2010 IEEE International Conference on Robotics and Automation
Anchorage Convention District
May 3-8, 2010, Anchorage, Alaska, USA

Performances of the Central-Axis Approach in Grasp Analysis

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Abstract— **In this paper, we consider a 3D grasping problem. In previous work, we presented the central-axis approach and proven its capability to analyze multifingered grasps. In the present paper, we extend our study by developing and analyzing modified force-closure algorithms, and giving rigorous theoretical demonstrations. Through numerical simulations, we show that the proposed approach is computationally efficient when comparing with the *qualitative* ray-shooting algorithm. The proposed *quantitative* force-closure test offers a good quality metric without computing the convex hull of the primitive contact wrenches in $\mathbb{R}^6$, which efficiently reduces the amount of computation. Several simulation examples showing the efficiency of the proposed approach are included in the paper.**

I. INTRODUCTION

BECAUSE force-closure grasps are reliable, it is desirable to design or synthesize such grasps. Hence, several researches in grasping and manipulation are interested in developing force-closure tests and associated grasp planners. This paper focuses on grasp analysis for which we present a fast quantitative test algorithm. The proposed test is computationally competitive compared with the known qualitative ray-shooting algorithm [13]. Moreover, the proposed approach can offer a quality measure without computing the convex hull of the primitive contact wrenches in $\mathbb{R}^6$.

The most useful characterization of grasp restraint is force-closure [1]-[4]. A given grasp achieves force-closure if the fingers can apply appropriate contact forces on the object to produce wrenches in any direction, and hence, they compensate any external wrench (up to a certain magnitude). Salisbury and Roth [5] characterized the force-closure by the geometric condition : *the primitive contact wrenches positively span the entire wrench space*. This condition is equivalent to saying that the origin of wrench space lies strictly inside the convex hull of the primitive contact wrenches [6]. Ponce et al. [7] illustrated that 4-finger force-closure grasps fall into three classes : concurrent, pencil and regulus grasps, and developed techniques for computing them. Jia-Wei Li et al. [8] proposed a geometric algorithm for computing 3-finger FC grasps. Chen et al. [9] developed a qualitative test algorithm for any number of contacts. Trinkle [10], [11] proposed a linear programming formulation of the force-closure problem by checking the existance of any positive null vector of the primitive contact wrench matrix. Han et al. [12] pointed out that friction constraints have the form of Linear Matrix Inequalities (LMIs) and formulated the grasping force optimization problem as a convex optimization problem involving LMIs. Liu [13] formalized qualitative test of force-closure grasps as linear programming problem based on the duality between convex hulls and convex polytopes. This qualitative test is considered as the most efficient algorithm [14]. For grasp quality, Kirkpatrick et al. [15] defined the quantitative measure as the radius of the largest sphere inscribed inside the convex hull of contact wrenches. This measure has been proposed in several forms, but it is best described by Ferrari and Canny [16]. In [19], we proposed to use the central-axis theory to study mutlifingered grasps and we developed a linear programming formulation of the force-closure problem.

In this paper, we will extend our study of the central-axis approach by developing modified force-closure algorithms and giving rigorous theoretical demonstrations. Through numerical simulations on multifingered grasping, we confirm the real-time efficiency of the proposed approach when compared with the qualitative ray-shooting algorithm [13]. The advantage of the proposed approach is its capability to give a good quality measure of the force-closure grasp without computing the sphere in six-dimensional wrench space, which efficiently reduces the computational cost.

The paper is organized as follow : In Section II, we present an overview of the relevant pieces of grasping and central axis theories. In Section III, we put forward the necessary and sufficient condition for n-finger equilibrium and force-closure grasps and we present the proposed algorithms. In Section IV, we show an implementation of the proposed approach with numerical simulations on multifingered grasping. Section V concludes the paper.

II. BACKGROUND

This section briefly describes the background of grasping, introducing the concept of the central-axis approach.

A. Grasp Wrench Space

This work assumes hard finger contact with Coulomb friction model. Hence, the finger contact force $\mathbf{f}_i$ must be within the friction cone at each contact point $\mathbf{c}_i$ (Fig. 1-a). The static friction coefficient $\mu = \tan(\alpha)$ depends on

The research leading to these results has received funding from Algerian government and from the European Community's Seventh Framework Program (FP7/2007-2013) under grant agreement no. 216239.

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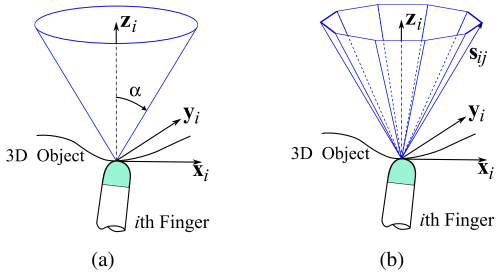


Fig. 1. Interpretation of the friction law : (a) Spatial-friction cone, (b) Quadratic cone approximated by an m-sided polyhedral cone.

materials that are in contact. In this situation, the tangential force f_i^t must satisfy the following constraints

$$f_i^t = \sqrt{f_{ix}^2 + f_{iy}^2} \leqslant \mu f_{iz} \quad , \quad (i = 1 \cdots n) \tag{1}$$

where f_{ix}, f_{iy} are the orthogonal tangent elements, and f_{iz} is the normal element of the ith fingertip contact force $\mathbf{f}_i$ at the contact $\mathbf{c}_i$.

The non-linear friction constraints given by (1) can be relaxed using polyhedral approximation. Each cone is linearized by an m-sided polyhedral convex cone (Fig. 1-b). Under this approximation, grasp force $\mathbf{f}_i$, expressed in the object coordinate frame, is given by

$$\mathbf{f}_i = \sum_{j=1}^{m} a_{ij}\mathbf{v}_{ij} \ ; \ \mathbf{v}_{ij} = R_i \ \mathbf{s}_{ij} \ ; \ a_{ij} \in \mathbb{R}^+ \tag{2}$$

The matrix R_i specifies the location of the ith coordinate frame w.r.t. the object coordinate frame. $\mathbf{s}_{ij}$ denotes the jth edge vector of the polyhedral convex cone expressed in the ith coordinate frame and satisfies $\mathbf{s}_{ij}^T\mathbf{z}_i = 1$. The sum $\sum_{j=1}^m a_{ij}$ specifies the amplitude of the normal component of the contact force $\mathbf{f}_i$.

A hard finger at $\mathbf{c}_i$ applies the moment $\mathbf{t}_{i/\mathbf{o}} = \mathbf{c}_i \times \mathbf{f}_i$ w.r.t. the origin $\mathbf{o}$. The force and the corresponding moment are stacked into a six-dimensional vector called *wrench*. The wrench induced on the object by the grasp force $\mathbf{f}_i$, denoted $\mathbf{w}_i$, applied at the origin $\mathbf{o}$, is given by

$$\mathbf{w}_i = \left[\mathbf{f}_i \ , \ \mathbf{t}_{i/\mathbf{o}}\right]^T = \sum_{j=1}^{m} a_{ij}\mathbf{w}_{ij} \tag{3}$$

Where $\mathbf{w}_{ij}$ denotes the primitive contact wrenches of the ith finger. They are given, w.r.t. the object coordinate frame, by

$$\mathbf{w}_{ij} = \left[\mathbf{v}_{ij} \ , \ \mathbf{c}_i \times \mathbf{v}_{ij}\right]^T \tag{4}$$

The net wrench applied by the hand on the grasped object is the sum of all primitive contact wrenches. It is given by

$$\mathbf{w}_g = \sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}\mathbf{w}_{ij} = \left[\mathbf{f}_g \ , \ \mathbf{t}_{g/\mathbf{o}}\right]^T \ ; \ a_{ij} \geqslant 0 \tag{5}$$

B. Equilibrium

In equilibrium study, we consider only the grasp forces applied by the n fingertips of the robotic hand. Therefore, a set of nm primitive contact wrenches are said to achieve equilibrium when there convex hull in $\mathbb{R}^6$ contains the origin [20]. In other words, a grasp achieves equilibrium when the equation $\mathbf{w}_g = \mathbf{0}$ admits a nontrivial and non-negative solution. The equilibrium condition w.r.t. $\mathbf{o}$ is given by

$$\begin{cases} \sum_{i=1}^n \sum_{j=1}^m a_{ij}\mathbf{v}_{ij} = \mathbf{0} \\ \sum_{i=1}^n \sum_{j=1}^m \mathbf{c}_i \times a_{ij}\mathbf{v}_{ij} = \mathbf{0} \end{cases} ; \ a_{ij} \geqslant 0 \tag{6}$$

C. Force-Closure

A grasp is force-closure when the contact forces $\mathbf{f}_i$ are able to produce any wrench in $\mathbb{R}^6$. In other term, the fingers can apply appropriate contact forces on the object to produce wrenches in any direction and hence, they compensate any external wrench. We notice that the force-closure property considers only the direction of wrenches while the magnitude is neglected. Intuitively, a system of wrenches achieves force-closure when any external load can be balanced by a non-negative combination of the primitive contact wrenches $\mathbf{w}_{ij}$. The term positively span $\mathbb{R}^6$ is reserved to represent force-closure property.

Definition 1 : *A set of vectors positively span $\mathbb{R}^6$ if any vector in $\mathbb{R}^6$ can be written as a positive combination of the given vectors.*

This is equivalent to saying that the origin of wrench space lies strictly inside the convex hull of the primitive contact wrenches. The most developed force-closure tests are based on this definition. The test proposed by Liu [13] uses the duality between convex hulls and convex polytopes to propose an equivalent ray-shooting problem. This test cannot provide a quality measure of a force-closure grasp because the primitive contact wrenches are normalized in order to save the time of computing the convex hull of the primitive contact wrenches. The most known quality measure is the radius of the largest sphere inscribed inside the convex hull of contact wrenches [15] [16], though computing the sphere in $\mathbb{R}^6$ space is computationally expensive [13].

D. Wrench Central-Axis

Poinsot's central-axis theorem states that every system of wrenches is equivalent to a single force plus a single moment acting on the same line. The central axis Δ_g of the grasp wrench $\mathbf{w}_g$ is defined as follows [17] :

$$\Delta_g = \left\{ \frac{\mathbf{f}_g \times \mathbf{t}_{g/\mathbf{o}}}{\| \mathbf{f}_g \|^2} + \lambda\mathbf{f}_g \ : \lambda \in \mathbb{R} \right\} \tag{7}$$

The moment about Δ_g is equal to the component of $\mathbf{t}_{g/\mathbf{o}}$ exerted by the system in the $\mathbf{f}_g$ direction, it is given by

$$\mathbf{t}_{g/\mathbf{I}} = \frac{\mathbf{f}_g^T \mathbf{t}_{g/\mathbf{o}}}{\| \mathbf{f}_g \|^2} \ \mathbf{f}_g \tag{8}$$

The main advantages in using the concept of the central axis is its ability to reduce any system of forces to an arbitrary point. We have shown in [18] that the central axis can construct directly the locus of resultant forces without any geometric transformation. In [19], we have given many examples that illustrate the relationship between force-closure property and the central axis of the grasp wrench.

Rewriting the equilibrium condition of (6) w.r.t. the kth contact point $\mathbf{c}_k$ yields

$$\begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}\mathbf{v}_{ij} = -\sum_{j=1}^{m} a_{kj}\mathbf{v}_{kj} \\ \sum_{i=1}^{n}\sum_{j=1}^{m}((\mathbf{c}_i - \mathbf{c}_k)\times a_{ij}\mathbf{v}_{ij}) = \mathbf{0} \end{cases} ;i \neq k\ ,\ a_{ij} \geqslant 0 \quad (9)$$

Poinsot's central-axis theorem allows to define the following two classes of central axes :

- Δ_k : the central axes of the wrench $\mathbf{w}_k$, which is associeted with the contact forces $\mathbf{f}_k = \sum_{j=1}^{m} a_{kj}\mathbf{v}_{kj}$. Δ_k define all pure forces in the kth friction cone.
- Δ_r : the central axes of the wrench $\mathbf{w}_r$ which is associeted with the other contact forces $\mathbf{f}_r = \sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}\mathbf{v}_{ij}$ where $i \neq k$ and the moment w.r.t. $\mathbf{c}_k$ is zero. Δ_r define all pure forces that passes through $\mathbf{c}_k$.

According to the equilibrium condition (9), we can immediately conclude the following proposition for n-finger equilibrium grasps

Proposition 1 : *A grasp can achieve equilibrium iff the $n-1$ first fingers can generate, at least, one central axis of class Δ_r that is opposite to one central axis of class Δ_k generated by the last kth finger.*

In planar grasps, a grasp that achieves non-marginal equilibrium also achieves force-closure [20]. This is equivalent to saying that, at least, one central axis of class Δ_r is pointing *strictly* within the negative kth friction cone. In other words, the grasp achieves non-marginal equilibrium when the central axes $\Delta_g^* = \Delta_r \cup \Delta_k$ positively span $\mathbb{R}^3$ ($\mathbb{R}^2$ for 2D grasps).

For the force $\mathbb{R}^3$ subspace, [21] shown that at least four vectors are needed to positively span $\mathbb{R}^3$. The following proposition describes a necessary and sufficient condition for four vectors to positively span $\mathbb{R}^3$ [21].

Proposition 2 : *Four vectors positively span $\mathbb{R}^3$ when the negative of any of these vectors lies inside the interior of the pyramid formed by the other three vectors.*

Proposition 2 indicates that a necessary and sufficient condition for force vectors to positively span $\mathbb{R}^3$ at a contact point $\mathbf{c}_k$ is the ability of the force cones to generate one force which is pointing strictly within the negative kth friction cone (cone pointing outside the object). This is equivalent to saying that a positive combination of the contact forces generates pure force along the unit vector $-\mathbf{z}_k$, the inverse normal at the contact point $\mathbf{c}_k$. Therefore, the contact forces positively span $\mathbb{R}^3$ at the first contact point $\mathbf{c}_1$ if they satisfy the following system of equations :

$$\begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}\mathbf{v}_{ij} = -\mathbf{z}_1 \\ \sum_{i=2}^{n}\sum_{j=1}^{m}((\mathbf{c}_i - \mathbf{c}_1)\times a_{ij}\mathbf{v}_{ij}) = \mathbf{0} \end{cases} ;\ a_{ij} \geqslant 0 \quad (10)$$

III. Central-Axis Approach

Clearly, a force-closure grasp achieves equilibrium, and the non-marginal equilibrium is a necessary and sufficient force-closure condition for 2D grasps. However, this condition is not sufficient to satisfy force-closure for 3D grasps. Hence, a generalization of (10) summarizes the 3D force-closure test in the following proposition :

Proposition 3 : *Given an arbitrary point (eg. $\mathbf{c}_i$), an n-finger grasp is force-closure iff :*
(i)- all grasp-wrench central axes of class $\Delta_g^* = \Delta_r \cup \Delta_k$ can positively span $\mathbb{R}^3$ at $\mathbf{c}_i$ (forces along Δ_g^* are noted $\mathbf{f}_g^*$), and
(ii)- the torque applied by the n fingers $\mathbf{t}_{g/\mathbf{c}_i}$, positively span $\mathbb{R}^3$ at $\mathbf{c}_i$.

Proof : if conditions 3-*(i)* and 3-*(ii)* are satisfied w.r.t. $\mathbf{c}_i$, the corresponding grasp wrench $\mathbf{w}_{g/\mathbf{c}_i}^* = \left[\mathbf{f}_g^*\ ,\ \mathbf{t}_{g/\mathbf{c}_i}\right]^T$ can balance any external wrench at point $\mathbf{c}_i$. Hence, the given grasp is force-closure at point $\mathbf{c}_i$. Now, w.r.t any other point $\mathbf{p} \in \mathbb{R}^3$, the grasp wrench $\mathbf{w}_g^*$ is writen as $\mathbf{w}_{g/\mathbf{p}}^* = \left[\mathbf{f}_g^*\ ,\ \mathbf{t}_{g/\mathbf{p}} = \mathbf{t}_{g/\mathbf{c}_i} + (\mathbf{c}_i - \mathbf{p})\times\mathbf{f}_g^*\right]^T$. The force $\mathbf{f}_g^*$ spans positively $\mathbb{R}^3$ at $\mathbf{c}_i$ and $\mathbf{p}$, that implies that the torque $(\mathbf{c}_i - \mathbf{p})\times\mathbf{f}_g^*$ can balance any external torques w.r.t. $\mathbf{p}$ except those around the axis $\mathbf{c}_i\mathbf{p}$. An external torque around the axis $\mathbf{c}_i\mathbf{p}$ can be balanced by the grasp torque $\mathbf{t}_{g/\mathbf{c}_i}$ because it spans positively $\mathbb{R}^3$ at $\mathbf{c}_i$. Therefore, the grasp wrench $\mathbf{w}_{g/\mathbf{p}}^*$ can balance any external wrench w.r.t any point $\mathbf{p} \in \mathbb{R}^3$ and, the conditions 3-*(i)* and 3-*(ii)* are sufficient. Furthermore, if one of the two conditions of the Proposition 3 is not satisfied, there exist external wrench that cannot be balanced by the grasp wrench $\mathbf{w}_g$. Hence, the two conditions are necessary. □

The second condition in the Proposition 3 states that the torque applied by the n fingers must positively span $\mathbb{R}^3$ at $\mathbf{c}_i$. Mechanically, this condition can be satisfied if the first condition 3-*(i)* is satisfied w.r.t. three non-collinear points. Accordingly, we put forward the following force-closure condition :

Proposition 4 : *An n-finger grasp is force-closure iff all grasp-wrench central axes of class Δ_g^* can positively span $\mathbb{R}^3$ w.r.t , at least, three non-collinear points (eg. $\mathbf{c}_i$, $i = 1,2,3$) (forces along Δ_g^* are noted $\mathbf{f}_{gi}^*$).*

Proof : When a grasp achieves force-closure, the contact forces positively span $\mathbb{R}^3$ w.r.t. any arbitrary point. Hence, spanning positively $\mathbb{R}^3$ w.r.t three points is a necessary condition. Further, if the central axes Δ_g^* spans positively $\mathbb{R}^3$ at point $\mathbf{c}_2$ then, the torque $(\mathbf{c}_1 - \mathbf{c}_2)\times\mathbf{f}_{g2}^*$ can balance any external torques w.r.t. $\mathbf{c}_1$ except those around the axis $\mathbf{c}_1\mathbf{c}_2$. Also, if the central axes Δ_g^* spans positively $\mathbb{R}^3$ at a third point $\mathbf{c}_3$ where $(\mathbf{c}_1 - \mathbf{c}_2)\times(\mathbf{c}_1 - \mathbf{c}_3) \neq \mathbf{0}$ (three non-collinear points) then, the torque $(\mathbf{c}_1 - \mathbf{c}_3)\times\mathbf{f}_{g3}^*$ can balance external moments around axis $\mathbf{c}_1\mathbf{c}_2$. Therefore, propositions 3 and 4 are equivalent and, the condition of Proposition 4 is sufficient to assure force-closure. □

A. Force-Closure Tests

Testing the force-closure is simplified using propositions 3 and 4. Based on Proposition 2 and the equations system

(10), we formulate the condition 3-*(i)* as the following LP :

$$\min_{\mathbf{x}=(a_{11},a_{12},\cdots,a_{2m},\cdots,a_{nm})^T} \left\{\mathbf{1}^T\mathbf{x} \ : \ A\mathbf{x} = -\mathbf{z}_1, \ \mathbf{x} \geqslant \mathbf{0}\right\} \quad (11)$$

the matrix A of dimension $(6 \times mn)$ is given by

$$A = \begin{pmatrix} \mathbf{v}_{11} & \cdots & \mathbf{v}_{1m} & \mathbf{v}_{21} & \cdots & \cdots & \mathbf{v}_{nm} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{t}_{21} & \cdots & \cdots & \mathbf{t}_{nm} \end{pmatrix} \quad (12)$$

Where $\mathbf{t}_{ij} = \mathbf{d}_i \times \mathbf{v}_{ij}$, $\mathbf{d}_i = \frac{1}{r}(\mathbf{c}_i - \mathbf{c}_1)$. We normalize the torques by r, the maximum radius from the wrench space origin, often the centre of mass. This ensures that the quality of a grasp will be independent of the object scale.

The second condition 3-*(ii)* states that the torques applied by the n fingers $\mathbf{t}_{g/\mathbf{c}_i}$ positively span $\mathbb{R}^3$ at $\mathbf{c}_1$. We can compute the sphere in $\mathbb{R}^3$ torque space to verify this condition. However, another equivalent mathematical statement is [23] [1] :

Proposition 5 : *A set of vectors $\mathbf{t}_{i\in[1\ldots k]} \in \mathbb{R}^3$ positively span $\mathbb{R}^{n_v}$ ($n_v \leqslant 3$) iff, there exists λ_i such that the following two conditions hold :*
(i) $\mathrm{rank}(M) = n_v$ with $M = [\mathbf{t}_1 \ldots \mathbf{t}_k] \in \mathbb{R}^{n_v \times k}$,
(ii) $\exists\ \lambda_i > 0$ such that $M\lambda_i = \mathbf{0}$.

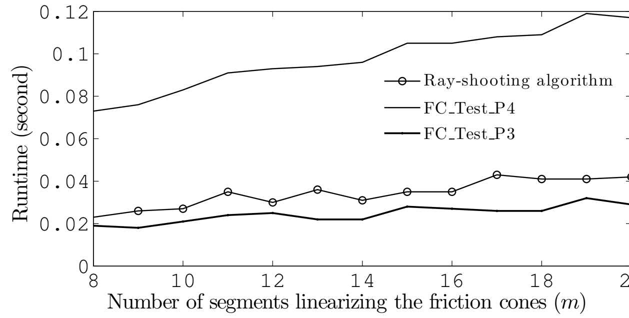
We notice that if 3-*(i)* is satisfied and using torque equations given by (10), there exist some non-zero torques ($\lambda_i > 0$) w.r.t. $\mathbf{c}_1$ that verify $\sum_{i=1}^{k} \lambda_i \mathbf{t}_i = \mathbf{0}$. Hence, according to Proposition 5, the condition 3-*(ii)* is verified if $\mathrm{rank}(M) = 3$. However, when $\mathrm{rank}(M) = 2$, we conclude that the non-zero torques ($\lambda_i > 0$) can positively span a plane $\mathscr{P}$ at $\mathbf{c}_1$. In this case, the condition 3-*(ii)* is verified if the other torques (with coefficients $\lambda_i = 0$) can generate positive and negative moments around the normal of $\mathscr{P}$ at $\mathbf{c}_1$ (noted $\mathbf{N}_{\mathscr{P}}$). When $\mathrm{rank}(M) < 2$, the condition 3-*(ii)* can be verified by computing the sphere in $\mathbb{R}^3$ torque space, which returns the minimim moment m_{ball} w.r.t $\mathbf{c}_1$.

We summarize the proposed tests for n-finger force-closure grasps as the following two algorithms (Alg. 1 and 2). The first algorithm Alg. 1, describes the two steps advanced in Proposition 3 where the condition 3-*(ii)* is verified using Proposition 5. For the second proposed force-closure algorithm (Alg. 2), we perform the resolution of n linear programs based on Proposition 4.

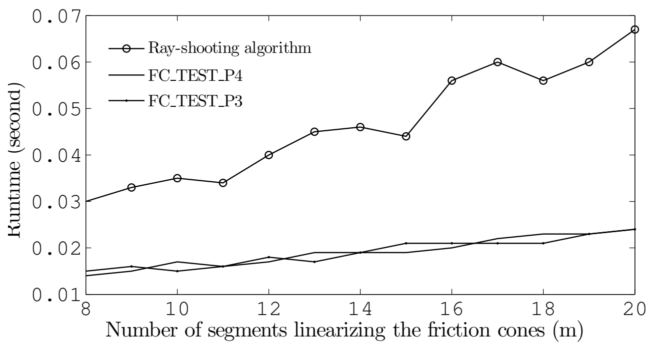
We notice that the proposed algorithms allow to test the force-closure of planar and spatial n-finger frictional and frictionless grasps. In contrast of the qualitative ray-shooting algorithm, the proposed approach gives a quality measure of the force-closure grasp, which is important in grasp synthesis and grasp optimization.

In order to give an idea about the execution run-time of the two proposed algorithms versus the number of sides linearizing the friction cones, we show in Fig. 2 two illustration examples from [13], [14]. The 4-fingered grasp achieves force-closure when $\mu = 0.5$ (Fig. 2(a)), the lower required run-time is obtained when the Alg. 1 is used. The force-closure test advanced in Alg. 2 takes more computational times. However, when the grasp is not force-closure, $\mu = 0.3$ (Fig. 2(b)), we can see that the two proposed tests have the same run-time and, the ray-shooting algorithm takes more computational times in the case of non-force-closure grasps [13], [14]. In general, a cone approximation with $12 \leqslant m \leqslant 20$ sides leads to acceptable practical results and, the percentage of discarded area is 4.51% with $m = 12$ and 1.64% when $m = 20$ [22].

[1] This is the main difference between the present paper and the algorithm advanced in our previous work [19]. The force-closure test algorithm Alg. 2 is also a new contribution of this paper.



(a) Force-closure grasp



(b) Non-force-closure grasp.

Fig. 2. The run-time for testing 4-finger grasps.

Algorithm 1 : Q_{P3} = `FC_Test_P3`(n, $\mathbf{c}_i$, $\mathbf{z}_i$, μ) ; ($i = 1 \ldots n$)

Ensure: Q_{P3}
Require: $n \geqslant 3$
if (11) is inconsistent **then**
  return $+\infty$ {unfeasible LP, not FC grasp}
else
  if $\mathrm{rank}(M) = 3$ **then**
    return $\mathbf{1}^T\mathbf{x}^*$ {the grasp is FC}
  else if $\mathrm{rank}(M) = 2$ **and** $\mathbf{t}_{g/\mathbf{c}_1}$ positively span $\mathbf{N}_{\mathscr{P}}$ **then**
    return $\mathbf{1}^T\mathbf{x}^*$ {the grasp is FC}
  else if $m_{ball} > 0$ **then**
    return $\mathbf{1}^T\mathbf{x}^*$ {the grasp is FC}
  else
    return $+\infty$ {$\mathbf{t}_{g/\mathbf{c}_1}$ cannot positively span $\mathbb{R}^3$}
  end if
end if

Algorithm 2 : Q_{P4} = FC_Test_P4(n, $\mathbf{c}_i$, $\mathbf{z}_i$, μ) ; $(i = 1 \ldots n)$

Ensure: Q_{P4}

Require: at least, three non-collinear points $\mathbf{c}_i$

```
for i = 1 to n do
    LP_i = min { 1^T x : A x = -z_i, x ⩾ 0 }
    if LP_i is inconsistent then
        return +∞ {unfeasible LP, not FC grasp}
    else
        Q(i) = (1^T x*) / n {if i ⩾ 3 then the grasp is FC}
    end if
end for
return Σ_{i=1}^{n} Q(i)
```

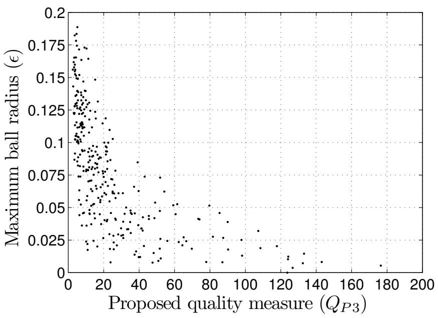


Fig. 3. The proposed quality Q_{P3} versus ε (4-contact grasps on ellipsoid).

B. Grasp Quality measure

When the grasp is force-closure, the optimal solution of the proposed LP is $Q_{P3} = \mathbf{1}^T\mathbf{x}^*$. The proposed quality gives the minimal contact forces that contribute to obtain the maximum of the force along axis $-\mathbf{z}_i$. The quality Q_{P4} in Alg. 2 is an average value that can give a better information about the distribution of the contact forces on the object's surface. The proposed qualities can measure how far a grasp is from loosing force-closure. We analyze the 4-fingered grasps in order to compare the proposed quality measures with the largest ball ε inscribed inside the convex hull of contact wrenches. The grasped object is a spheroid with the lengths of the principal axes set to $R = 2$ and $r = 1$, respectively. The friction coefficient is set to $\mu = 0.4$ and, the friction cone is linearized by a pyramid of $m = 12$ sides. We vary randomly the locations of 4 contact points on this ellipsoid and we compute 1000 grasps. The quantitative check is carried out successfully and all grasps are tested in a run-time of 0.735 seconds using algorithm 1. The qualitative ray-shooting algorithm takes 0.858 seconds to test all grasps. However, computing the sphere in $\mathbb{R}^6$ wrench space needs 94.141 seconds to check these grasps. Hence, the proposed test reduces efficiently the computational cost. In Fig. 3, we plot the maximum radius of the sphere ε (using qhull library [27]) versus the proposed quality measure Q_{P3}, we can remark that the minimization of Q_{P3} leads to maximizing ε. Therefore, the proposed force-closure test can be considered as a quantitative one without computing convex hull.

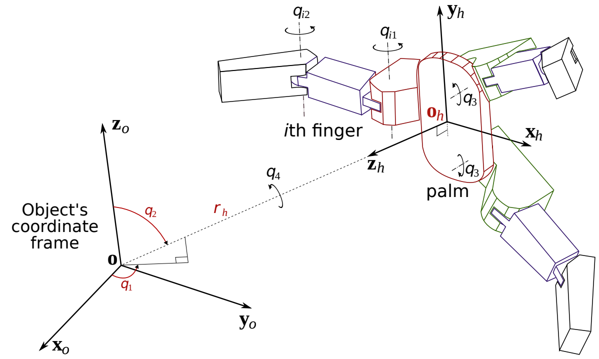
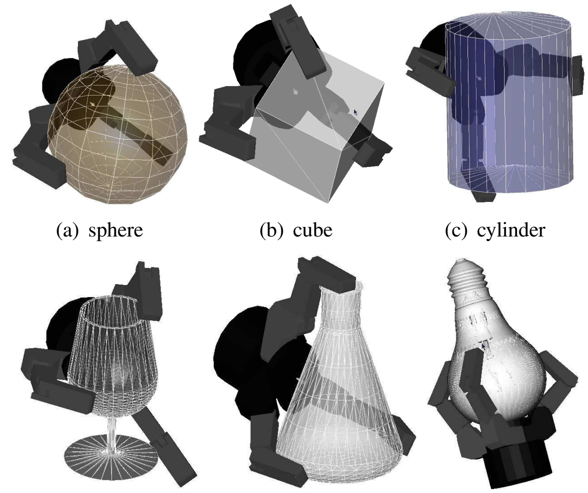


Fig. 4. The different parameters defining the starting posture of the Barrett w.r.t the object coordinate frame : $\{q_1, q_4\} \in [0, 2\pi]$, $\{q_2, q_3\} \in [0, \pi]$, $q_{i1} \in [0, 144]^\circ$ and $q_{i2} = 0.333 * q_{i1} + 40\ ^\circ$; $i = 1 \ldots 3$ finger's number.



(a) sphere (b) cube (c) cylinder

(d) glass (e) flask (f) lamp

Fig. 5. Some good grasps generated randomly. The obtained qualities $[Q_{P3}, \varepsilon]$ are : (a)$[0.875, 0.604]$, (b)$[0.741, 0.492]$, (c)$[1.960, 0.302]$, (d)$[1.430, 0.347]$, (e)$[1.120, 0.379]$, (f)$[1.724, 0.308]$.

IV. Numerical Results

In this Section, we use the 3-fingered Barrett hand [28] to study the performances of the proposed approach by evaluating multifingered grasps on various objects[2]. The simulations were performed under the public simulator GraspIt 2.0 [24]. To generate a grasp, the Barrett hand is randomly positioned at a starting configuration which is defined by four parameters $q_{i=1\ldots4}$ (refer to Fig. 4). The hand is then translated along the coordinate axis $\mathbf{z}_h$ until it is prevented from moving further by contact. Third, the fingers are closed around the object until contacts or joint limits prevent from further motion. If at least three contacts between the hand and the object exist, the obtained grasp is evaluated. Fourth, the angles of the proximal links are initialized, the hand is slightly backed away from the object along the axis $\mathbf{z}_h$ with a small distance $d_{back} = 5mm$ and the fingers are closed again. Finally, This backing off iteration continues until the fingers reach the object and the corresponding grasp can be

[2]In this paper, all numerical results are obtained on a Pentium-M laptop (processor 1.7 GHz, 1.5 Go of RAM, OS. Linux).

TABLE I

PERFORMANCES OF THE PROPOSED QUANTITATIVE TEST

Object	Grasps	cpu_{CA}(s)	cpu_{RS}	cpu_{CH}
sphere	209	0.179	0.244	32.879
cube	347	0.263	0.360	29.740
cylinder	409	0.327	0.407	38.169
glass	533	0.485	0.608	67.604
flask	565	0.470	0.609	66.287
lamp	578	0.534	0.677	68.296

run-time unit is second. CA : central-axis (Alg. 1).
RS : ray-shooting algorithm [13], [14].
CH : computing convex hull in $\mathbb{R}^6$ [15].

evaluated. The friction coefficient is set to $\mu = 1.0$ assuming rubber finger covers. Friction cones are approximated by $m = 12$ sides. The distance r_h (Fig. 4) is selected sufficiently high to assure collision avoidance.

Using this method, we generate 200 starting postures randomly and we compute the resulting grasps on various polyhedral objects. In figure 5, we present the best obtained grasps and their corresponding quality measures. All grasps in Fig. 5 have $\varepsilon > 0.1$, which confirm that they are good power grasps [25]. Note that the possible ranges of the largest ball ε depend on μ. The computational performances of the proposed central-axis approach are depicted in Table I. The number of tested grasps is depicted in the second column. We can easily remark that the proposed approach is computationally competitive compared to the qualitative ray-shooting test and, more efficient then computing the maximum sphere inscribed in the wrenches convex hull.

V. CONCLUSIONS AND FUTURE WORKS

This paper focuses on the proposition of modified force-closure test algorithms based on the central-axis theory and using friction cone linearization.

Other important research topic within the area of grasping is grasp synthesis [23]. It is the problem of choosing the posture of the robotic hand and contact point locations to optimize a grasp quality metric. Many work are concerned with this topic [24], [25], [26], and we must notice that the the most significant amount of computation is spend checking the feasibility and evaluating the quality of each generated grasp. Therefore, The main contribution of the present paper is the proposition of a real-time efficient quantitative force-closure test. Through numerical simulations on multifingered grasps, we have confirmed the efficiency of the central-axis approach in testing the force-closure property. The proposed approach can be applied on-line to evaluate the quality measures of force-closure grasps, which efficiently reduces the amount of computation for grasp synthesis.

As generated grasps must perform different tasks in the environment, our future works will be concentrated on the development of oriented task qualities.

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