, \overline{V})}{\overline{\beta}} \frac{\partial C_{\varrho} / \partial \beta}{C_{\varrho} / \beta} \bigg|_{(\overline{\lambda}, \overline{V}, \overline{\beta})} \end{split}$$

In pitch-varying control, import pitch. Pitch power can be described as $\dot{\beta} = -\frac{1}{\tau}\beta + \frac{1}{\tau}\beta_d$ in its linear manipulation field.

Then, LPV model of the whole wind speed field can be expressed:

$$G: \begin{cases} \dot{x} = A(\theta)x + B_{v}(\theta)\hat{V} + B(\theta)u, \\ \hat{T}_{s} = C_{t}x, \\ y = Cx + Du, \end{cases}$$
Where: $x = \begin{bmatrix} \hat{\theta}_{s} & \hat{\Omega}_{r} & \hat{\Omega}_{g} & \hat{\beta} \end{bmatrix}$
 $u = \begin{bmatrix} \hat{\beta}_{d} & \hat{\Omega}_{z} \end{bmatrix}$
 $y = \begin{bmatrix} \hat{\Omega}_{g} & \hat{T}_{g} \end{bmatrix}.$
(2)



Figure 1. LPV model block diagram.

LPV model matrix is:

$$A(\theta) = \begin{bmatrix} 0 & 1 & -1 & 0 \\ -\frac{K_s}{J_r} & -\frac{B_r(\theta) + B_s}{J_r} & \frac{B_s}{J_r} & \frac{K_{r,\beta}(\theta)}{J_r} \\ \frac{K_s}{J_g} & \frac{B_s}{J_g} & -\frac{B_s + B_g}{J_g} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix},$$
$$B_v(\theta) = \begin{bmatrix} 0 & \frac{k_{r,V}(\theta)}{J_r} & 0 & 0 \end{bmatrix}^T, \quad B(\theta) = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{\tau} \\ 0 & 0 & \frac{B_g}{J_g} & 0 \end{bmatrix}^T,$$
$$C = \begin{bmatrix} K_s & B_s & -B_s & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = 0.$$

Where: $\theta = \begin{bmatrix} \overline{V} & \overline{\Omega} & \overline{\beta} \end{bmatrix}^T$

III. LPV CONTROL DESIGN

Speed: $V_{\min} \leq V \leq V_{\max}$, pitch: $\beta_{\min} \leq \beta \leq \beta_{\max}$, rotated speed: $\Omega_{\min} \leq \Omega \leq \Omega_{\max}$. The following design different LPV controllers for different wind speed fields. As for low wind speed (wind speed is less than rating speed 12m/s), the pitch angle is zero, and there are tow variable in the scheduling parameters which compose a convex tetrahedron whose vertexes are $\phi_{m1} = (V_{\max}, \Omega_{\max})$, $\phi_{m2} = (V_{\min}, \Omega_{\max})$, $\phi_{m3} = (V_{\min}, \Omega_{\min})$, $\phi_{m4} = (V_{\max}, \Omega_{\min})$. The same as high speed (wind speed is higher rating wind speed 12m/s) whose rotated speed is constant, and there are also tow variables in the scheduling parameters. We will take low speed for example, by convex decomposition technique, and drive system LPV expressions can be expressed through four vertexes of convex tetrahedron:

$$\dot{x} = \left[\sum_{i=1}^{4} \rho_i(\theta) A(\phi_{mi})\right] x + \left[\sum_{i=1}^{4} \rho_i(\theta) B(\phi_{mi})\right] u$$
(3)
Where:

$$\rho_1(\theta) = \frac{(\Omega_{\max} - \Omega)(V_{\max} - V)}{(\Omega_{\max} - \Omega_{\min})(V_{\max} - V_{\min})}$$
$$\rho_2(\theta) = \frac{(\Omega_{\max} - \Omega)(V - V_{\min})}{(\Omega_{\max} - \Omega_{\min})(V_{\max} - V_{\min})}$$

$$\rho_{3}(\theta) = \frac{(\Omega - \Omega_{\min})(V - V_{\min})}{(\Omega_{\max} - \Omega_{\min})(V_{\max} - V_{\min})}$$
$$\rho_{4}(\theta) = \frac{(\Omega - \Omega_{\min})(V_{\max} - V)}{(\Omega_{\max} - \Omega_{\min})(V_{\max} - V_{\min})}$$
$$\sum_{i=1}^{4} \rho_{i}(\theta) = 1$$

According to convex polyhedron structure LPV model (3), design separately state feedback gain K1, K2, K3 and K4 to meet requirements. Similar to LPV model convex decomposition structure, take vertexes feedback gain K1-K4 as four vertexes of convex polyhedron LPV controller. At any point θ in convex polyhedron, synthesize designed feedback controller of each vertex to get LPV controller of convex polyhedron structure.

$$K = \rho_1(\theta)K_1 + \rho_2(\theta)K_2 + \rho_3(\theta)K_3 + \rho_4(\theta)K_3 + \rho_4(\theta)K_4$$
(4)



Figure 2. LPV controller design theory.

At each vertex, design controller using LMI method to satisfy requirements. Therefore, give the following definitions firstly:

Definition 1 (LMI field) suppose that D is a subset of complex plane, if there is a symmetry matrix $\alpha = [\alpha_{kl}] \in \mathbb{R}^{m \times m}$ and $\beta = [\beta_{kl}] \in \mathbb{R}^{m \times m}$, meeting $D = \{z \in C : f_D(z) < 0\}$, where

$$f_D(z) \coloneqq \alpha + z\beta + \bar{z}\beta^T = \left[\alpha_{kl} + \beta_{kl}z + \beta_{kl}\bar{z}\right]_{1 \le k, l \le m}$$
(5)

It is the eigenfunction of D, and then D is a LMI field.

A LMI field can also be charactered by an $m{\times}m$ block matrix

$$M_D(A, X) = \left[\alpha_{kl} X + \beta_{kl} A X + \beta_{lk} X A^T \right]_{l \le k, l \le m}$$
(6)

Definition 2 matrix A is D steady, iff exist an symmetry matrix X to make

$$M_D(A,X) > 0, X > 0$$
 (7)

At each vertex of convex polyhedron, LPV model (3) can be described as LTI system.

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t)$$
(8)

$$Z(t) = C_1 x(t) + D_{11} w(t) + D_{12} u(t)$$
(9)

$$y = C_2 x(t) + D_{21} w(t)$$
(10)

Where: $x(t) \in R^{n_p}$ is state vector of generalized object; $u(t) \in R^{n_u}$ is control input; $w(t) \in R^{n_w}$ is outer input (include object disturbance and measure yawp); $z(t) \in R^{n_z}$ is adjusted output; $y(t) = \in R^{n_y}$ is measure output. Given steady field D and H ∞ performance guideline $\gamma > 0$, search for a LTI control rule u=K(s) y to make:

- Closed-loop pole is in field D;
- $||T_{wz}||_{\infty} < \gamma$, T_{wz} stands for closed-loop transfer function.

Suppose that controller parameter matrix are A_c, B_c, C_c, D_c , and then define new variables:

$$B_{c} = NB_{c} + SB_{2}D_{c}$$

$$\overline{C}_{c} = C_{c}M^{T} + D_{c}C_{2}R$$

$$\overline{A}_{c} = NA_{c}M^{T} + NB_{c}C_{2}R + SB_{2}C_{c}M^{T} + S(A + B_{2}D_{c}C_{2})R$$
(11)

Where M, N satisfying

$$I - RS = MN^T$$
12)

(

Theory 1 Suppose that D is arbitrary LMI field of left-open plane, and take formula (8) as its eigenfunction. H ∞ synthesis problem with pole restriction can be solved iff exist matrixes $R = R^T \in R^{n \times n}, S = S \in R^{n \times n}$ and $\overline{A}_c, \overline{B}_c, \overline{C}_c, D_c$ to make the following LMI feasible

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0 \tag{13}$$

$$\left[\alpha_{kl} \begin{bmatrix} R & I\\ I & S \end{bmatrix} + \beta_{kl} \phi + \beta_{lk} \phi^T + \beta_{lk} \phi^T \right]_{k,l} < 0$$
(14)

$$\begin{array}{c} \psi_{11} & \psi_{21}^{T} \\ \psi_{21} & \psi_{22} \end{array} \Big| < 0$$
 (15)

Where:

$$\phi = \begin{bmatrix} AR + B_2 \overline{C}_c & A + B_2 D_c C_2 \\ \overline{A}_c & SA + \overline{B}_c C_2 \end{bmatrix}$$
(16)

$$\psi_{11} = \begin{bmatrix} AR + RA^T + B_2 \overline{C}_c + \overline{C}_c^T B_2^T & B_1 + B_2 D_c D_{21} \\ (B_1 + B_2 D_c D_{21})^T & -\gamma I \end{bmatrix}$$
(17)

$$\psi_{21} = \begin{bmatrix} A_c + (A + B_2 D_c C_2)^T & SB_1 + B_c D_{21} \\ C_1 R + D_{12} \overline{C}_c & D_{11} + D_{12} D_c D_{21} \end{bmatrix}$$
(18)

$$\psi_{22} = \begin{bmatrix} A^T S + SA + \overline{B}_c C_2 + C_c^T \overline{B}_c & (C_1 + D_{12} D_c C_2)^T \\ C_1 + D_{12} D_c C_2 & -\gamma \end{bmatrix}$$
(19)

R, S can be solved by using LMI toolbox of Matlab, and then obtain controller coefficient A_c, B_c, C_c, D_c by using singular values decomposition and solving linear equation (8). We can also obtain immediately the solution of controller by using LMI toolbox instruction of Matlab.

By using theory 1 we can obtain feedback gain of each vertex satisfying requirement. According to convex polyhedron LPV controller (4), we can get feedback control K of arbitrary θ . It is valuable to notice that the obtaining of each vertex feedback gain is all going on offline. Only (4) is actual online computation. So at the actual control, computation capacity of the controller is small, and easy to realize.

IV. SIMULATION RESEARCH

According to 1MW wind turbine parameters, we can obtain drive-train stiffness and damping $K_s = 1.566 \times 10^6 Nm$, $B_s = 1421.1$. Adopt double-fed machine, according to rating revolution of 1650r/min, rating voltage of 690V, rating frequency of 50Hz, and then we can obtain generator damping $B_g = 415.9935$ by calculating. We can also obtain generator and aerodynamic moment of inertia $J_g = 4.95 KGM^2$, $J_r = 830000 KGM^2$ by calculating. The density of air is $\rho = 1.225 kg/m^3$. The blade radius is R = 30.3m. And take $\tau = 10 \mu s$ for delay time.

The following are simulations for drive system in low and high wind speed field separately.

A. The simulation of low wind speed

In the low wind speed field, pitch is invariable ($\beta = 0$), and there are two variables in the scheduling parameter $\theta = [\overline{\nu} \quad \overline{\Omega}]^{T}$. Then state variable is $x = [\theta_s \quad \Omega_r \quad \Omega_g]$, and control input $u = [\Omega_r]$. So LPV model matrixes are:

$$A(\theta) = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{K_s}{J_r} & -\frac{B_r(\theta) + B_s}{J_r} & \frac{B_s}{J_r} \\ \frac{K_s}{J_g} & \frac{B_s}{J_g} & -\frac{B_s + B_g}{J_g} \end{bmatrix}, \\ B_v(\theta) = \begin{bmatrix} 0 & \frac{k_{r,V}(\theta)}{J_r} & 0 \end{bmatrix}^T, \ B(\theta) = \begin{bmatrix} 0 & 0 & \frac{B_g}{J_g} \end{bmatrix}^T, \\ C_t = \begin{bmatrix} K_s & B_s & -B_s \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, D = 0$$

The aerodynamic torque of low speed field can be linearized as:

$$\begin{split} \hat{T}_{r} &= -B_{r}(\overline{\Omega}, \overline{V}) \cdot \hat{\Omega}_{r} + k_{r,V}(\overline{\Omega}, \overline{V}) \cdot \hat{V} \\ B_{r}(\overline{\Omega}, \overline{V}) &= -\frac{\partial T_{r}}{\partial \Omega_{r}} \bigg|_{(\overline{\Omega}, \overline{V})} = -\frac{T_{r}(\overline{\Omega}, \overline{V})}{\overline{\Omega}} \frac{\partial C_{\varrho} / \partial \lambda}{C_{\varrho} / \lambda} \bigg|_{(\overline{\lambda}, \overline{V})} \\ k_{r,V}(\overline{\Omega}, \overline{V}) &= \frac{\partial T_{r}}{\partial V} \bigg|_{(\overline{\Omega}, \overline{V})} = \frac{T_{r}(\overline{\lambda}, \overline{V})}{\overline{V}} \left(2 - \frac{\partial C_{\varrho} / \partial \lambda}{C_{\varrho} / \lambda} \bigg|_{(\overline{\lambda}, \overline{V})} \right) \end{split}$$

In the low speed field, the range of turbine rotated speed is $1.36rad/s \le \overline{\Omega} \le 2.41rad/s$, the range of wind speed is $(V_{\min})3m/s \le V \le 12m/s(V_{\max})$. Calculate separately $B_r(\theta)$ and $k_{rV}(\theta)$ of four vertexes (show as table 1).

Introduce data calculated into LPV model, select appropriate $H\infty$ performance index γ by using LMI toolbox, and then we can obtain feedback gains of four vertexes satisfying requirement K1=0.5027, K2=0.5535, K3=0.5032, K4=0.5470. Set up control frame as figure 3 in the simulation.



Where:
$$\Omega_{ref} = \begin{cases} \lambda_0 \vec{V} / R & V_{\min} \leq \vec{V} \leq V_{\Omega_N}, \\ \Omega_N & V_{\Omega_N} \leq \vec{V} \leq V_{\max}, \end{cases}$$

Input a set of data of wind speed of 6m/s < V < 10m/s (show as figure 4). The sampling is 0.05s. From the simulation figure5, we can see that with the action of the convex polyhedron structure LPV controller, the output machine rotated speed vary with wind speed, and the track performance is all right.

B. The simulation of high wind speed

In the high wind speed, turbine rotated speed achieves rating rotated speed. So it is invariable ($\Omega_N = 2.25 rad/s$). Also there are tow variables in the scheduling parameters $\theta = \begin{bmatrix} \overline{\beta} & \overline{V} \end{bmatrix}^T$. Then state variable is $x = \begin{bmatrix} \theta_s & \Omega_r & \Omega_g & \beta \end{bmatrix}$, and control input $u = \begin{bmatrix} \Omega_z & \beta_d \end{bmatrix}$.

So LPV model matrixes are:

$$\begin{split} A(\theta) &= \begin{bmatrix} 0 & 1 & -1 & 0 \\ -\frac{K_s}{J_r} & -\frac{B_s}{J_r} & \frac{B_s}{J_r} & \frac{k_{r,\beta}(\theta)}{J_r} \\ \frac{K_s}{J_g} & \frac{B_s}{J_g} & -\frac{B_s + B_g}{J_g} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, \\ B_v(\theta) &= \begin{bmatrix} 0 & \frac{k_{r,V}(\theta)}{J_r} & 0 & 0 \end{bmatrix}^T, \quad B(\theta) = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{\tau} \\ 0 & 0 & \frac{B_g}{J_g} & 0 \end{bmatrix}^T, \\ C &= \begin{bmatrix} K_s & B_s & -B_s & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = 0. \\ \hat{T}_r &= k_{r,V}(\overline{\beta}, \overline{V}) \cdot \hat{V} + k_{r,\beta}(\overline{\beta}, \overline{V}) \cdot \hat{\beta}. \end{split}$$

$$\begin{split} k_{r,V}(\overline{\beta},\overline{V}) &= -\frac{\partial T_r}{\partial V} \bigg|_{(\Omega_N,\overline{\beta},\overline{V})} = \frac{T_r(\Omega_N,\overline{\beta},\overline{V})}{\overline{V}} \Biggl(2 - \frac{\partial C_\varrho/\partial\lambda}{C_\varrho/\lambda} \bigg|_{(\overline{\lambda},\overline{V},\overline{\beta})} \Biggr) \\ k_{r,\beta}(\overline{\beta},\overline{V}) &= \frac{\partial T_r}{\partial\beta} \bigg|_{(\Omega_N,\overline{\beta},\overline{V})} = \frac{T_r(\Omega_N,\overline{\beta},\overline{V})}{\overline{\beta}} \frac{\partial C_\varrho/\partial\beta}{C_\varrho/\beta} \bigg|_{(\overline{\lambda},\overline{V},\overline{\beta})} \end{split}$$

In the high wind speed, the range of pitch is $(-0.0348 \text{rad}) - 2^0 \le \beta \le 24^0 (0.4187 \text{rad})$, the range of wind speed is $(V_{\min}) 12m/s \le V \le 25m/s(V_{\max})$. Calculate separately $k_{r,\beta}(\theta)$ and $k_{r,V}(\theta)$ of four vertexes (show as table 2). Also obtain feedback gains K1=0.2644, K2=0.5098, K3=0.2748, K4=0.5509.

Input a set of wind speed 18m/s < V < 24m/s and a set of pitch $14^0 < \beta < 22^0$ (show as figure 6 and 7). We can obtain the following simulation figure 8.

In pitch-varying control, through control the variety of pitch angle, make output machine rotated speed to fluctuate near its rating rotated speed. It is fine to control the running of wind power generator.

V. CONCLUSION

The paper combines gain scheduling and $H\!\infty$ theory, using LPV synthesis technique to put forward a new gain scheduling LPV robust Ho controller design method that place closedloop pole into the field satisfying dynamic response. The method transforms wind turbine model into LPV model with polyhedron structure through LPV convex convex decomposition technique. Design separately feedback gain of each vertex to meet Hoo performance and dynamic character for convex polyhedron. Synthesize each vertex feedback gain to obtain LPV controller with convex polyhedron structure. The simulation results validate that the LPV controller designed has good follow performance, and not large fluctuation. The method makes up the shortage of PID control and makes wind power generator to possess well robust stability and dynamic performance.

TABLE I.	$B_r(\theta)$ and $k_{r,V}(\theta)$ of four vertexes through convex decomposition in low wind speed	
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vertexes $(\overline{\Omega}, \overline{V})$	$\lambda = \frac{wR}{V}$	$C_p(C_p=C_p/\lambda)$	$T_r = \frac{1}{2} \rho \pi R^3 C_Q(\lambda, \beta) V^2$	$\frac{\partial Cp}{\partial \lambda}$	$B_r(\overline{\Omega},\overline{V})$	$k_{r.V}(\overline{\Omega},\overline{V})$	
<i>ф</i> (2.41,12)	6.08525	0.4799	6.0757e+005	0.04295	1.1480e+005	1.2432e+005	
<i>φ</i> (2.41,3)	24.341	0	0	0	0	0	
<i>ф</i> (1.36,3)	13.736	0.25494	8.9368e+003	-0.05056	2.4472e+004	1.7052e+004	
<i>\phi</i> (1.36,12)	3.434	0.21622	4.8509e+005	0.1752	-6.3580e+005	8.7914e+003	

TABLE II. $k_{r,\theta}(\theta)$ AND $k_{r,V}(\theta)$ of four vertexes through convex decomposition in high wind speed

vertexes $(\overline{\beta}, \overline{V})$	$\lambda = \frac{wR}{V}$	$C_p(C_Q = C_p/\lambda)$	$T_r = \frac{1}{2} \rho \pi R^3 C_Q(\lambda, \beta) V^2$	$\frac{\partial Cp}{\partial \beta}$	$\frac{\partial Cp}{\partial \lambda}$	$k_{r,\beta}(\overline{\beta},\overline{V})$	$k_{r.V}(\overline{oldsymbol{eta}},\overline{V})$
<i>\phi</i> (24,25)	2.7297	0.079746	9.7687e+005	0.0307	-0.4752	3.7607e+005	8.5370e+004
<i>ф</i> (24,12)	5.6868	0.67155	9.0978e+005	2.2354	-0.7441	3.0284e+006	1.6312e+005
<i>φ</i> (-2,12)	5.6868	0.44649	6.0488e+005	0.5344	-0.5559	7.2398e+005	1.0644e+005
<i>φ</i> (-2,25)	2.7297	0.082564	1.0114e+006	0.7757	-0.4605	9.5022e+006	8.8185e+004



Figure 4. Input a set of wind speed of $6 \sim 10$ m/s.



Figure 5. Output machine rotated speed.



Figure 6. Input a set of wind speed of $18 \sim 24$ m/s.



Figure 7. Input a set of pitch of $14^{\circ} \sim 22^{\circ}$.



Figure 8. Output machine rotated speed.

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