2008 IEEE International Conference on
Robotics and Automation
Pasadena, CA, USA, May 19-23, 2008

An Efficient Decentralized Learning by Exploiting Biarticular Muscles — A Case Study with a 2D Serpentine Robot —

Wataru Watanabe, Takahide Sato and Akio Ishiguro

Abstract— **This study is intended to deal with the interplay between control and mechanical systems, and to discuss the "brain-body interaction as it should be" particularly from the viewpoint of learning. To this end, we have employed a decentralized control of a two-dimensional serpentine robot consisting of several identical body segments as a practical example. The preliminary simulation results derived indicate that the convergence of decentralized learning of locomotion control can be significantly improved even with an extremely simple learning algorithm, *i.e.*, a gradient method, by introducing biarticular muscles compared to the one only with monoarticular muscles. This strongly suggests the fact that a certain amount of computation should be offloaded from the brain into its body, which allows robots to emerge various interesting functionalities.**

I. INTRODUCTION

In robotics, traditionally, a so-called *hardware first, software last* based design approach has been employed, which seems to be still dominant. Recently, however, it has been widely accepted that the emergence of intelligence is strongly influenced by not only control systems but also their embodiments, that is the physical properties of robots' body [1][2]. This strongly suggests that a certain amount of computation for generating the behavior should be *offloaded* from the control system (*i.e.* brain-nervous systems) into its body system (*i.e.* musculo-skeletal systems). In order to directly indicate this kind of "embodied" computation, Pfeifer *et al.* have recently proposed a concept that they referred to as *morphological computation*, which is expected to be a guiding principle for building intelligent robotic agents [3]. Despite its appealing concept, there still remains much to be understood about how such "computational offloading" can be achieved so as to emerge useful functionalities.

In light of these facts, this study is intended to deal with the interplay between control and mechanical systems, and to analytically and synthetically discuss the "brain-body coupling as it should be". More specifically, the goal of this study is to clearly answer the following questions:

- To what extent computational offloading from the control system to the mechanical system should be done?
- What sort of the body's properties should be focused on so as to effectively exploit the morphological computation?

Since this research field is still in its infancy, it is of great worth to accumulate various case studies at present. Based on this consideration, here, we investigate the questions above particularly from the viewpoint of learning.

W. Watanabe, T. Sato and A. Ishiguro are with the Department of Electrical and Communication Engineering, Tohoku University, 6-6-05, Aoba, Aramaki, Aoba-ku, Sendai, 980-8579, Japan {watanabe/tsato}@cmplx.ecei.tohoku.ac.jp, ishiguro@ecei.tohoku.ac.jp

As a practical example, we demonstrate decentralized control of a two-dimensional serpentine robot consisting of several identical body segments. In particular, this study verifies how the convergence of decentralized learning of locomotion is influenced by effectively introducing a *long-distant physical interaction* between the body segments. The preliminary simulation results derived indicate that the convergence of decentralized learning of locomotion control can be significantly improved by introducing biarticular muscles. More specifically, we have found that the serpentine robot driven by biarticular muscles shows considerably rapid learning even with an extremely simple learning algorithm, *i.e.*, a gradient method, compared to the one driven only by monoarticular muscles. This strongly supports the necessity of exploiting morphological computation in generating the behavior.

In what follows, we firstly introduce "the insects' compound eyes and wing design" which are good instantiations of well-balanced couplings between control and body systems. Then, we illustrate our method by taking a two-dimensional serpentine robot antagonistically driven by pairs of muscles as a practical example. We subsequently discuss how the way of implementing muscles, *i.e.*, monoarticular and biarticular muscles, significantly influences the performance of learning.

II. LESSONS FROM BIOLOGICAL FINDINGS

Before explaining our approach, it is highly worthwhile to look at some biological findings. Beautiful instantiations of well-balanced couplings between control and body systems can be found particularly in insects. In what follows, let us briefly illustrate some of these instantiations.

Compound eyes of some insects such as houseflies show special *facet*, *i.e.*, vision segment, distributions; the facets are densely spaced toward the front whilst widely on the side. Franceschini *et al.* demonstrated with a real physical robot[1] that this non-uniform layout significantly contributes to detect easily and precisely the movement of an object without increasing the complexity of neural circuitry [6].

Another elegant instantiation can be observed in insects' wing design [7][8]. As shown in Fig. 1(a), very roughly speaking, insects' wings are composed of hard and soft materials. It should be noted that the hard material is distributed asymmetrically along the moving direction. Due to

[1]Another interesting robot can be found in [4][5].

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this material configuration, insects' wings show complicated behavior during each stroke cycle, *i.e.*, twist and oscillation. This allows them to create useful aerodynamic force, and thus they can realize agile flying. If they had symmetrical material configuration as shown in Fig. 1(b), the complexity of neural circuitry responsible for flapping control would be significantly increased.

III. THE MODEL

A. A Robotic Case Study with a 2D Serpentine Robot

In order to investigate brain-body interaction as it should be, we investigate a decentralized learning of locomotion control of a two-dimensional serpentine robot as a case study. The reason why we have focused on a serpentine robot is that the body structure as well as the type of locomotion is one of the simplest in robots with fixed morphology. This simplicity allows us to effectively investigate how the brain-body interaction should be.

B. Body Structure

Fig. 2 schematically illustrates the structure of the serpentine robot. As shown in the figure, this robot consists of several identical body segments. Each joint is antagonistically driven by a pair of muscles, *i.e.*, flexor and extensor, within a prespecified range of rotation. This serpentine robot moves forward in a two-dimensional surface by lateral undulation, in which waves of lateral bending of the body are propagated along the body from head to tail. For effectively generating a propulsive force, we assume that the friction between each body segment and the ground is relatively low along the longitudinal direction compared to the latitudinal direction.

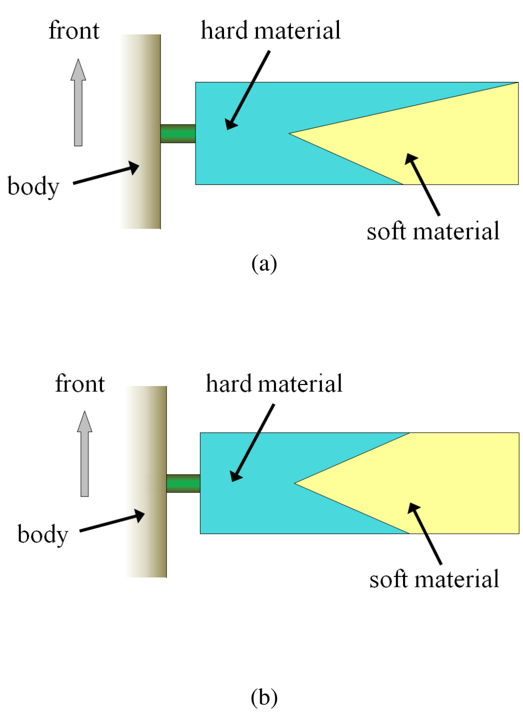


Fig. 1. Material configuration in insects' wings.

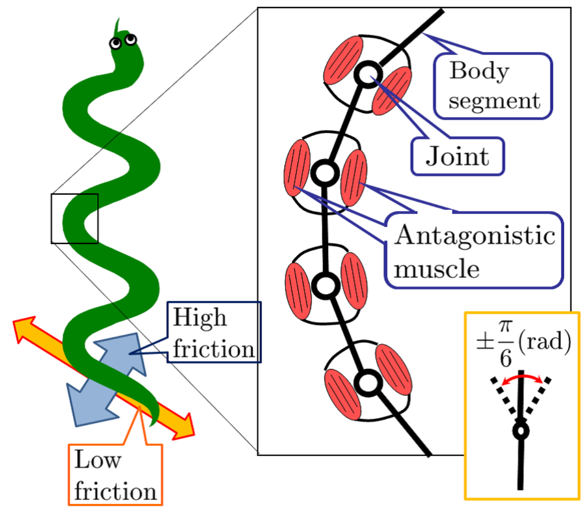


Fig. 2. A schematic of the structure of the two-dimensional serpentine robot employed in this case study.

C. Control Mechanism

In order to generate the bend in each part of robot for lateral undulation, a decentralized control mechanism which independently drives each pair of antagonistic muscles is employed. Here, for simplicity, we have introduced the following assumptions: each of these muscles generates a prespecified "periodic impulsive force"; and each joint is driven by applying impulsive forces alternatively to the corresponding flexor and extensor in an *anti-phase* manner. In addition, the period and the impulsive force applied to all the muscles are assumed to be identical. For convenience, hereafter the phase of the impulsive force applied to the ith body segment's flexor is denoted as θ_i $(i = 1, 2, \cdots, n)$[2]. Thus, the control (learning) parameters in this model end up to be the set of the phases $\theta_1, \theta_2, \cdots, \theta_n$ (n is the number of pairs of antagonistic muscles). Note that each pair of the antagonistic muscles is driven in a fully decentralized manner. To realize the setup explained the above, a simple rhythm generator which controls each pair of antagonistic muscles independently according to the rhythm, $\sin(\omega t + \theta_i)$, is implemented. The detail is shown in Fig. 3, where ω and T are the frequency and the period, τ and Δt are the amplitude and the time interval of the prespecified periodic impulsive force, respectively.

IV. PROPOSED METHOD

Based on the above arrangements, this section analytically discusses how the control and body systems influence the learning convergence. The task of this robot is to realize convergence which leads to a locomotion with minimum energy cost of transport from arbitrary initial relative-phase conditions.

[2]Since the impulsive forces are alternatively applied to the corresponding flexor and extensor in an anti-phase manner, we can consider only the phase of the flexor concerned.

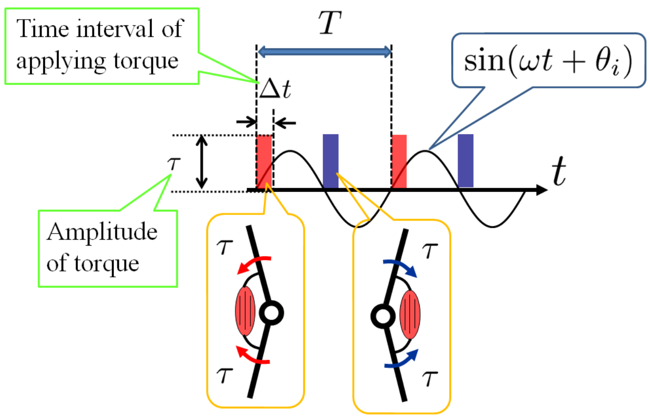


Fig. 3. A schematic of the control mechanism on a pair of antagonistic muscles.

A. Analysis of the Learning Convergence

Let P be the energy cost of transport of this robot, then P can be expressed as a function of the phases as:

$$P = P(\boldsymbol{\theta}), \tag{1}$$
$$\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_n)^T. \tag{2}$$

Here, for the purpose of simplified analysis, a simple learning scheme based on a *gradient method* is employed, which is denoted by

$$\Delta\boldsymbol{\theta}^{(k)} = -\boldsymbol{\eta} \left. \frac{\partial P(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}^{(k)}}, \tag{3}$$

where $\Delta\boldsymbol{\theta}^{(k)}$ is the phase modification at time step k, $\boldsymbol{\eta}$ is an $n \times n$ matrix which specifies how the learning will exploit the information about the gradient in its determination of the phase modification. Based on (3), the set of the phases at time step k is expressed in the following form:

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} + \Delta\boldsymbol{\theta}^{(k)} = \boldsymbol{\theta}^{(k)} - \boldsymbol{\eta} \left. \frac{\partial P(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}^{(k)}}. \tag{4}$$

Let $\boldsymbol{\theta}^{(\infty)}$ be a set of converged phases. By performing the Taylor series expansion for $P(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ around $\boldsymbol{\theta}^{(\infty)}$, the following equation is obtained:

$$P(\boldsymbol{\theta}) \approx P\left(\boldsymbol{\theta}^{(\infty)}\right) + \left(\left. \frac{\partial P(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}^{(\infty)}} \right)^T \left(\boldsymbol{\theta} - \boldsymbol{\theta}^{(\infty)}\right), + \frac{1}{2} \left(\boldsymbol{\theta} - \boldsymbol{\theta}^{(\infty)}\right)^T \left(\left. \frac{\partial^2 P(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}^{(\infty)}} \right) \left(\boldsymbol{\theta} - \boldsymbol{\theta}^{(\infty)}\right). \tag{5}$$

Noting the fact that $\left. \frac{\partial P(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}^{(\infty)}} = 0$ and that $P\left(\boldsymbol{\theta}^{(\infty)}\right)$ is scalar, the partial derivative of $P(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ is:

$$\frac{\partial P(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \boldsymbol{C}\left(\boldsymbol{\theta} - \boldsymbol{\theta}^{(\infty)}\right), \tag{6}$$
$$\boldsymbol{C} \equiv \left. \frac{\partial^2 P(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}^{(\infty)}}, \tag{7}$$

where $\boldsymbol{C}$ is an $n \times n$ *Hessian* matrix. Hence, the substitution of (6) into (4) yields:

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} - \boldsymbol{\eta}\boldsymbol{C}\left(\boldsymbol{\theta}^{(k)} - \boldsymbol{\theta}^{(\infty)}\right). \tag{8}$$

For the sake of the following discussion, a *residual vector* $\boldsymbol{e}^{(k)}$ is introduced, which is equivalent to $\boldsymbol{\theta}^{(k)} - \boldsymbol{\theta}^{(\infty)}$. Then, (8) can be rewritten as:

$$\boldsymbol{e}^{(k+1)} = \boldsymbol{A}\boldsymbol{e}^{(k)}, \tag{9}$$
$$\boldsymbol{A} = \boldsymbol{I} - \boldsymbol{\eta}\boldsymbol{C}, \tag{10}$$

where $\boldsymbol{I}$ is an $n \times n$ unit matrix. $\boldsymbol{A}$ in (9) is a matrix which characterizes the property of learning convergence. This will automatically lead to the following fact: for rapid convergence, it is necessary that the spectral radius of matrix $\boldsymbol{A}$ is less than 1.0. What should be stressed here is the fact that as shown in (10) matrix $\boldsymbol{A}$ is composed of the two matrices: $\boldsymbol{\eta}$ and $\boldsymbol{C}$.

B. Meanings of Matrices $\boldsymbol{\eta}$ and $\boldsymbol{C}$

Based on the above consideration, a well-balanced coupling is investigated by tuning the parameters in matrices $\boldsymbol{\eta}$ and $\boldsymbol{C}$. As has been already explained, matrix $\boldsymbol{\eta}$ specifies the information pathways (or neuronal/axonal interconnectivity) between the pairs of antagonistic muscles, which will be used to calculate the phase modification (a schematic of this is shown in Fig. 4). This implies that matrix $\boldsymbol{\eta}$ does relate to the design of the control system. On the other hand, obviously from the definition (see (7)), $\boldsymbol{C}$ is a matrix whose off-diagonal elements will be salient as the *long-distant physical interaction* between the body segments through the physical connections becomes significant. This strongly suggests that the property of this matrix is remarkably influenced by the design of the embodiment, *i.e.* the body system.

The design of the control system can be easily done by tuning the elements of matrix $\boldsymbol{\eta}$, while much attention has to be paid to the design of the body system. This is simply because one cannot *directly* access the elements of matrix $\boldsymbol{C}$ nor tune them unlike matrix $\boldsymbol{\eta}$. Although the design of the body system can be significant to the convergence of learning same as the one of the control system, there are few study about the effect of embodiment for the reason of difficulty. Consequently, it seems to be highly worthwhile to investigate about the design of body system.

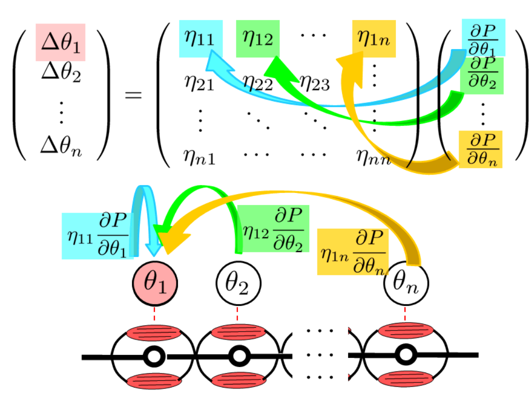


Fig. 4. A schematic of information pathways between the pairs of antagonistic muscles specified by matrix $\boldsymbol{\eta}$.

C. On the Effective Body Design

As explained the above, the long-distant physical interaction between the body segments significantly influences the convergence of learning, since this is closely related to the property of the off-diagonal elements of matrix $\boldsymbol{C}$. In order to investigate this, here, we have focused on monoarticular and biarticular muscles which are widely observed in the animal kingdom, ranging from insects to mammals. As a first step of the investigation, here, we have employed the following two types of embodiments: one is a robot antagonistically driven only by pairs of monoarticular muscles, while the other antagonistically driven only by pairs of biarticular muscles. Note that biarticular muscles induce more long-distant physical interaction between the body segments compared to the case with monoarticular muscles. Fig. 5 illustrates these two embodiments considered in this study.

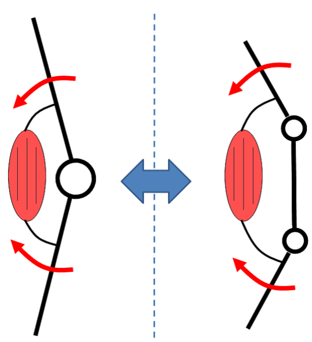
V. PRELIMINARY SIMULATION RESULTS

A. Simulator

In order to efficiently investigate the well-balanced coupling, a simulator has been developed. The following simulations have been conducted with the use of a physics-based, three-dimensional simulation environment [9]. A view of the simulator is shown in Fig. 6. This system simulates both the internal and external forces acting on the agent and objects in its environment, as well as various other physical properties such as contact between the agent and the ground, and torque applied by pairs of the antagonistic muscles on the joints.

B. Experimental Design

In order to verify the difference of the learning performance between the two robots shown in Fig. 5, we have carried out simulations. The simulation conditions employed are as follows: the number of body segments was 10 for the monoarticular muscle-driven robot, whilst 11 for the biarticular muscle-driven robot. This is simply because the number of control parameters, *i.e.*, $\theta_1, \theta_2, \cdots, \theta_n$, is set to be identical. Therefore, the number of control parameters in each robot was in total 9. Since the number of body segments



(a) In the case of monoarticular muscles.

(b) In the case of biarticular muscles.

Fig. 5. Two embodiments considered in this study. Note that biarticular muscles can effectively induce "long-distant physical interaction" between the body segments compared to the one only with monoarticular muscles.

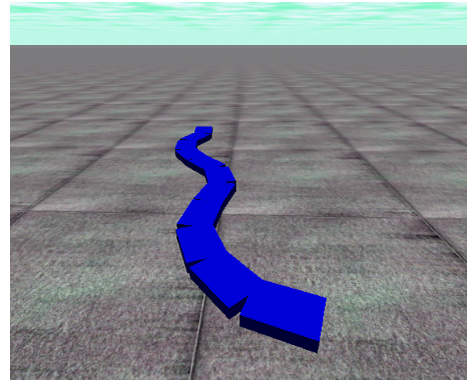


Fig. 6. A view of the developed simulator.

is different between these two robots, we measure the energy efficiency by a dimensionless *specific cost of transport* [10] for the fair comparison, which is given by

$$P = \frac{E_{total}}{m \times d}, \tag{11}$$

where E_{total} is the total energy consumption of the robot, m is the robot mass, and d is the distance traveled of the robot, respectively. The detail how to calculate E_{total} is the following equation:

$$E_{total} = \sum_i \int \{\delta(\tau_i(t)\dot{\Theta}_i(t)) + \gamma\tau_i^2(t)\}dt, \tag{12}$$

where $\delta(x) = 0$ for $x \leq 0$ and $\delta(x) = x$ for $x > 0$, τ_i and $\dot{\Theta}_i$ are the ith joint torque and the angular velocity respectively, γ is a constant value [11]. The other parameters, such as the amplitude and frequency of periodic force (torque) generated by a muscle, were set such that the robot generates a traveling wave when the set of phases achieves an appropriate relative-phase condition. Table I shows the detail of body and control parameters in each robot (each control parameter is also shown in Fig. 3). Moreover, the gradient for the phase modification was calculated by employing the scheme of central difference as follows:

$$\frac{\partial P}{\partial \theta_i} = \frac{P(\cdots, \theta_i + \Delta\theta, \cdots) - P(\cdots, \theta_i - \Delta\theta, \cdots)}{2\Delta\theta}, \tag{13}$$

where $\Delta\theta$ is tiny amount of the phase variation, set to be $0.02\pi(\mathrm{rad})$. In addition, the number of phase modification is 100.

TABLE I

REPRESENTATIVE BODY AND CONTROL PARAMETERS

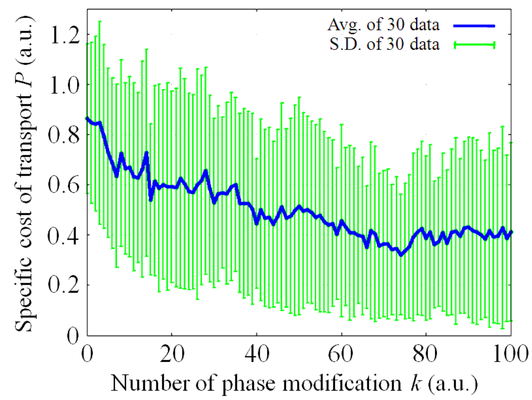
	monoarticular	biarticular
n	9	9
m (kg)	8.60	9.46
ω (rad/s)	2.0	2.0
τ (Nm)	0.5	0.5
Δt (s)	0.25	0.25

C. Verification of Learning Convergence

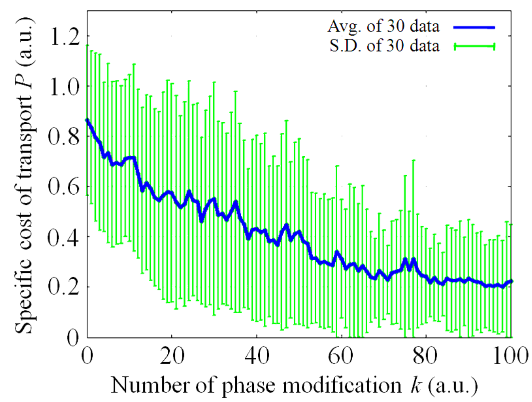
Based on the above setup, in what follows, we discuss the effect for learning convergence induced by control and body systems. As a first step of the investigation, here, we have employed three types of experimental configurations: (i) the robot with diagonal matrix $\boldsymbol{\eta}$ driven only by monoarticular muscles; (ii) the robot with *tridiagonal* matrix $\boldsymbol{\eta}$ driven only by monoarticular muscles; (iii) the robot with diagonal matrix $\boldsymbol{\eta}$ driven only by *biarticular* muscles. The robot with the configuration (i) consists of the simplest control and body systems since each pair of antagonistic muscles employs only its own gradient information for phase modification and each body segment physically interacts only its nearest neighbors. On the other hand, in the case of tridiagonal matrix $\boldsymbol{\eta}$, each pair of antagonistic muscles exploits not only its own gradient information but also its nearest neighbors' one, and each body segment also physically interacts its second-nearest neighbors in the case of biarticular muscles. Therefore, the effect of "brain-body interaction" can be verified by comparing the learning convergence between (i) and (ii), and between (i) and (iii). In addition, for the ease of highlighting the effect of the body design, diagonal and tridiagonal matrix $\boldsymbol{\eta}$ are set to be the following simple structure: all of the diagonal elements have same constant value 0.4905 in both cases; all of the tridiagonal ones have same constant value 0.1962 in the case of tridiagonal matrix; the other are set to be zero.

Fig. 7 is representative data which shows the difference between the three configurations obtained; in each graph, the vertical axis denotes the dimensionless specific cost of transport, while the horizontal one is the number of phase modification (time step k) conducted in the learning process. Note that each graph was obtained by averaging over 30 data simulated under different initial relative-phase conditions, in which each initial phase was independently and randomly set to be within the range $-\pi < \theta_i^{(0)} \leq \pi(\mathrm{rad})$, and the error bars indicate the standard deviation (S.D.) of the learning performance. Very interestingly, the convergence of learning is significantly different: the robot driven by biarticular muscles (configuration (iii)) realizes considerably rapid and consistent learning performance, compared with the ones driven by monoarticular muscles with both diagonal and tridiagonal matrix $\boldsymbol{\eta}$ (configuration (i) and (ii)). Note that the improvement achieved by increasing the "complexity of control system", *i.e.*, changing the structure of matrix $\boldsymbol{\eta}$ from diagonal to tridiagonal, is far less than the one by introducing the biarticular muscles. This strongly suggests the importance of "computational offloading" from the brain to its body in generating the behavior.

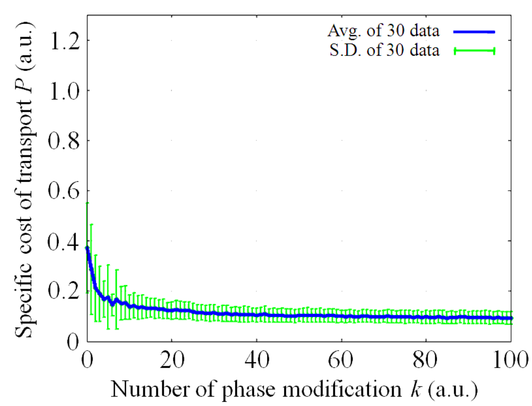
Fig. 8 also illustrates the snapshots of the locomotion generated after three characteristic phase modifications: (I) the initial phase; (II) after 25 phase modifications; and (III) after 50 phase modifications. As the figures explain, the robot driven by biarticular muscles exhibits an efficient locomotion generating a traveling wave from the head to the tail after the 25th learning step. On the other hand, the robot driven



(i) In the case of the robot with diagonal matrix $\boldsymbol{\eta}$ driven only by monoarticular muscles.



(ii) In the case of the robot with *tridiagonal* matrix $\boldsymbol{\eta}$ driven only by monoarticular muscles.

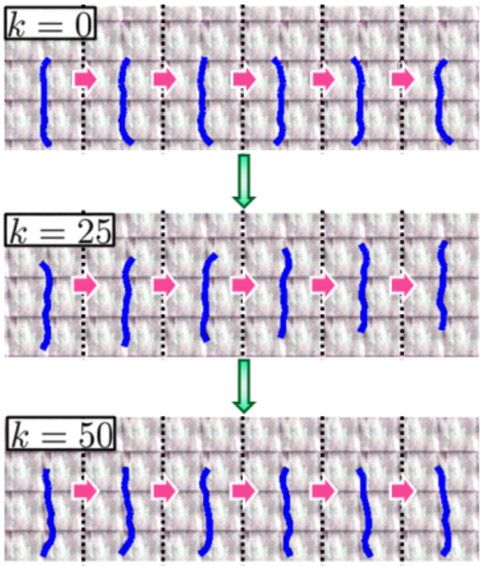


(iii) In the case of the robot with diagonal matrix $\boldsymbol{\eta}$ driven only by *biarticular* muscles.

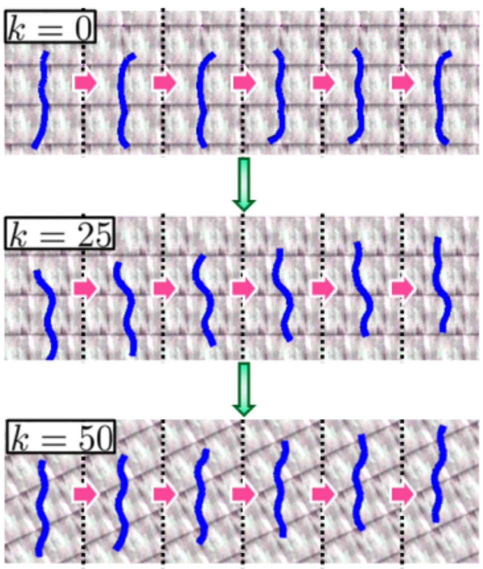
Fig. 7. Comparison of the learning performance between the robots with different configurations obtained by averaging over 30 data simulated under different initial relative-phase conditions.

only by monoarticular muscles shows a wriggling behavior and does not move forward effectively even around the 50th learning step.

These derived results strongly support the conclusion that the body system imposes significant influence on the learning convergence. In other words, it can be one of the grounds for our hypothesis, *i.e.*, a certain amount of computation should be offloaded from control system into its body system. Despite its simplicity, the results clearly show the



(a)In the case of monoarticular muscles.



(b)In the case of biarticular muscles.

Fig. 8. Representative data of locomotion generated after three characteristic phase modifications (see from left to right in each figure).

computational offloading stemming from the exploitation of biarticular muscles can significantly improve the learning surface.

VI. CONCLUSIONS AND FUTURE WORKS

This study has intensively investigated "brain-body interaction as it should be" particularly from the viewpoint of learning. For this purpose, a decentralized learning of locomotion control of a two-dimensional serpentine robot antagonistically driven by pairs of muscles was employed as a case study. The preliminary experiments conducted in this paper support several conclusions and have clarified some interesting phenomena for further investigation, which can be summarized as: first, the convergence of the decentralized learning of locomotion control can be significantly improved by introducing biarticular muscles; second, "brain-body interaction as it should be" in this case study can be analytically discussed in terms of the spectral radius of a matrix which specifies the property of learning convergence; third and finally, a certain amount of computation required for generating the behavior should be offloaded to its body, *i.e.*, the mechanical system, which is sometimes referred to as morphological computation.

Another important point to be stressed is closely related to the concept of *emergence*. One of the crucial aspects of intelligence is the *adaptability* under hostile and dynamically changing environments. How can such a remarkable ability be achieved under limited/finite computational resources? The only solution would be to exploit *emergence phenomena* created through the interaction dynamics between control system, body system, and the environment. This research is a first step to shed light on this point in terms of balancing control system with their body system. Finally, in future studies we plan to numerically analyze the matrix $\boldsymbol{C}$ which specifies the long-distant physical interaction between the body segments, and to discuss the design policy of the physical interaction (body system) and of the neuronal interconnectivity (control system).

VII. ACKNOWLEDGMENTS

This work has been partially supported by a Grant-in-Aid for Scientific Research on Priority Areas "Emergence of Adaptive Motor Function through Interaction between Body, Brain and Environment" and "Tohoku Neuroscience Global COE Basic & Translational Research Center for Global Brain Science" from the Japanese Ministry of Education, Culture, Sports, Science and Technology.

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