Reliable Low-Cost Telecardiology: High-Sensitivity Detection of Ventricular Beats using Dictionaries

B. S. Chandra, C. S. Sastry, S. Jana
Indian Institute of Technology, Hyderabad,
Yeddumailaram-502205, India
Email: {bschandra, csastry, jana}@iith.ac.in

***Abstract*—Can reliable telecardiology be achieved at low bandwidth cost? In response, we propose a detector at the user end so that only beats found to be anomalous are transmitted to a diagnostic center, where all received beats are correctly (re)classified. In this framework, high reliability is achieved by detectors with high sensitivity. Having laid the design framework, we then realize desired high-sensitivity detection using a dictionary learning approach. Specifically, using patient records from the MIT-BIH arrhythmia database, we detect ventricular ectopic beats (VEBs), which are known to be precursors to various serious arrhythmic conditions in the heart. In particular, we achieve a reliability of one undetected VEB in one thousand while saving 78.2% bandwidth using dictionaries with 240 atoms. With larger dictionaries with 420 atoms, we achieve an even higher bandwidth savings of 79.2% while allowing no (less than one in 1766) undetected VEB. Finally, we compare our results with performances a large set of reported heartbeat classifiers, and demonstrate the suitability of our approach in the context of telecardiology.**

Keywords*—*ECG, Ventricular Beats, Telecardiology, High-Sensitivity Detection, Dictionary Learning.

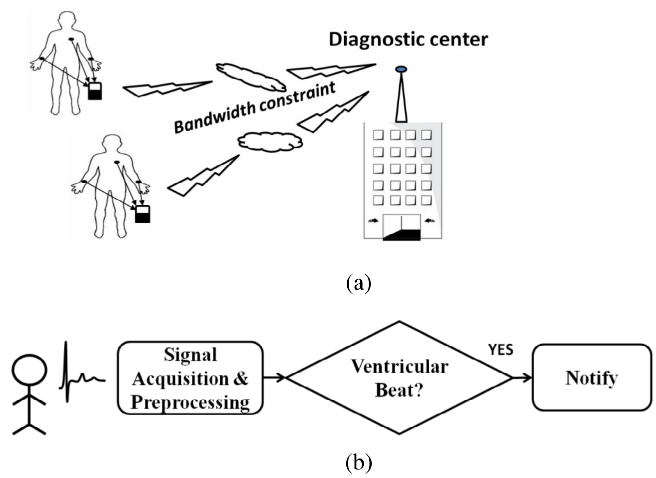

Fig. 1: (a) Telecardiology architecture under consideration; (b) Block diagram of anomalous heartbeat detector.

I. Introduction

Electrocardiogram (ECG) is an indispensable aid in diagnosing, monitoring and managing cardiovascular diseases, which account for 30% of the global death [1]. In certain scenarios, including high-risk-patient care, ECG from a subject is continuously monitored to detect any deviation from normal sinus rhythm. Additional complexities arise when the subject requires remote monitoring [2]. Consider a telecardiology architecture, depicted in Fig. 1a, where ECGs of remote users are transmitted over bandwidth constrained links to a diagnostic center equipped to accurately detect anomalous beats. Traditionally, the entire signal would be transmitted, resulting in perfect reliability, albeit with the attendant high bandwidth requirement. In this context, with a view to realizing a low-cost system, one would ask: Can reliable telecardiology, in terms of accuracy of anomalous beat detection, be achieved with significantly lower bandwidth?

In response, we propose automated heartbeat classification at the user device (see Fig. 1b), and transmission of only those beats that are detected as abnormal. Indeed, assuming an (unrealizable) ideal classifier with both sensitivity and specificity unity, one would achieve perfect reliability with only α fraction of the original bandwidth, where α denotes the prevalence rate of anomalous beats. In practice, we shall achieve a high reliability target using suitable high-sensitivity classifiers. Not surprisingly, bandwidth requirement increases with decreasing specificity subject to sensitivity constraint. Thus, the usual sensitivity-versus-specificity trade off in the underlying classifier maps to the reliability-versus-bandwidth trade off in the telecardiology system, albeit nonlinearly. In this paper, we propose a natural design framework for telecardiology system design based on the latter trade off, and make explicit and illustrate the aforementioned nonlinear mapping, while indicating the target high reliability (equivalently, high sensitivity) region.

Having laid down the design framework, we demonstrate high-sensitivity detection with acceptable specificity using class-specific dictionaries, and hence reliable low-cost telecardiology. In this paper, we shall consider anomaly resulting from only ventricular ectopic beats (VEB) for the sake of simplicity. Although such beats do occur occasionally even in healthy individuals, those could indicate onset of serious conditions, especially, in vulnerable individuals [3]. Specifically, we train individual dictionaries for normal beats and VEBs, respectively, based on well established interval and morphological features. Given a test heartbeat, such features are represented using both the dictionaries, and we assign to it that class, whose dictionary provides sparser representation. Our main idea here is that a VEB should finds a better representation in the VEB dictionary, rather than in the normal beat dictionary, and *vice versa*. Using the proposed classi-

978-1-4799-6644-8/14/$31.00 ©2014 IEEE

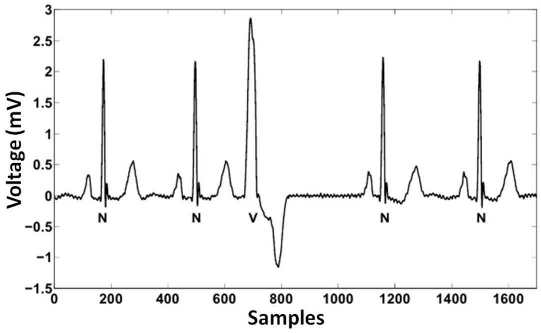

Fig. 2: ECG record containing normal and ventricular beats. Beats annotated "N" indicate normal, and "V" indicate VEBs.

fication rule, desired high-sensitivity detection was achieved for appropriate dictionary sizes. Specifically, we demonstrate the effectiveness of the proposed scheme using the MIT/BIH arrhythmia database. Each heartbeat gives rise to signal vector of approximate size one thousand, from which we extract 66 features for training respective dictionaries. In this framework, using a dictionary size 240, we demonstrate 78.2% savings in bandwidth, while allowing one undetected VEB in one thousand. Further, larger dictionaries of size 420 achieve 79.2% bandwidth savings while allowing no undetected VEB beats (more accurately, less than one in 1766). Finally, we illustrate the relative suitability of the proposed high-sensitivity classifier vis-à-vis previously reported algorithms.

The rest of the paper is organized as follows. Motivation and main contributions are described in Sec. II in appropriate medical and engineering contexts. Sec. III formally states the problem, while the proposed dictionary-based solution is provided in Sec. IV. Experiments and results are reported in Sec. V. Finally, Sec. VI concludes the paper with a discussion.

II. Motivation and Contribution in Context

At this point, we provide detailed motivation by placing our contribution in medical and engineering contexts.

A. *Clinical Motivation*

A heterogeneous set of serious conditions, symptomatized by abnormal electrical activity in the heart, are categorized as cardiac arrhythmia [3]. Arrhythmias originating in the atria include atrial fibrillation, atrial flutter, and supraventricular tachycardia, whereas those originating in the ventricles include ventricular fibrillation, ventricular tachycardia, and ventricular flutter. While a normal heartbeat is triggered by the sinoatrial node, certain abnormal ventricular conditions trigger a premature ventricular contraction (PVC) beat ahead of the usual sinoatrial trigger. Such PVC beats could be either benign, or a precursor of aforementioned serious arrhythmic conditions, especially in subjects with compromised heart. Abnormal beats also occur when the usual sinoatrial trigger does not materialize, and the contraction is instead initiated by ventricular pacemaker cells as a backup. Such a ventricular escape beat also either occurs in a healthy individual (skipped beats), or could be a harbinger of serious arrhythmic conditions

Feature	Description
Heartbeat interval features	• Number of samples between current R_peak location and Previous R_peak location • Number of samples between current R_peak and the next R_peak • QRS_offset-QRS_onset • R_peak-Q_peak • S_peak-R_peak • Magnitude of Q_peak • Magnitude of R_peak • Magnitude of S_peak • P_offset-P_onset • Magnitude of P_peak • P_peak-P_onset • P_offset-P_peak • T_offset-T_onset • Magnitude of T_peak • T_peak-T_onset • T_offset-T_peak
Morphological features	• 30 uniformly sampled data points within 60ms window with R_peak as center • 20 uniformly sampled data points within 80ms window with T_peak as center

TABLE I: Feature vector has length 66, comprising of 16 heartbeat interval features, and 50 morphological features.

in cardiac patients. Additionally, since the morphologies of both PVC and ventricular escape beats are approximately the same, the Association for the Advancement of Medical Instrumentation (AAMI EC57:1998) standard describes both as ventricular ectopic beats (VEBs) [4]. In this backdrop, we propose to detect VEBs, and use those as markers to potentially initiate medical intervention.

B. *Motivation for High Sensitivity Classifiers*

Consider a telecardiology system depicted in Fig. 1a, where each user is equipped with a heartbeat classifier as shown in Fig. 1b, so that only beats detected as anomalous are transmitted. As mentioned earlier, we shall consider VEBs as the only anomaly. Further, denote by Se and Sp, respectively, the sensitivity and the specificity of the classifier. We also assume that the diagnostic center has the resources to validate and correct, if necessary, the class of each beat it receives. Thus one fails to detect a VEB only if that beat is originally classified as normal and never transmitted. Thus the fraction of undetected VEBs, $1 - Se$, measures the reliability of the system. The lower the above fraction, the more reliable is the system, and perfect reliability is achieved when such fraction equals zero. Correspondingly, the fraction B of bandwidth usage is give by

$$B = Se \times \alpha + (1 - Sp)(1 - \alpha), \qquad (1)$$

where α is the prevalence rate of VEBs, and the bandwidth requirement without any classifier is taken as the reference. Of course, no classification is equivalent to $Se = 1$ and $Sp = 0$,

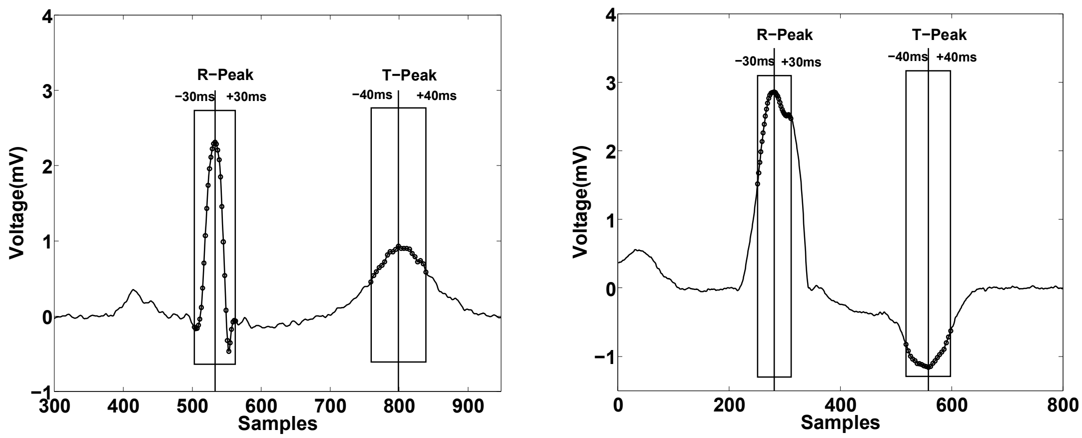

Fig. 3: Morphologocal features: (left) normal beat; (right) VEB.

where, although perfect reliability is achieved ($1 - Se = 0$), one does not save bandwidth ($B = 1$). On the other hand, perfect reliability would be achieved by an ideal classifier ($Se = 1$, $Sp = 1$) with required bandwidth fraction equal to $Se \times \alpha$, amounting to substantial savings. Unfortunately, such an ideal classifier is not realizable. In practice, we seek to save bandwidth while still achieving high reliability (e.g., no more than two undetected VEBs in one thousand, i.e., $Se \geq 99.8\%$).

C. Proposed Solution vis-à-vis Engineering Choices

In classifying each heartbeat into two classes, normal and VEB, various engineering choices arise. For instance, classification algorithms have been reported based on characteristic points of ECG signals (P, Q, R, S, and T) [5], [6], as well as fractal dimension and Hurst exponent [7], [8]. However, we seek to design classifiers using labeled historic data, and hence limit to only machine learning techniques. In this regard, linear discriminant analysis and neural network have been employed [9], [10]. Further, unsupervised methods of dimensionality reduction have been used in conjunction with compressively sampled ECG data, whence anomaly detection has been successfully demonstrated [11], [12]. In this backdrop, keeping practical implementation in view, we additionally desire a method where classification performance can seamlessly be traded off against compute requirement. Accordingly, in this paper we adopt dictionary learning so that the above trade off could be achieved by varying the dictionary size.

The proposed dictionary learning solution enjoys intimate theoretical connection with sparse coding, where a signal is expressed as a linear combination of relatively few basis vectors (equivalently, atoms of a dictionary) [13]. Indeed, we propose sparse coding, using dictionaries learnt using the K-SVD (singular value decomposition) algorithm [14]. Of course, other dictionary learning techniques, such as the method of optimal directions (MOD), also exist alongside K-SVD, and find applications in areas including image restoration, denoising and texture classification [15]. Specific techniques apart, effectiveness of dictionary learning has in general not been demonstrated for classification of ECG beats. The present paper fills this gap by demonstrating dictionary-based high-sensitivity classification and its effectiveness in the context of high-reliability telecardiology.

III. Problem Statement

We begin by mathematically formulating the problem of classifying an ECG beat into the normal and VEB categories. Denote by x any signal vector representing an ECG beat. A candidate classifier specifies two mutually exclusive and exhaustive subsets Γ_1 and Γ_2 of set Γ of all possible x such that if a beat $x \in \Gamma_1$, it is declared normal, else if $x \in \Gamma_2$, it is declared a VEB. We wish to find Γ_2 (and hence Γ_1) such that the sensitivity, i.e., fraction of VEB beats detected as VEB beats, is high (say, above 99.9%). Subject to this, we desire to maximize specificity, i.e., fraction of normal beats declared as normal beats. Recall that the sensitivity (Se) determines the reliability ($= 1 - Se$), whereas both sensitivity and specificity determine the bandwidth requirement according to (1). Generally, two approaches are taken towards designing such classifier: based on (i) stochastic model under each hypothesis (normal and VEB), and (ii) historic data making use of appropriate learning method. As mentioned earlier, we adopt the latter in view of abundant labeled data, and propose a dictionary based solution.

IV. Proposed Dictionary based Classifier

To proceed, we need the mathematical notions of compressive sensing and dictionary learning.

A. Mathematical preliminaries

1) Compressive Sampling: Compressive sampling (CS) aims at recovering high dimensional sparse vector $x \in \mathcal{R}^n$ from a few of its measurements $y == \Phi x \in \mathcal{R}^m$ with $m < n$, where Φ denotes the measurement matrix [13]. Formally, we seek to solve

$$\min_x \|x\|_0 \quad subject \quad to \quad \Phi x = y, \tag{2}$$

where $\|\cdot\|_0$ indicates the l_0 (counting) norm. In general, (2) is intractable. Fortunately, under certain technical conditions, solution to (2) remains unaltered if $\|\cdot\|_0$ is replaced by the

l_1 norm $\|\cdot\|_1$, where new problem requires more tractable l_1 solvers. Among the existing l_1 solvers, orthogonal matching pursuit (OMP), a simple and effective (although greedy) algorithm, will be used in our paper [13]. The aforementioned technical condition relates to the sufficiency of the set of measurements as a function of signal sparsity, which is often empirically estimated through repeated experimentation.

CS theory also applies to signal recovery from noisy (inaccurate) measurements

$$y = \Phi x + e \quad \|e\|_2 < \epsilon,$$

for some $\epsilon > 0$. Specifically, we seek recovered signal

$$\hat{x} = \arg\min_x \parallel y - \Phi x \parallel_2 + \tau \parallel x \parallel_1, \tag{3}$$

for appropriate τ under certain technical conditions. The optimization problem (3) is often solved by iterative soft-thresholding method [13].

2) Dictionary Learning: The method of dictionary learning identifies a tunable selection of basis vectors providing sparse representation. Given a set of signals $\{x_i\}_{i=1}^n$, K-SVD [14] obtains the dictionary D that provides the sparsest representation for each example in this set. It involves a two-step procedure. In the first step, for a given dictionary D, we obtain matrix Ψ with sparse columns by solving the following optimization problem:

$$\Psi = \arg\min_\Theta \sum_l \parallel \Theta_l \parallel_1 \text{ subject to } X = D\Theta, \tag{4}$$

where Θ_l is the l^{th} column of Θ, and X is the matrix whose columns are x_i's. Using the above Ψ, the pair (D, Ψ) is then updated as

$$(\hat{D}, \hat{\Psi}) = \arg\min_{D,\Psi} \|X - D\Psi\|_F^2 \text{ subject } \text{ to } \|\Psi_i\|_0 \leq T_0 \forall i, \tag{5}$$

where Ψ_i denotes the i^{th} column of Ψ, T_0 the sparsity parameter, and $\|\cdot\|_F$ indicates the Frobenius norm. In view of CS theory, thus the K-SVD algorithm alternates between sparse coding (4), solved using an l^1 solver such as OMP, and dictionary update (5), solved using iterative soft-thresholding, till there is a convergence in the dictionary so learnt.

B. Proposed Solution

Armed with the preceding mathematical background, we now propose a dictionary based heartbeat classification method that exploit labeled historic data. Denote such labeled dataset by $\{\{x_{il}\}_{i=1}^{N_l}\}_{l=1}^{K}$. Here l indicates the class label: $l = 1$ indicates normal, and $l = 2$ indicates VEB, with number K of classes equaling two at present. Further, i indicates the signal index and takes values up to N_l, the number of beats present in class l. Now, as detailed above, we learn the dictionary $\hat{D}_l$ for class l from $\{y_{i,l}\}_{i=1}^{N_l}$ for both $l = 1$ (normal) and $l = 2$ (VEB). All such dictionary learning is performed offline.

In real time, when a heartbeat vector x is presented, the proposed classifier assigns the class label, the dictionary corresponding which provides the sparsest representation. Specifically, we set an accuracy level $\epsilon > 0$, and find the sparsest representation $\hat{\alpha}_l$ of x using each dictionary $\hat{D}_l$ $(l = 1, ..., K)$ by solving

$$\hat{\alpha}_l = \arg\min \|\alpha_l\|_1 \text{ subject to } \|x - \hat{D}_l\alpha_l\|_2 < \epsilon.$$

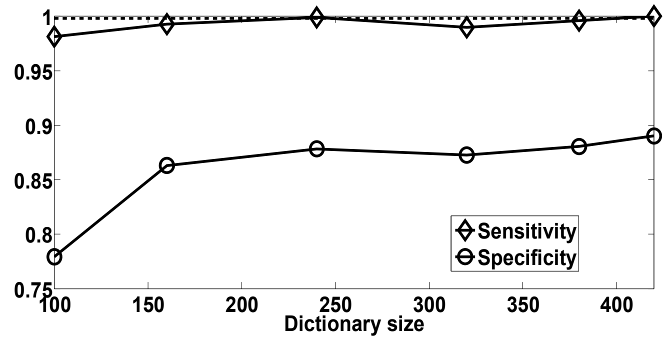

Fig. 4: Sensitivity and Specificity of classifier for different dictionary sizes

	Actual	
Labled	V	N
V	1764	2
N	215	1551

(a) Dictionary size 240

	Actual	
Labled	V	N
V	1766	0
N	194	1572

(b) Dictionary size 420

TABLE II: Confusion matrix for proposed classifiers. Here V indicate VEB and N indicates Normal classes.

Finally, we assign to x the class label

$$\hat{l} = \arg\min \|\hat{\alpha}_l\|_0, \tag{6}$$

i.e., the index of the sparsest representation. If (6) results in a tie between two indices i and j, we pick i such that $\|x - \hat{D}_i\hat{\alpha}_i\|_2 < \|x - \hat{D}_j\hat{\alpha}_j\|_2$. If dictionary size is small, it may not be possible to obtain ϵ-accurate representation using any rival dictionary. In that case, we shall only make use of the tie-breaking mechanism. Finally, notice that one may use smaller dictionaries, potentially incurring classification accuracy loss, in order to reduce compute requirement within the proposed framework. Although our solution applies to any number K of classes, in this paper we confine to $K = 2$.

V. Experiments and Results

We use MIT-BIH Arrhythmia Database available in the PhysioBank archives[1] [16], consisting of 30-minute excerpts of two channel ambulatory ECG recordings digitized at 360 samples per second per channel with 11-bit resolution over a 10 mV range. Each beat in the database is annotated by two or more cardiologists independently. Prior to classification, we remove baseline wander using median filters of window size 200ms and 600ms. Such filters remove P-waves, QRS complexes and T-waves leaving behind the baseline wander [9]. We then subtract the baseline wander from the original signal.

A. Proposed Features

Towards desired classification, we first generate two sets of features: (i) heartbeat interval features, and (ii) morphological features [9]. As a first step we used the following heuristic

[1] Available at http://physionet.org/physiobank/database/mitdb

	Sensitivity (%)	Specificity (%)
Chow *et al.*[2] [19]	97.4	99.2
Hu *et al.*[1] [20]	78.9	96.8
Christov *et al.* [21]	96.9	96.7
G Bortolan *et al.* [22]		
Neural networks (NN)	95.8	98.3
K-th nearest neighbour (kNN)	91.3	98.7
Discriminant analysis (DA)	97.0	94.4
Fuzzy logic (FL)	92.8	98.4
Chazal *et al.*[1] [9]	77.5	98.9
Gómez-Herrero *et al.* [23]	98.5	97.2
Inan *et al.*[2] [24]	85.3	99.1
Jiang *et al.*[1] [25]	94.3	99.4
Ince *et al.*[2] [26]	93.4	99.2
Proposed method		
Dictionary size 240	**99.9**	**87.8**
Dictionary size 420	**100**	**89**

[1] Classifiers proposed for multi class classification.
[2] Specificity calculated by assuming prevalence as 11%.

TABLE III: Comparison of the proposed method with rival methods in terms of classification performance.

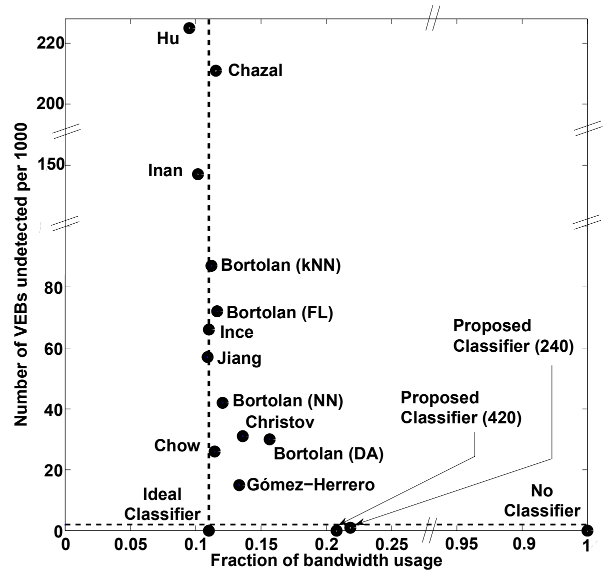

Fig. 5: Comparison of various classifiers in the context of telecardiology.

segmentation. Consider an R_peak located at time t_0, and suppose the durations of the pre-RR and the post-RR intervals are T_{pre} and T_{post}. Then the interval $[t_0 - 0.5T_{pre}, t_0 + 0.75T_{post}]$ provides the estimated beat segment corresponding to an R_peak located at t_0. Here we made use of the locations of the R_peaks given in the Physionet database annotations.

Next we obtain fiducial points of heartbeat such as, onset and offset of QRS complex, P_wave, and T_wave, position and magnitude of P_peak, Q_peak R_peak S_peak and T_peak using a fiducial point identifier algorithm [17]. In order to improve accuracy, we resampled our signals at 1024 Hz. From these points, we compute a set of heartbeat interval features given in Table I. We also compute morphological feature vectors consisting of 30 uniformly spaced samples within a window of 60ms with R_peak as center, 20 uniformly spaced samples within a window of 80ms with T_peak as center. Such morphological features within normal and ventricular ectopic beats are depicted in Figure 3.

B. Learning Class-specific Dictionaries

The experiment is performed using ECG signals pertaining to 11 patient records in MIT-BIH Arrhythmia database. Each patient data is divided into training and test sets. For any given patient, the number of normal beats are significantly higher compared to that of VEB beats. For training, we choose the same number of normal beats as that of VEB beats for each subject. Further, we learn dictionaries for both the ECG beat classes under consideration on the basis of training data using K-SVD algorithm as described earlier. Next each test beat is projected onto both dictionaries and the beat is assigned to the class whose dictionary provides the sparser representation. The dictionaries of both normal and ventricular beats are trained using 1755 beats and the classification performance is evaluated on 1766 beats from the same set of patients.

C. Classification Performance

Fig. 4 depicts the performance of the proposed classifier in terms of sensitivity and specificity for various sizes of dictionaries. Note that our method achieves high sensitivity for a range of dictionary sizes. To highlight this, we draw a dashed line indicating a sensitivity of 99.8%, and observe multiple points above that line in the sensitivity plot. As expected [13], the specificity is acceptable when the dictionary size is about three times the feature vector length or more. For a dictionary size of 66x240, sensitivity and specificity of 99.9% and 87.8%, respectively, are achieved, and the corresponding confusion matrix is presented in Table IIa. For a larger dictionary size of 66x420, we achieve sensitivity and specificity of 100% and 89%, respectively and the corresponding confusion matrix is presented in Table IIb. Note the improvement in the classifier performance is achieved at the cost of higher compute requirement.

Table III compares the classification performance of our method with various reported algorithms in terms of sensitivity and specificity. While our technique achieves higher sensitivity than rival algorithms, the latter in general achieve higher specificity, making a fair comparison difficult. Yet, devoid of context (such as telecardiology), one sometimes wishes to keep both sensitivity and specificity roughly equal, while maximizing that equal quantity. According to such criterion, certain reported classifiers, especially, due to [19], [21], [22], [23], [25] and [26], do appear attractive. Unfortunately, an application such as telecardiology does not lead to the aforementioned criterion.

To highlight the importance of telecardiology context, in Fig. 5 we make comparison between the same classifiers as earlier, but now with respect to the number of VEBs undetected per one thousand beats vis-à-vis the fraction of original bandwidth used. Here we assume an 11% prevalence rate of VEBs[2]. As mentioned earlier, we use as reference the bandwidth requirement when no classifier is deployed. On the other hand, an ideal classifier would use only 11% of the reference bandwidth (shown by vertical dashed line). In this backdrop, notice that a number of reported classifiers do operate close to, or even less than, such ideal bandwidth. However, those do not perform close to our reliability limit of two undetected VEBs in one thousand (shown by horizontal dashed line). The nearest in this respect, the classifier proposed by Gómez-Herrero *et al.* [23], requires only 13% of the reference bandwidth, but fails to detect about 15 VEBs in 1000, which is 7.5 fold higher than the acceptable limit. In comparison, the proposed classifier with dictionary size 66x240 would use 21.8% of bandwidth, while missing only 1 anomalous beat per 1000. A larger dictionary size of 66x420 leads to only 20.8% of bandwidth with no (less than one in 1766) VEB misclassification.

VI. Discussion

In this paper, we consider VEB versus normal heartbeat classification in the context of bandwidth constrained telecardiology. Specifically, we desire a high reliability of two undetected beats in one thousand or less (i.e., sensitivity greater than 99.8%). Subject to such reliability constraint, we sought to minimize the bandwidth usage. In this backdrop, we demonstrated such high-sensitivity classification (99.9% and 100%) using dictionary learning techniques, while achieving substantial bandwidth savings (78.2% and 79.2%, respectively). Additionally, proposed classifiers are scalable in terms of compute requirement (dictionary sizes of 240 and 420, respectively), and hence assume practical significance. In theory, one may achieve high classification accuracy as well as high class-specific compression and hence low transmission bandwidth, by simply enlarging the feature vector to include the entire signal vector. However, the prohibitive compute requirement for both offline training of a large dictionary, and real-time signal representation as a linear combination of large number of dictionary atoms could make such schemes impractical. In summary, the trade off is not merely between reliability and bandwidth, but involves the compute requirement as well.

References

[1] World Health Organization, Cardiovascular diseases (CVDs), Fact sheet $N^0$317, March 2013.

[2] Sekar, Prem, and V Vilvanathan, "Telecardiology: effective means of delivering cardiac care to rural children." *Asian Cardiovascular and Thoracic Annals,* vol. 15, no.4, pp. 320–323, 2007.

[3] M. S. Thaler, *The Only EKG Book Youll Ever Need*, 5th ed. Philadelphia, PA: Lippincott Williams & Wilkins, 2007.

[4] Recommended Practice for Testing and Reporting Performance Results of Ventricular Arrhythmia Detection Algorithms Association for the Advancement of Medical Instrumentation, Arlington, VA, 1987, ANSI ECAR:1987.

[5] C. Li, C. Zheng and C Tai, "Detection of ECG characteristic points using wavelet transforms," *IEEE Trans. in Biomed. Engg.,* vol. 42, no. 1, pp. 21–28, January 1995.

[6] P. Ranjith, P.C. Baby and P. Joseph, "ECG analysis using wavelet transform: application to myocardial ischemia detection," *Innovation and Research in Biomed. Engg,* vol. 24, no. 1, pp. 44–47, February 2003.

[2]As the statistics for VEB prevelance is not directly available, we take as a representative figure the CVD prevalence rate (which is 11% in the USA [18])

[7] D. Zhang, G. Tan, J. Hao,"Fractal Random walk and classification of ECG signal," *Int. J. of Hybrid Inform. Technology,* vol. 1, no. 1, pp. 1–10, January 2008.

[8] B. Paolo, DePetrilloa, S. dArmond and U. E. Ruttimann, "Determining the Hurst exponent of fractal time series and its application to electrocardiographic analysis," *Computers in Biology and Medicine,* vol. 29, no. 6, pp. 393–406, November 1999.

[9] P. D. Chazal, R. B. Reilly, "A Patient-Adapting Heartbeat Classifier Using ECG Morphology and Heartbeat Interval Features," *IEEE Trans. on Biomed. Engg.,* vol. 53, no.12, pp. 2535–2543, December 2006.

[10] W. Jiang, S. G. Kong, G.D. Peterson, "ECG signal classification using block-based neural networks," *IEEE Int. Joint Conf. on Neural Networks,* vol.1, pp. 326–331, August 2005.

[11] M. Fira, L. Goras, N. Cleju and C. Barbasa, " On the projection matrices influence in the classification of compressed sensed ECG signals," *Int. J. of Adv. Computer Science and Applications,* vol. 3, no. 8, 2012.

[12] H. Ding, H. Sun and Kun-mean Hou, "Abnormal ECG signal detection based on Compressed Sampling in Wearable ECG sensor," *Int. Conf. on Wireless Comm. and Sig. Proc. (WCSP),* pp. 1–5, November 2011.

[13] M. Elad, *Sparse and Redundant Representations: From Theory to Applications in Signal and Image Processing,* New York: Springer, 2010.

[14] M. Aharon, M. Elad and A. Bruckstein, "K-SVD: An algorithm for designing overcomplete dictionaries for sparse representation," *IEEE Trans. on Signal Processing,* vol. 54, no. 11, November 2006.

[15] K. K. Delgado, J. F. Murray, B. D. Rao, K. Engan, T. Lee, and T. J. Sejnowski."Dictionary learning algorithms for sparse representation." *Neural computation,* vol. 15, no. 2, pp. 349–396, 2003.

[16] A. L. Goldberger, L. A. N. Amaral, L. Glass, J. M. Hausdorff, P. Ch. Ivanov, R. G. Mark, J. E. Mietus, G. B. Moody, C.K. Peng and H.E.Stanley, "PhysioBank, PhysioToolkit, and PhysioNet: Components of a New Research Resource for Complex Physiologic Signals," *Circulation,* vol. 101, no. 23, pp. e215–e220, 2000.

[17] E.B. Mazomenos E.B, D. Biswas, A. Acharyya, T. Chen, K. Maharatna, J. Rosengarten, J. Morgan, N, Curzen, "A Low-Complexity ECG Feature Extraction Algorithm for Mobile Healthcare Applications," *IEEE J. of Biomed. and Health Informatics,* vol. 17, no. 2, pp. 459–469, March 2013.

[18] National Center for Health Statistics (NCHS). Summary health statistics for U.S. adults: National Health Interview Survey, 2012. Vital Health Stat. 10(260). http://www.cdc.gov/nchs/data/series/sr_10/sr10_260.pdf

[19] H. S. Chow, G. B. Moody, R.G. Mark, "Detection of ventricular ectopic beats using neural networks," *Computers in Cardiology*, pp. 659–662, 1992.

[20] Y Hu, S. Palreddy, W. J. Tompkins, "A patient-adaptable ECG beat classifier using a mixture of experts approach," *IEEE Transactions on Biomedical Engineering,* vol. 44, no. 9, pp. 891–900, Sept. 1997.

[21] I. Christov, I. Jekova and G. Bortolan, " Premature ventricular contraction classification by the Kth nearest-neighbours rule," *Physiol. Meas.,* vol. 26, no. 123–130, January 2005.

[22] G. Bortolan, I. Jekova, I. Christov, "Comparison of four methods for premature ventricular contraction and normal beat clustering," *Computers in Cardiology*, pp. 921–924, Sept. 2005.

[23] G. Gómez-Herrero, I. Jekova, V. Krasteva, I. Christov, A. Gotchev, and K. Egiazarian, "Relative estimation of the Karhunen-Love transform basis functions for detection of ventricular ectopic beats.", *IEEE Computers in Cardiology,* vol. 33, pp. 569–572, 2006.

[24] O. T. Inan, L. Giovangrandi, G. T. A. Kovacs, "Robust Neural-Network-Based Classification of Premature Ventricular Contractions Using Wavelet Transform and Timing Interval Features," *IEEE Transactions on Biomedical Engineering,* vol.53, no.12, pp. 2507–2515, December 2006.

[25] W. Jiang, S. G. Kong, "Block-Based Neural Networks for Personalized ECG Signal Classification," *IEEE Transactions on Neural Networks,* vol.18, no. 6, pp. 1750–1761, November, 2007

[26] T. Ince, S. Kiranyaz, M. Gabbouj, "Automated patient-specific classification of premature ventricular contractions," *Engineering in Medicine and Biology Society,* pp. 5474–5477, August 2008.