Inconsistencies in Health Care Knowledge

Diana Costa
and
Manuel A. Martins
CIDMA - Center for R&D Mathematics and Applications
Department of Mathematics, Univ. Aveiro, Portugal
{dianafcosta,martins}@ua.pt

***Abstract*—In this paper we focus in health care knowledge, specified by hybrid formulas, representing flows of medical assistance in the care delivery process in a hospital. As in standard knowledgebases inconsistencies may arise. In fact, Medical Informatics is one field where the ability to reason with inconsistent information is crucial. Patients can receive different, and moreover contradictory, diagnoses from different physicians, and the same can happen with medical treatments: they can exhibit contradictory symptoms.**

We introduce a paraconsistent version of multimodal hybrid logic to help with this medical issue, specially through the diagnosis.

Keywords*—*Paraconsistent Logic, Hybrid Logic, Health Care.

I. Introduction

Management of knowledge and information is a key issue in all infra-structures underlying modern information-centric societies. Hence, the study and development of flexible logical systems able to handle heterogeneous and complex data has become more and more relevant for the last decades, resorting to interdisciplinary research in linguistics, computer science, mathematics and even philosophy. For example, to achieve a successful decision-support and knowledge management approach to medical representation (for some examples see [12]).

Data collection introduces very often inconsistencies. Being a common fact rather than a queer anomaly, inconsistencies have to be suitably addressed. In particular, one needs to handle data exhibiting, at the same time, assertions of the form q and $\neg q$, configuring local inconsistencies, but without producing global inconsistency. This phenomenon appears frequently in knowledge representation in the area of health care, considering patient data, medical guidelines and the care delivery process. Medical Informatics deals with health care knowledge that represents the daily behavior of a patient in the health system and an effective procedure to manage such flow of information should be studied. Moreover, Medical Informatics is one field where the ability to reason with inconsistent information is crucial. Through the health care processes in a hospital, patients can receive different, and even contradictory, diagnoses from different physicians, and the same can happen with medical treatments: they can exhibit contradictory symptoms. Therefore, it is worth developing easy mechanisms that offer a safe way to 'live' with inconsistency. Such is the domain of *paraconsistent* reasoning — a natural way to deal with inconsistencies allowing both affirmative and negative sentences to be true or false, depending on the context. Such logic should help in the prevention, diagnosis and therapy of patients. More precisely, a paraconsistent logic is a kind of non-classical logical system that violates the Principle of Non-Contradiction ([6]) which states that from contradictory premises any formula can be derived. Łukasiewicz was the first to discuss the possibility of violating this ancient principle, however he did not develop any logical system to formalize his studies. His disciple, Stanisław Jaśkowski, was the one who constructed the first system of propositional paraconsistent logic ([10]). Afterwards, for the last sixty years, many philosophers, logicians and mathematicians have become involved in the area.

Modal logics ([2]) have been successfully used to model state transition systems as well as to model flows of information systems. Modal logics have interesting algorithmic properties, and, moreover, can naturally be translated into first-order logic which allows the use of efficient provers. However, they lack the ability to explicitly refer to specific states, or stages of interpretation, which, in a number of cases, is a desirable feature. Hybrid logics [1], on the other hand, overcome this limitation by introducing a new set of propositional symbols, called *nominals*, each of them holding only in a specific state — the state it *names*.

Several variants of paraconsistent logic have been studied, often to meet different aims or target specific applications ([11]). Research has been driven not only by theoretical interest, but also by genuine problems in different scientific domains, namely Computer Science, Medicine and Robotics. This paper proposes a new mathematical procedure to reason about knowledge bases formalized in hybrid logic that may contain inconsistencies arising from the way data was obtained. A paraconsistent hybrid logic, following the work of Grant and Hunter in [9], is presented in this paper. An important result that makes this generalization possible is the existence of Robinson diagrams in (global) hybrid logic. This implies that hybrid models can be represented by a set of hybrid formulas. In [4] we have already presented the mathematical foundations of the single modality version of this logic. Here we go further by providing the multimodal version needed for applications, namely for the application to the case study of the health care delivery process we developed in Section 4.

Outline of the paper.

Section 2 will introduce hybrid multimodal languages and hybrid diagrams. In Section 3, we introduce Quasi-hybrid logic over formulas in negation normal form to avoid double negation. Then, in Section 4, we present an application of this logic in the medical informatics area and we also give an illustrative case study.

II. Hybrid Logics

Hybrid logics ([1]) are a brand of modal logics that add the possibility to describe transition structures and the ability to refer to specific states. Hybrid logics were introduced by Arthur Prior in the 50's. If modal logics have been successfully used for specifying reactive systems, the hybrid component adds the possibility to refer to individual states and to reason about the system's local behaviour at each of them. A very important feature that will be central in our approach is the fact that multimodal hybrid logic can specify Robinson Diagrams. Namely, $@_i p$ says that the proposition p is true at the state named by i, while $\neg @_i p$ (logically equivalent to $@_i \neg p$) denies this; $@_i j$ says that the states named by i and j are identical, while $\neg @_i j$ (logically equivalent to $\neg @_i \neg j$) states that they are distinct; finally, $@_i \langle \pi \rangle j$ says that the state named by j is a successor of the state named by i with the modality π, and $\neg @_i \langle \pi \rangle j$ (logically equivalent to $@_i [\pi] \neg j$) denies this. Consequently, in hybrid logic we are able to completely describe models. This way of looking at models as sets of formulas will be used to measure the inconsistencies in a model by conforming to the Quasi-classical case.

A. Multimodal Hybrid Logic.

Let $L = \langle \mathrm{Prop}, \mathrm{Nom}, \mathrm{Mod} \rangle$ be a *hybrid similarity type* where Prop is a set of *propositional symbols*, Nom is a set of *nominals* and Mod is a set of *modality labels*. The well-formed formulas over L, $\mathrm{Form}_@(L)$, are defined by the following grammar:

$$WFF := \ i \mid p \mid \bot \mid \top \mid \neg \varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \langle \pi \rangle \varphi \mid [\pi] \varphi \mid @_i \varphi$$

Definition 1. *A* hybrid structure $\mathcal{H}$ *over* L *is a tuple* $(W, (R_\pi)_{\pi \in \mathrm{Mod}}, N, V)$*. Here,* W *is a non-empty set called* domain *whose elements are called* states *or* worlds*, and* $(R_\pi)_{\pi \in \mathrm{Mod}}$ *is a family of binary relations such that* $R_\pi \subseteq W \times W$ *for each* $\pi \in \mathrm{Mod}$*, those relations are called* accessibility relations*.* $N : \mathrm{Nom} \to W$ *is a function called* hybrid nomination *that assigns nominals to elements in* W *such that for any nominal i,* $N(i)$ *is the element of* W *named by i. We call this element the denotation of i under* N*.* V *is a* hybrid valuation*, which means that* V *is a function with domain* Prop *and range* $Pow(W)$ *such that* $V(p)$ *tells us at which states (if any) each propositional symbol is true.*

The satisfaction relation is a generalization of multimodal satisfaction between a hybrid structure $\mathcal{H}$, a state $w \in W$ a and a hybrid formula defined by adding the following conditions for nominals and satisfaction statements:

$\mathcal{H}, w \models i$ iff $w = N(i)$;

$\mathcal{H}, w \models @_i \varphi$ iff $\mathcal{H}, w' \models \varphi$, where $w' = N(i)$.

If $\mathcal{H}, w \models \varphi$ we say that φ is satisfied in $\mathcal{H}$ at w. If φ is satisfied at all states in a structure $\mathcal{H}$, we write $\mathcal{H} \models \varphi$.

For $\Delta \subseteq \mathrm{Form}_@(L)$, we say that $\mathcal{H}$ is a *model of* Δ iff for all $\theta \in \Delta, \mathcal{H} \models \theta$.

B. Hybrid Diagrams.

In order to define the diagram of a hybrid structure we have to define the concept of literal. For a hybrid similarity type $L = \langle \mathrm{Prop}, \mathrm{Nom}, \mathrm{Mod} \rangle$, we define:

Hybrid atoms over L: $HAt(L) = \{@_i p, \ @_i j, @_i \langle \pi \rangle j \mid \ i, j \in \mathrm{Nom}, p \in \mathrm{Prop}, \pi \in \mathrm{Mod}\}$;

Hybrid literals over L: $HLit(L) = \{@_i p, \ @_i \neg p, \ @_i j, @_i \neg j, @_i \langle \pi \rangle j, \ @_i [\pi] \neg j \mid i, j \in \mathrm{Nom}, p \in \mathrm{Prop}, \pi \in \mathrm{Mod}\}$.

An important feature of hybrid logics is the fact that we can specify Robinson diagrams. As in first-order logic, in order to define the diagram of a hybrid structure, we expand the hybrid similarity type L by adding new nominals for the elements of the domain W. We write $L(W)$ for this new hybrid similarity type; in other words, $L(W) = \langle \mathrm{Prop}, \mathrm{Nom} \cup W, \mathrm{Mod} \rangle$.

Given a hybrid structure $\mathcal{H} = (W, (R_\pi)_{\pi \in \mathrm{Mod}}, N, V)$ over L, we denote by $E(W)$ the natural expansion of $\mathcal{H}$ to $L(W)$ by taking N the identity on the new symbols.

The diagram of a hybrid structure $\mathcal{H}$ over L (*i.e.*, the set of literals over $L(W)$ that are valid in $\mathcal{H}(W)$) plays a very important role in the syntactical representation of hybrid models (see [4] for details).

III. Paraconsistency in Multimodal Hybrid Logic

In this section we will study paraconsistency in multimodal hybrid logic. We start by defining a *Quasi-hybrid Multimodal Logic*. In [9], it is assumed that all formulas are in *Prenex Conjunctive Normal Form*, but here, we will assume, without loss of generality, that all formulas are in *Negation Normal Form*. As in [9], we define a bistructure, the decoupled and strong satisfaction and QH models as sets of quasi-hybrid literals whose definition is to appear. We also present, as expected, the paraconsistent diagram of a bistructure. The inconsistency measure, the central goal of this paper is introduced at the end of this section.

A. Quasi-hybrid Multimodal Logic.

In order to generalize the approach in [3] to the multimodal hybrid case, we have to consider formulas in *negation normal form* (*i.e.*, formulas in which the negation symbol occurs immediately before propositional symbols/nominals). There is no loss of generality; actually, a simple adaptation on the proof in [4] by considering multiple modalities can testify it.

Definition 2. *The set of* NNF *formulas over* L*,* $\mathrm{Form}_{\mathrm{NNF}(@)}(L)$*, is recursively defined as follows. For* $p \in \mathrm{Prop}, \ i \in \mathrm{Nom}, \pi \in \mathrm{Mod}$*,*

- $\bot, \top, p, \ i, \ \neg p, \ \neg i$ *are in* NNF*;*

- *If* $\varphi, \ \psi$ *are formulas in* NNF*, then* $\varphi \vee \psi, \ \varphi \wedge \psi$ *are in* NNF*;*

- *If* φ *is in* NNF *and* $i \in \mathrm{Nom}$*, then* $[\pi]\varphi, \ \langle \pi \rangle \varphi, @_i \varphi$ *are in* NNF*;*

It can be shown that every formula $\varphi \in \mathrm{Form}_@(L)$ is logically equivalent to a formula $\varphi^* \in \mathrm{Form}_{\mathrm{NNF}(@)}(L)$.

Actually, the proof of this proposition is only slightly different from the proof in [4], and through it a recursive procedure that transforms a formula φ into its negation normal form can be formulated. Therefore, without loss of generality, we will assume that all formulas are in negation normal form.

Let θ be a formula in NNF and let $\sim$ be a complementation operation such that $\sim \theta = nnf(\neg\theta)$. The $\sim$ operator is not part of the object hybrid similarity type but it makes some definitions clearer.

In order to accommodate the inconsistencies in a model, we have to consider two valuations for propositions: V^+ and V^-. A *hybrid bistructure* (or a *paraconsistent hybrid model*) is a tuple $(W, (R_\pi)_{\pi\in\mathrm{Mod}}, N, V^+, V^-)$ where $(W, (R_\pi)_{\pi\in\mathrm{Mod}}, N, V^+)$ and $(W, (R_\pi)_{\pi\in\mathrm{Mod}}, N, V^-)$ are hybrid structures. The map V^+ is the interpretation for positive propositional symbols, and V^- is the interpretation for the negative ones.

Definition 3. *For a hybrid bistructure* $E = (W, (R_\pi)_{\pi\in\mathrm{Mod}}, N, V^+, V^-)$ *we define a satisfiability relation* $\models_d$ *called* decoupled satisfaction *at* $w \in W$ *for propositional symbols and nominals as follows:*

- $E, w \models_d p$ *iff* $w \in V^+(p)$; *-* $E, w \models_d \neg p$ *iff* $w \in V^-(p)$;

- $E, w \models_d i$ *iff* $w = N(i)$; *-* $E, w \models_d \neg i$ *iff* $w \neq N(i)$.

Since we allow both a positive and a negative propositional symbol to be satisfiable, we have decoupled, at the level of the structure, the link between a formula and its complement. In contrast, if a classical hybrid structure satisfies a propositional symbol at some world, it is forced to do not satisfy its complement at that world. This decoupling gives us the basis for a semantic for paraconsistent reasoning.

The satisfiability relation $\models_s$, called *strong satisfaction*, is defined as in the hybrid case except for the atoms and for disjunctions. For the atoms $p, \neg p, i, \neg i$ it is defined as the decoupled satisfaction and for the disjunction we have:

$E, w \models_s \theta_1 \vee \theta_2$ iff,
$[E, w \models_s \theta_1$ *or* $E, w \models_s \theta_2]$ *and* $[E, w \models_s \sim \theta_1 \Rightarrow E, w \models_s \theta_2]$ *and* $[E, w \models_s \sim \theta_2 \Rightarrow E, w \models_s \theta_1]$;

We define strong validity by $E \models_s \theta$ iff for all $w \in W, E, w \models_s \theta$.

B. Quasi-hybrid Models.

Analogously to the definition in the multimodal hybrid case of a model for a set Δ of formulas, we say that E is a *quasi-hybrid model* of Δ iff for all $\theta \in \Delta, E \models_s \theta$.

To make it easier to follow, we will assume that N maps nominals to themselves; hence W will always contain all the nominals in L. This also means that all nominals are mapped to distinct elements, *i.e.*, N is an inclusion map. Hence, for a given hybrid similarity type $L = \langle \mathrm{Prop}, \mathrm{Nom}, \mathrm{Mod}\rangle$ and a domain W of a bistructure we must have $\mathrm{Nom} \subseteq W$.

As we pointed out before, hybrid logic can specify Robinson diagrams. Following our assumption that N is injective, to define diagrams we do not need the hybrid literals regarding equality between nominals, *i.e.*, $@_i j$ and $@_i \neg j$. Therefore, in this context, we reformulate the notion of atom and literal. Namely, for a hybrid similarity type $L = \langle \mathrm{Prop}, \mathrm{Nom}, \mathrm{Mod}\rangle$, we define:

$Quasi - hybrid\ atoms$ over L: $QHAt(L) = \{@_i p,\ @_i\langle\pi\rangle j \mid i, j \in \mathrm{Nom}, p \in \mathrm{Prop}, \pi \in \mathrm{Mod}\}$;

$Quasi - hybrid\ literals$ over L: $QHLit(L) = \{@_i p,\ @_i\neg p, @_i\langle\pi\rangle j,\ @_i[\pi]\neg j \mid i, j \in \mathrm{Nom}, p \in \mathrm{Prop}, \pi \in \mathrm{Mod}\}$.

To build the paraconsistent diagram, we add new nominals for the elements of W which are not named yet, and we denote this expanded similarity type by $L(W)$, *i.e.*, $L(W) = \langle \mathrm{Prop}, W, \mathrm{Mod}\rangle$ (recall that $\mathrm{Nom} \subseteq W$). As in the standard case, $E(W)$ denotes the natural expansion of the bistructure E to the hybrid similarity type $L(W)$, by taking N the identity for the new nominals. Moreover, we will assume that Prop, Nom, Mod are finite sets for any hybrid similarity type $L = \langle \mathrm{Prop}, \mathrm{Nom}, \mathrm{Mod}\rangle$, as well as the domain W of any bistructure.

The *elementary paraconsistent diagram of* E is the set of quasi-hybrid literals over $L(W)$ that hold in $E(W)$, *i.e.*,

$$Pdiag(E) = \{\alpha \in \mathrm{QHLit}(L(W)) \mid E(W) \models_s \alpha\}.$$

The paraconsistent diagram $Pdiag(E)$ completely defines the bistructure E in the sense that, fixing the domain W and N being the identity, there is an unique model of $Pdiag(E)$ (over $L(W)$) with domain W and a hybrid nomination N, which is $E(W)$. Therefore, in the sequel, we will represent a bistructure $E = (W, (R_\pi)_{\pi\in\mathrm{Mod}}, N, V^+, V^-)$ by its (finite) elementary diagram $Pdiag(E)$. This syntactical representation will play an important role throughout this paper.

Let $L = \langle \mathrm{Prop}, \mathrm{Nom}, \mathrm{Mod}\rangle$ be a hybrid similarity type, $\Delta \subseteq \mathrm{Form}_{\mathrm{NNF}(@)}(\mathrm{L})$ and W be a finite set. We write $\mathrm{QH}(L, \Delta, W)$ for the set of representations (*i.e.*, paraconsistent diagrams) of hybrid bistructures that are models of Δ with domain W. Recall that the domain and the hybrid similarity type are considered to be finite. This implies that the bistructures are finite and consequently the representations of QH models are also finite. This fact is relevant in the next section when discussing the measure of inconsistency in a model.

The syntactic representations of models will be denoted by $\mathcal{M}$, $\mathcal{M}_1$, etc. Let $\mathcal{M}$ be the representation of E with domain W. For $w \in W$, we write $\mathcal{M}, w \models_s \varphi$ if $E, w \models_s \varphi$. Analogously we define $\mathcal{M} \models_s \varphi$ by $E \models_s \varphi$.

As we have already pointed out, we can represent bistructures by the quasi-hybrid literals that are true there. Therefore, we will consider models to be representations of bistructures and consequently, we will want to build models as sets of quasi-hybrid literals.

Example 1. *Let* $L = \langle\{p\}, \{i\}, \{\pi\}\rangle$, $W = \{i\}$ *and* $\Delta = \{@_i[\pi]\neg p, @_i[\pi]p\}$. *Two examples of QH models for* Δ *with domain* $W = \{i\}$ *are:* $\mathcal{M}_1 = \{@_i[\pi]\neg i\}$; $\mathcal{M}_2 = \{@_i\langle\pi\rangle i, @_i p, @_i\neg p\}$ *(see Figure 1).* ∎

Example 2. *Let* $L = \langle\{p, q\}, \{i, j\}, \{\pi, \pi'\}\rangle$, $W = \{i, j\}$, *and* $\Delta = \{@_i\langle\pi\rangle j \vee @_j\langle\pi'\rangle i, @_i\langle\pi\rangle(p \vee$

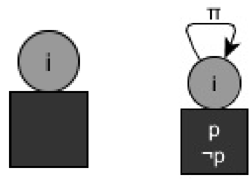

Fig. 1. The QH models $\mathcal{M}_1$ and $\mathcal{M}_2$.

$q), @_i[\pi']q, @_i[\pi]\neg j, @_i\neg q\}$. A QH model for Δ with domain $W = \{i, j\}$ is, for example: $\mathcal{M} = \{@_i[\pi]\neg j, @_j\langle\pi'\rangle i, @_i\langle\pi\rangle i, @_i q, @_i\neg q, @_j[\pi]\neg i, @_j[\pi]\neg j, @_i[\pi']\neg i, @_i[\pi']\neg j, @_j[\pi']\neg j\}$ (see Figure 2). ■

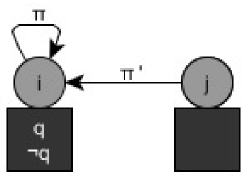

Fig. 2. The QH model $\mathcal{M}$.

C. The Inconsistency Measure.

Now we will introduce a way to measure the inconsistencies in a QH model. This measure is a ratio between 0 and 1 whose numerator is the number of inconsistencies in the model, and whose denominator is the total possible number of inconsistencies there.

To make the notation in the next definition simpler, let us consider the set of *inconsistency literals over L and W* as $IL(L, W) = \{@_i p \mid i \in W, p \in \text{Prop}\}$. For a QH model $\mathcal{M}$, $Conflictbase(\mathcal{M}) = \{@_i p \in IL(L, W) \mid @_i p \in \mathcal{M}\ \&\ @_i\neg p \in \mathcal{M}\}$. The inconsistency measure comes in the form:

Definition 4. *The measure of inconsistency for a model $\mathcal{M}$ in the context of a hybrid similarity type L and domain W is given by the $ModelInc$ function giving a value between 0 and 1 as follows:*

$$ModelInc(\mathcal{M}, L, W) = \frac{|Conflictbase(\mathcal{M})|}{|IL(L, W)|}$$

Example 3. *Some examples over the use of the $ModelInc$ function:*

- *In Example 1, $ModelInc(\mathcal{M}_1, L, W) = 0$ and $ModelInc(\mathcal{M}_2, L, W) = 1$.*
- *In Example 2, $ModelInc(\mathcal{M}, L, W) = \frac{1}{4}$* ■

IV. Applications

Several variants of paraconsistent logic have been proposed to answer different problems in specific applications. Research has been driven not only by theoretical interest, but also by genuine challenges in different scientific domains, namely Computer Science, Medicine and Robotics (see for example [7], [8]). In the medical practice, for example, consulting two or more physicians may lead to (partially) contradictory diagnoses, none of them to be dismissed. In Physics, paraconsistent logic was used to deal with some aspects of quantum mechanics. In Computer Science, subdomains like requirements engineering, artificial intelligence and automated reasoning within information processing knowledge bases, are among the most relevant areas in which paraconsistent logic can address theoretical difficulties raised by inconsistent data. There are many fields where paraconsistency regarding a hybrid logic is welcome. Inconsistent information can appear as we hear from different sources. A very important application of this subject concerns medicine, and is about the health care flow of a patient.

A. Health Care Flow of a Patient.

Figure 3 represents a fragment of the clinical flow of patients in a central hospital. A patient coming into a hospital

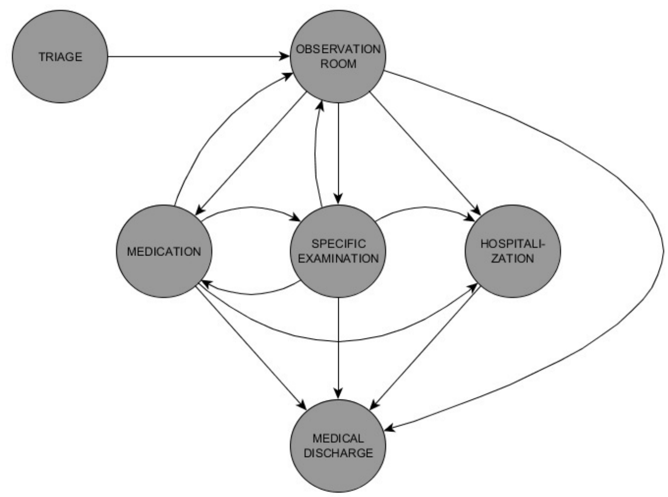

Fig. 3. The care delivery process.

is consulted at the triage station. The next step in the care delivery process is the observation of the patient by physicians at an observation room. From this stage, several things can happen: (1) the patient may need to take some medication, (2) the patient may need to take an examination, (3) the patient may need to be hospitalized, or (4) the patient may be discharged. If the medication has no effect, or the examination is inconclusive, the patient returns to the previous state. The following situations may also occur: (i) the patient takes medication and after takes an examination or vice-versa, (ii) the patient after being medicated or examined needs to be hospitalized, (iii) the patient only needed one of the following — medication/examination/hospitalization — and is discharged after that treatment. This representation corresponds to the set Δ, which must be satisfied in every model, and that comes in the form of:

$$\Delta = \{@_{Triage}(\Box Obs.room \wedge \Diamond Obs.room), @_{Obs.room}\Diamond Med \vee @_{Obs.room}\Diamond Exam \vee @_{Obs.room}\Diamond Hosp \vee @_{Obs.room}\Diamond Med.disch, @_{Med}\Diamond Exam \vee @_{Med}\Diamond Hosp \vee @_{Med}\Diamond Med.disch, @_{Exam}\Diamond Med \vee @_{Exam}\Diamond Hosp \vee @_{Exam}\Diamond Med.disch, @_{Hosp}\Box Med.disch\}$$

The pathway of cares of the patient can be represented by a Kripke frame and the reports made at each stage are represented resorting to a decoupled valuation. Note that the decouple of the valuation is mandatory since very often the diagnosis is not deterministic and we have to allow inconsistencies; actually, a team of physicians may not agree in the diagnosis of a specific disease or even an exam can be inconclusive (for example a CT Screening for lung cancer may hold inconclusive evidence).

The propositional variables are used to represent the data in the patient report that may vary from one observation to another. More specifically, a propositional variable can be seen as a health feature observed in the patient (for example fever, cancer, cough, pallor). Some of them are classical, however some others are paraconsistent. Nominals are used to name referential states (*i.e.* important moments of diagnosis), while modalities are used to label transitions in the flow, for example transitions induced by the administration of a certain medicine or by a specific examination.

B. Practical Example.

Figure 4 represents the pathway of care of the patient A

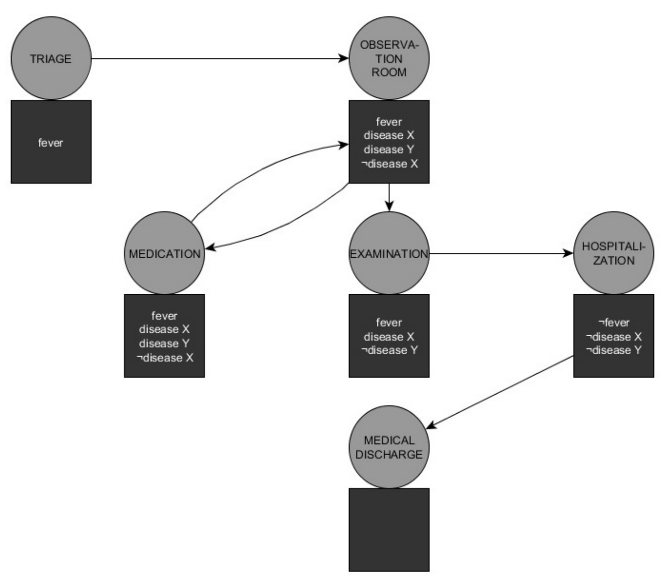

Fig. 4. The QH model $\mathcal{M}$ of a patient A.

that appears at the triage with fever. At the observation room, he is diagnosed with disease X. Another physician disagrees and diagnoses him not with disease X but with disease Y. The patient takes some medication, and the fever does not go away so he returns to the observation room, where the diagnoses are maintained. The patient takes an examination that is conclusive: the patient has disease X but not disease Y. He is hospitalized, loses his fever and is finally discharged. The medical pathway of the patient A can be seen as a paraconsistent model, where $\text{Prop} = \{fever, disease\ X, disease\ Y\}$ and $W = \{Triage, Obs.\ room, Med, Exam, Hosp, Med.Disch\}$ and the valuations are given in Figure 4.

We can measure the inconsistency of this model in order to compare it with others. Assuming that the propositional variable $fever$ can not be paraconsistent, and that the paraconsistency relies only on the medical diagnoses, and also that at the triage there is no paraconsistency as well as at the medical discharge because there are not diagnoses to make, the measure of inconsistency for this model is $\frac{2}{8}$.

Another example is given in Figure 5, which represents

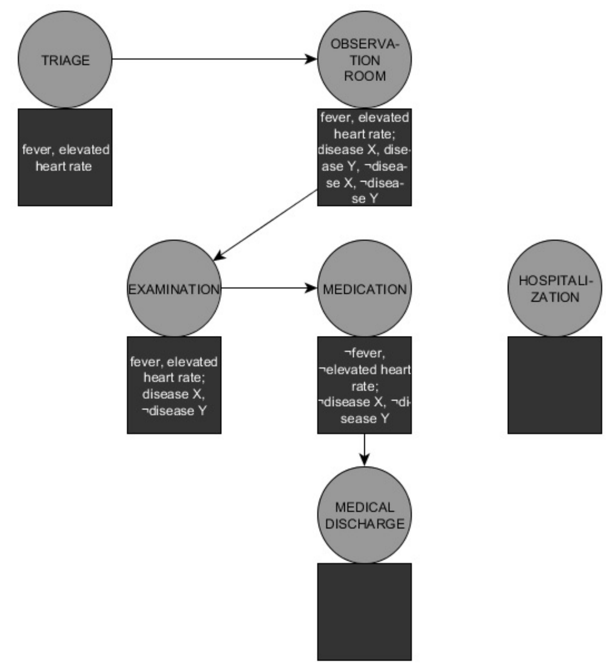

Fig. 5. A QH model $\mathcal{M}'$ of the patient B.

the pathway of care of the patient B that enters in triage with fever and an elevated heart rate. At the observation room, two physicians disagree with the diagnose, one keeping that the patient has disease X but not disease Y and the other stating the converse. The patient takes an examination where it is concluded that he has disease X but not disease Y. The patient takes a certain medicine that cures his fever and elevated heart rate and is discharged. Again, the patient's medical pathway can be seen as a paraconsistent model, where $\text{Prop} = \{fever, elevated\ heart\ rate, disease\ X, disease\ Y\}$ and $W = \{Triage,\ Obs.\ room,\ Med,\ Exam,\ Hosp,\ Med.Disch\}$ and the valuations are given in Figure 5.

Assuming that the propositional variables $fever$ and $elevated\ heart\ rate$ are not paraconsistent, and that the paraconsistency relies only on the medical diagnoses, and also that at the triage there is no paraconsistency, as well as at the medical discharge because there are not diagnoses to make, the measure of inconsistency for this model is $\frac{2}{8}$.

Comparing the two situations, we conclude that $\mathcal{M}'$ is as inconsistent as $\mathcal{M}$.

Further considerations.

Transitions between states can also be labeled with modalities that might correspond to specific medications or examinations. If we had the chance to fully axiomatize the medical guideline, we would have the perfect case. The axiomatization of the medical guideline would include: (1) the complete (not a fragment) clinical flow of patients in a central hospital,*i.e.*, all the possible transitions between different stages (formulas of the form $@_i\langle\pi\rangle j$, i, j nominals, π a modality), (2) the action of specific medication in the problems verified in the patient, for example $p \rightarrow \langle A\rangle\neg p$ means that a patient with problem p would take medicine A and get cured, and furthermore, (3) the diagnose of a disease by means of a specific examination, *i.e.*, for any disease there would be an examination conclusive — the disease is present or not.

V. Conclusion and Further Work

Formal methods, *i.e.* mathematical tools, have been advocated as a means of increasing the reliability of systems, specially those which are safety critical, as is the case of medical knowledge representation. The scientific community has been working on providing efficient representations, technologies, and tools to reason over electronic health records, as well as over health care information systems. Our contribution in this paper goes in this direction, namely we present a paraconsistent version of multimodal hybrid logic that provides models adequate to represent medical knowledge. As the examples we presented showed, it is worth to integrate the above mentioned methods as part of the solution of problems in Medical Informatics (e.g. [12]). The mathematical developments of this paper are the use of quasi-hybrid multimodal logic - the paraconsistent version of multimodal hybrid logic - to reason about inconsistencies in health care knowledge. We define two valuations, so that we can accommodate inconsistencies, and define decoupled and strong satisfaction. Since QH models can be viewed as sets of quasi-hybrid literals, we can use them to represent the care pathways of patients in a hospital and consequently compare them with respect to inconsistency. In practice, each state of observation does not consist on a unique procedure, in almost all of them a sequence of decisions and consequent diagnosis are made, so it is worth to consider themselves as structured states (for example state transition systems that can be modeled by a Kripke model). This suggests the use of hierarchical hybrid logic or, even better, some paraconsistent version of it, to model the health flow.

Acknowledgment

This work was partially supported by the FCT projects FCOMP-01-0124-FEDER-028923 and PEst-OE/MAT/UI4106/2014 and the project GetFun with reference FP7-PEOPLE-2012-IRSES.

References

[1] P. Blackburn. Representation, reasoning, and relational structures: A hybrid logic manifesto. *Log. J. IGPL*, 8(3):339–365, 2000.

[2] P. Blackburn, M. de Rijke, and Y. Venema. *Modal logic.* Cambridge: Cambridge University Press, 2001.

[3] T. Braüner. Axioms for classical, intuitionistic, and paraconsistent hybrid logic. *J. Logic Lang. Inf.*, 15(3):179–194, 2006.

[4] D. Costa and M. A. Martins. Paraconsistency in hybrid logic, 2014. (Technical Report available at http://sweet.ua.pt/martins/phl14.pdf).

[5] D. Costa and M. A. Martins. Quasi-hybrid logic, 2014. Proceedings of Advances in Modal Logic - Groningen, The Netherlands, August 2014.

[6] N. C. da Costa, J.-Y. Béziau, and O. A. Bueno. Aspects of paraconsistent logic. *Bull. IGPL*, 3(4):597–614, 1995.

[7] J. I. da Silva Filho, G. Lambert-Torres, and J. M. Abe. *Uncertainty treatment using paraconsistent logic. Introducing paraconsistent artificial neural networks.* Amsterdam: IOS Press, 2010.

[8] N. A. Ernst, A. Borgida, I. J. Jureta, and J. Mylopoulos. Agile requirements engineering via paraconsistent reasoning. *Information Systems*, 43(0):100 – 116, 2014.

[9] J. Grant and A. Hunter. Measuring inconsistency in knowledgebases. *Journal of Intelligent Information Systems*, 27(2):159–184, 2006.

[10] S. Jaśkowski. A propositional calculus for inconsistent deductive systems. *Log. Log. Philos.*, 7:35–56, 1999.

[11] C. A. Middelburg. A survey of paraconsistent logics. *CoRR*, abs/1103.4324, 2011.

[12] D. Riano, R. Lenz, S. Miksch, M. Peleg, Reichert, and A. M., ten Teije. *(Eds.) Process Support and Knowledge Representation in Health Care*. Lecture Notes in Computer Science, Vol. 8268. 2013.