# Two-Dimensional Dynamic Modeling of a Sliding Motion of a Soft Fingertip Focusing on Stick-to-Slip Transition

Van Anh Ho and Shinichi Hirai

*Abstract*—We describe here our proposed method to investigate the sliding motion of a soft fingertip in the dexterous manipulation. This paper focuses on analyzing dynamic sliding motion of a 2-dimensional (2-D) soft fingertip on a plane. To investigate the deformation of the fingertip during this process, we consider the soft fingertip as if it was composed of a finite number of elastic cantilevers which are compressible and bendable. Simulation will be carried out firstly on this 2-D model of soft fingertip, focusing on the analysis of incident slippage. After that, various experimental results will be shown to verify this model.

## I. INTRODUCTION

Recently, there has been a large number of robotic researches focusing on dexterous manipulation of objects by using soft fingered robotic hands. These researches are categorized into two main groups. First, studies focused on analysis of contact mechanics between various soft fingers and objects [1]. On the other hand, human imitated tactile sensing systems have been developed, together with developments of many kinds of sensors, to emulate human abilities in object grasping and dexterous handling [2]. Whistle the former researches emphasize on analyzing stable grasping or object posture controlling by utilizing soft fingertips' compliance during their pushing or rolling motion on surface of objects; the latter ones concentrated tactile texture perceptions of sensory fingertips, especially incipient slip detection, during the sliding motions on the objects' surface. Among the various types of motion of soft fingertips, it is widely known that sliding motion takes an important role in robotics manipulation. For example, to assess the texture of an object's surface, the fingertip needs to slide slightly on the surface to extract information about roughness or friction. The trend of sliding motion of the object between fingertips during grasping, or the incipient slip, is recognized as a crucial factor in stable object manipulation. Nevertheless, there has not been much research which bridges theoretical and experimental research of sliding motion of the soft fingertip. Kao et al. [3] proposed a method for modeling sliding fingers by combining compliance and friction limit surface. However, only quasistatic simulation was mentioned, not yet the dynamic one. Therefore, a proper theory to explain and verify experimental results is therefore needed.

We have been working on proposing a theoretical background to explain the experimental results of the sliding motion of a soft fingertip. The initial results of this work,



Fig. 1. Proposed model of a soft fingertip comprising of virtual elastic cantilevers. (a) Distribution of normal forces when the fingertip is pushed. (b) Virtual cantilevers and linkage springs are introduced.

which are described in this paper, are mainly on dynamic sliding motion of a 2-D model of a soft fingertip with a finite number of elastic cantilevers. By observing the dynamic sliding of an individual cantilever's free-end, we can assess the dynamic transition between stick state to slip state of the fingertip, *i.e.* the incipient slip. After simulation, we conducted many experiments to validate the simulated model by employing commercial load cell, developed micro force/moment sensor, and high speed camera.

## II. PROPOSED MODEL OF THE SLIDING SOFT FINGERTIP

Inoue et al. [1] proposed a soft fingertip model which comprised of an infinite number of vertical elastic virtual springs to investigate the deformation of the fingertip during pushing or rolling motion on the object. However, this model is not sufficient to demonstrate sliding motion with appearance of frictional force. Therefore, instead of virtual springs, we proposed a new model in which the soft fingertip is composed of finite virtual elastic cantilevers which are compressible, tensile, and bendable (Fig. 1). This new model assures the ability of representing the diverse deformations of the soft fingertip during a sliding motion in which the soft fingertip pushes and slides at the same time. These cantilevers are fixed on the equatorial surface of the fingertip, and the free ends are on the outer surface of the fingertip. Each cantilever has a uniform cross sectional area which is circular with radius  $d_r$ ; whereas the length of cantilevers differ depending on their coordinates within the fingertip.

Conventionally, previous theoretical researches eliminated the role of frictional force in their dynamic model. However, frictional force plays a crucial role in the stabilizing of objects grasped by robotic hands. By dividing the soft fingertip into many cantilevers in our advanced model of the fingertip, surface-to-surface contact between the fingertip and the objects is split into point-to-surface contacts between free ends of cantilevers and the surface of the objects. Therefore,

V.A. Ho and S. Hirai are with the Department of Robotics, Biwako-Kusatsu Campus, Ritsumeikan University, Kusatsu, Shiga, 525-8577 Japan gr048071, hirai@se.ritsumei.ac.jp



Fig. 2. Proposed model of a soft fingertip comprised of virtual elastic cantilevers.

Coulomb's friction law can be applied to each free end of the cantilever on the contact surface [4]. As a result, by observing dynamic motion of all free ends of the cantilevers the fingertip with appearances of frictional forces, we can assess the transient responses, especially the incipient slip, during the sliding motion of the fingertip. In this paper, we will report on the 2-D model of the soft fingertip, as a preliminary step of our research. Let *r* be soft fingertip's radius. Firstly, let us analyze the distribution of forces inside the soft fingertip when it is pushed perpendicular to the plane with contact depth  $d_n$ . In this case, all cantilevers are also pushed vertically with different normal deformations. Normal force  $F_k^n$ , acting on one free end of a cantilever whose coordinates are determined by *x*, and  $\theta$  (Fig. 2(a)), is calculated based on the equation originated in [1]:

$$F_k^{\ n} = k(\sqrt{r^2 - x^2} - (r - d_n)) \tag{1}$$

with k being the stiffness coefficient of the cantilever.

It was also proved in [1] that the highest value of normal force is at the *center* cantilever at which the maximum compression is recorded; while it decreases gradually toward the edge (Fig. 2(a)). Therefore, when the fingertip's rigid base starts to move (but it is not the gross sliding), it is intuitive that regions near the edges will give way and slide a short distance before the center starts to slip. This motion around the peripheral zone appears in the form of vibrations, and if the center zone is still fixed, overall motion does not occur ([6]). Thus, it is necessary to observe dynamic motion of each free end of cantilever on the contact surface.

For the sake of simplicity, three assumptions are given as follows:

- 1) When the cantilever is bent, its deformation is significant only at the free end.
- Interactions between continuous cantilevers only happen mainly between their free ends on the contact surface.
- Only cantilevers whose free ends are acting on the contact surface are taken into account in this investigation.



Fig. 3. Forces acting on k cantilever.

Cantilevers positioning outside the contact surface are irrelevant to the sliding motion of the fingertip.

Based on above assumptions, we propose a complete 2-D model of soft fingertip contacts and slides on a rigid plane. In the model mentioned in [1], all beams were considered independent, unrelated to their neighboring ones. One beam could deform without any interaction with the others. Therefore, we will introduce these interactions among virtual cantilevers as well, to demonstrate more precise behavior of the fingertip during its manipulating task. We consider that between two continuous cantilevers' free ends there is a small horizontal virtual spring, hereafter called linkage spring (Fig. 2(b)). These springs represent interaction between cantilevers, and they will take an important roles to form vibrations on the outer surface of the fingertip on the contact surface. Stiffness of each spring is similar and depends on the distance between the axes of two contiguous cantilever beams, as well as the cross section.

Consequently, by using this model, it is possible to observe the motion of each cantilever's free end on the contact surface. By doing so, we can assess a more detailed transient period between the stick and slip states of the fingertip when sliding along the plane surface, especially at the moment of transition.

#### III. SIMULATION

We implemented a simulation of the soft fingertip using the aforementioned model in the transient period between the stick to slip state during the sliding motion. In this simulation, a fingertip which has the radius r is first pushed with contact depth  $d_n$ . After that, it starts to move with constant velocity v. By observing the movement of each cantilever's free end, we can perceive the entire motion of the fingertip in this period.

# A. Parameters Calculation

Firstly, the Young's modulus E of the soft fingertip used in the simulation was measured by conducting a compression test on polyurethane gel and implementing a linear approximation. After that, we had E=0.2032 MPa [1]. To specify the number of cantilevers, the radius of the cross sectional area of the cylindrical cantilever is set to dr. Moreover, this number is obviously dependent on the contact depth  $d_n$  as well. Let  $l_0$  equal to distance between two continuous cantilevers' neutral axes (Fig. 2). Thus, number n of cantilevers on the contact surface is calculated based on the length PQ (Fig. 2(a)), which is defined as  $PQ/l_0$ . Let number of cantilevers be n = 2N + 1. The *center* cantilever is numbered 0. The cantilevers which distributed on the right side of *center* one are numbered from 1 to N, whereas the left ones are from -1to -N (Fig. 2(a)). Bending stiffness of the k-th ( $k \in [-N,N]$ ) cantilever is also calculated based on its parameters, as well as based on the equation:

$$k_k^{\ b} = \frac{3EI}{l_k^{\ 3}} = \frac{3E\pi dr^4}{l_k^{\ 3}} \tag{2}$$

with  $l_k$  being the natural length of the *k*-th cantilever. The stiffness of each *linkage spring* is similar and specified by:

$$k_s = \frac{Es_0}{l_0} = \frac{E\pi dr_0^2}{l_0}$$
(3)

where  $dr_0$  is the radius of the cross sectional area of the *linkage spring*.

#### B. Force/moment analysis

Applied loads in this model are given by constraints. The plane is set to stay fixedly; while the 2-D fingertip is given vertical and horizontal constraints. It is firstly pushed with constant contact depth  $d_n$ . By doing so, each cantilever will receive specified normal compression, and based on equation (1) value of normal force  $F_n^k$  acting on it can be calculated. After that, under one specific contact depth, the fingertip is given a constant horizontal velocity v. As a result, a 2-D model of a sliding fingertip on a plane is assessed.

Let us detach the *k*-th cantilever to assess external forces acting on it on moment *t*. Fig. 3 shows the detached *k*-th cantilever with interactions by the two neighbored (k - 1)-th, and *k*-th *linkage* springs. The normal force  $F_k^n$  is caused by normal deformation of the fingertip, or cantilever. The bending force:

$$F_k^{bending} = k_k^b s = k_k^b(vt) \tag{4}$$

appears when the cantilever is bent. Two elastic forces  $F_k^s$ ,  $F_{k-1}^s$  are caused by deformations of the (k-1)-th and k-th springs when the free end of the k-th cantilever starts to move; and are calculated as follows:

and:

$$F_k^s = k_s (u_{k+1} - u_k)$$
(5)

$$F_{k-1}^s = k_s(u_k - u_{k+1}) \tag{6}$$

in which  $u_k$ ,  $u_{k-1}$ , and  $u_{k+1}$  are the displacements of free ends of k-th, (k-1)-th, and (k+1)-th cantilevers, respectively.

Moreover, there is tangential frictional force acting on the freed end of the cantilever  $F_k^{friction}$  (Fig. 3). Therefore, motion equation of the cantilever is formulated as the following:

$$F_k^{bending} + F_k^s - F_{k-1}^s - F_k^{friction} = \Gamma_k \tag{7}$$

or:

$$k_{s}u_{k-1} - (k_{k}^{b} - 2k_{s})u_{k} + k_{s}u_{k+1} = \Gamma_{k} - F_{k}^{friction} + k_{k}^{b}s \quad (8)$$

with  $\Gamma_k$  varies depending on states of contact of the cantilever's free end. During the sticking period, it is obvious that  $\Gamma_k = 0$ . When  $F_k^{friction}$  reaches value of  $F_k^n \mu$ , the free end of cantilever starts to move based on Coulomb's law,  $\Gamma_k$  equals to  $m_k \ddot{u_k}$  in which  $m_k$  is mass of k - th cantilever,  $u_k$  is a deviation of the free end after starting to slide. As a result, motion of this cantilever has two stages depending on the states of the dynamic frictional force. In the first stage, free end of the cantilever sticks to the surface, and the friction force keeps increasing. When the friction force reaches its maximum defined by  $F_{k}^{n}\mu$ , where  $\mu$  is friction coefficient, the free end starts to slide. After this moment, the frictional force keeps unchanged. For other cantilevers, the acting force/moment are similar, except the (-N)-th cantilever and the N-th cantilever at which there are no -(N-1)-th linkage spring and (N+1)-th linkage spring, respectively. Motion equations for all the cantilevers, *i.e.* soft fingertip, during sliding motion are summarized as followed:

$$\mathscr{K}\mathbf{u} = m\ddot{\mathbf{u}} - \sum_{k=-N}^{N} F_{k}^{friction} + \mathbf{K}_{\mathbf{b}}s$$
(9)

with s = vt is moved distance of fingertip at time t;  $\mathbf{u} = [u_{-N}, ..., u_k, ..., u_N]^t$ ,  $\mathbf{K_b} = [k_{-N}^b, ..., k_k^b, ..., k_N^b]^t$ ,

$$\mathcal{K} = \begin{pmatrix} -k_{-N}^{b} - k_{s} & k_{s} & & \\ k_{s} & -k_{-N+1}^{b} - 2k_{s} & k_{s} & & \\ & \vdots & \vdots & \vdots & \\ & & & -k_{N}^{b} - k_{s} & k_{s} \end{pmatrix}$$

This equation can not be solved analytically, therefore, we employed numerical method to assess the movements. During the simulation time, value of frictional force acting on each cantilever's free end is calculated and based on that, stick or slip state of this free end will be decided. The overt slip of soft fingertip on the contact surface will happen when all the free ends slide.

## C. Simulation results

To conduct the simulation, let us have radius of soft fingertip r =10 mm; contact depth  $d_n =2$  mm; dr=1 mm,  $dr_0=1$  mm. In this case, N = 6. Fig. 4 shows the response of the total friction force acting over the contact surface during the stick-to-slip period of the fingertip which is moved with velocity v = 2 mm/s. It is evident that when the fingertip starts to move with constant velocity, it still sticks to contacting plane causing its deformation. Therefore, the value of the friction force increases. This value is calculated by sum up all the friction forces acting on cantilevers' free ends. We can observe the fluctuation on the increasing slope of the total friction force. It is caused by vibrations of free ends on the contact surface during this period. These vibrations of cantilevers cause the directions of friction forces acting on free-ends to change frequently, *i.e.* signs of fiction force



Fig. 4. Simulation results: Response of frictional force acting on the contact surface during stick-to-slip transition.

values. Let us take a deeper look on the value of friction force acting on one cantilever's free end in Fig. 5. The friction force changes its saturation from positive value to negative one continuously, corresponding to the vibration in position of this free end. At the moment of  $t_A = 2$  s (Fig. 4), the friction force reaches its highest value, corresponding to the moment at which the last free end started to slide. After this, all the free ends stop vibrating, and move with constant velocity v. Consequently, vibration on the contact surface of the soft fingertip happens and takes a crucial role in transition stick-to-slip period. Researches beforehand also showed experimentally that by detecting vibrations on the surface contact of soft fingertips, moment of incipient slip could be perceived [5]. As aforementioned, Howe *et al.* [6] suggested that during the stick-to-slip transition the center of the contact surface, *i.e.* the position of the 0-th cantilever's free end, was the last one slide right before the gross slippage of the moved fingertip on the object's plane. Nevertheless, our result is different. Fig. 6 shows the order of first slide of all cantilevers' free ends on the contact surface before the gross slide of the fingertip. As a result, the last cantilever's free-end started to move is the 5-th cantilever. The 0-th cantilever's free end, i.e. center one, had started to move some moment before that. As a result, there is a delay time between the moment of sliding of the center cantilever's free end and the overt slip of entire fingertip. This delay time is small at high speed, and vice versa. Thus, even the normal pressure is highest at the center zone of the contact surface; it seems to be not the last one to move before overt slip of the fingertip. This delay time is considered important for the real application; in which the incipient slip of the fingertip, while contacting with object, needs to be detected properly. By using some sensory receptors to detect the sliding of the center area on the contact surface, we can judge the overt slippage of the fingertip prior actual one, thus more timely; because it is usually not fast enough to react to the slippage right after this moment as common ways did.

Consequently, by conducting the dynamic simulation using the proposed model, we had a deeper look at the stick-toslip transition of soft fingertip. Vibrations of the outer skin



Fig. 5. Simulation results: Magnification of response of frictional force acting on an arbitrary cantilever's free-end.



Fig. 6. Orders of movement of virtual cantilevers' free-ends during the stick-to-slip transition.

of the fingertip on the contact surface have been observed. Moreover, orders of sliding of cantilevers' free ends, *i.e.* various contacting areas, as well as moment of gross slide of the fingertip are assessed in detail. Unlike the conclusions of previous researches, center zone of contact surface, which is under interaction with other regions, is not the last sliding before the overt slide of fingertip. If this result is confirmed, it will take an important role in detecting incipient slip of the soft fingertip. In the next section, experiment results will be shown to validate these simulation results.

#### IV. EXPERIMENT

#### A. Experimental setup

To validate the proposed model, we conducted experiment on an objective soft fingertip. This is a semi-cylindrical soft fingertip which has the radius of 20 mm, and the thickness equals to 4 mm. This soft fingertip was made from polyurethane rubber (KE-12, Exseal, Japan) after an 8-hour curing phase at room temperature. On the outer surface of the fingertip, there are many ridges distributed uniformly (Fig. 7(a)). Each ridge has the shape of  $90^{\circ}$  arc of a circle which has diameter of 0.5 mm. Distance of two continuous arcs (or ridges)' centers is 1 mm. These distribution of ridges represent epidermis ridges on the human's fingertips, and free ends of cantilevers in the model reported in section II. Fig. 7(b) illustrates set-up of the experiment. The soft fingertip is attached on a 2-DOF (degree of freedom) xz-motorized linear stage (XMSG615 and ZMSG413, Misumi, Japan) to give vertical and horizontal translation of the fingertip on the flat rigid plane. There are two kinds of force/moment sensors employed to measure force/moment acting on the fingertip



Fig. 7. Experiment set-up a) Semi-cylindrical soft fingertip and the distributed ridges on the surface. b) Entire view of the experimental apparatuses.

during the experiment. The 3-DOF load cell PD3-32-10-105 (Nitta, Japan) is fixed upon the fingertip to measure the total forces (normal force and friction force) acting on the fingertip during sliding motion. This load cell is compact, and able to suffer up to 15 N. Moreover, a developed 3-DOF micro force/moment sensor utilizing MEMS (Micro Electro Mechanic System) technology was also used in this experiment. This sensor includes a sensing core (2 mm in length, 2 mm in width, and 0.5 mm in height) which can detect simultaneously one component of force  $(F_7)$  and two components of moments ( $M_x$ , and  $M_y$ ) up to 0.5 N and 2 Nmm, respectively. This sensor is placed at the center point of the rigid plane of the fingertip. Results in last paper [7] showed that with this configuration, responses of the micro sensor will reflect contact states of the center zone on the contact surface. Moreover, to observe the detailed movement on the contact surface, we used a high speed camera (MV2-D1280-640 CMOS camera, Photonfocus) which takes successive images of the contact surface during sliding motion of the fingertip with frame rate of 500 fps (frames per second). To do so, the soft fingertip is slid on a transparent rigid plane made of acrylic resin ACBTA (Misumi, Japan), and the camera is placed underneath the transparent plane. In order to help the camera to realize ridges on the surface of the soft fingertip easily, we coated the ridges by very thin layer of black painting.

## B. Experimental results analysis

1) Force/moment analysis: In this experiment, the fingertip was slid along x-axis, therefore signals of moment  $M_y$  from the load cell or the sensor represent tangential frictional forces acting on the contact surface during the sliding motion; whereas the signals of moment  $M_x$  are basically zero. Fig. 8(a) shows the responses of the load cell during the sliding motion of the fingertip. These signals are total external force/moment exerting on the fingertip. We can see that the signal of normal force is unchanged during this period because the fingertip is kept with constant contact depth; whereas the moment  $M_y$  changes remarkably. This signal represents frictional force acting on the contact



Fig. 8. Signals of normal forces  $(F_z)$  and tangential moments  $(M_y)$  from: (a) Load cell. (b) Sensor.



Fig. 9. Model of force/moment acting on the fixed-end of the 0-th cantilever: (a) Contact States of the cantilever. (b) Graphs of force and moment acting the cantilever.

surface, thus, it also includes two stage. First, due to the sticking phase, it increases gradually. After it reaches its top, it keeps at the stable value, corresponding to the sliding phase of the fingertip. These response are similar to simulation results in Fig. 4. Moment of transition from stick to slip state is slightly different between experiment ( $t_B$ =2.4 s in Fig. 8(a)) and simulation ( $t_A$ =2 s in Fig. 4). It is due to the fact that the experimental soft fingertip has unavoidable thickness which is ignored in the 2-D model.

In Fig. 8(b), the signal of moment  $M_y$  from the micro sensor also acts similarly with those of load cell and simulation result. The moment of transition  $t_B$  is just slightly different. However, the signal of normal force is dissimilar to that of load cell. When the fingertip starts to slide, it decreases gradually. After reaching its bottom value at  $t_A = 1.6$  s, it jumps up a little and keeps unchanged afterward. As aforementioned, location of the micro sensor is at the fixed-

end of the 0-th cantilever, we supposed that responses of the sensor would be similar to reactive force/moment acting on fixed-end (Fig. 9(a)). Let us detach the center cantilever, and observe reactive force/moment on the fixed-end during the sliding motion of the fingertip. Fig. 9(b) shows the plots of normal force and moment  $M_{y}$ . Initially, the fingertip is pushed with specific contact depth, the center cantilever is compressed as well resulting normal force. When the fingertip begins to slide, the free end of the center cantilever still sticks to the surface, while the fixed-end moves. This makes the cantilever less compressed resulting decent of normal force; whereas the tangential force increases. After the tangential force (friction force) reaches its threshold, the cantilever's free-end starts to move and the normal force keeps unchanged. As a result, the moment at which the normal force reaches its bottom is coincident with initial slide of the *center* cantilever's free end. Base on this, we consider that the moment at which the signal of normal force of the sensor reaches the minimum value (Fig. 8(b)) is moment of slippage of the *center* cantilever's free end. Nevertheless, unlike the past conclusion that when the center zone of contact surface was the last one slide before gross slippage of the fingertip ([6]), there is a delay time  $t_d$  between slide of *center* zone (at  $t_A$ ) and the entire slide of fingertip (at  $t_B$ ) (Fig. 8). This  $t_d$  is small at high moving speed of the fingertip, and vice versa. Therefore, let us conclude that the center zone might not be the last zone moving before gross slide of the entire contact surface of the fingertip. This is also similar to simulation result reported in section III.

2) Vision analysis: We controlled the operation of the camera simultaneously with the sliding motion of the fingertip. Fig. 10(a) shows the example of a grayscale image of ridges on the fingertip's skin and the contact surface. By conducting image processing using OpenCV, we obtained the binary image with coordinates of contours (Fig. 10(b)). After that, we observed the movements of center points of contacting ridges on the contact surface (Fig. 10(c)). To do so, the *i*-th ridge was extracted into two continuous vertical edges by using edge-emphasizing filter. Central points' coordinates of these edges, say  $(x_i, y_i)$  and  $(x_{i+1}, y_{i+1})$ , are perceived afterward. Ultimately, tracked point of *i*-th ridge is defined as  $1/2(x_i+x_{i+1},y_i+y_{i+1})$ . As a result, by tracking these movements of central points of ridges, we could assess the orders of movement of these points, i.e. cantilevers' free ends.

Because the *y*-coordinate of each central point keeps unchanged during the sliding motion of the fingertip, we only need to observe movements basing on *x*-coordinate *vs*. time. Fig. 11 shows graphs of movements of some featured ridges during stick-to-slip transition. After the fingertip was slid, the most outside one, *i.e.* the (-6)-th, or the (-*N*)-th, ridge moves firstly. In sequence, inner ridges begin to slide, for example the (-4)-th,  $-3^{rd}$  ridges. At the time  $t_A = 1.9$  s, the 0-th ridge starts to move, whereas the 4-th, 5-th, or 6-th ridges have not slid yet. At time  $t_B = 2.3$  s, the 5-th ridge moves resulting all the ridges give the way, *i.e.* the gross sliding of entire soft fingertip on the surface happens after



Fig. 10. Images for processing: (a) Grayscale image. (b) Binary image with extracted edges. (c) Image with tracked points.



Fig. 11. The *x*-coordinates show the orders of movements of some represented tracked points.

this moment. Once again, experimental results confirm that the center zone of the contact surface is not the last one moving before gross slippage of the fingertip. Besides, we also observe fluctuations (vibrations) of the ridges during the transition. It was vibration on the skin of the fingertip featured incipient slip detections in some researches [5].

# V. DISCUSSION

Elaborating transition from sticking to slipping state of the soft fingertip has been considered important in dexterous manipulation. Role of frictional force acting on the contact surface is obviously undeniable in effort to model this transition. Obeying to those, we have described our work in proposing a model in which stick-to-slip transition of a soft fingertip was investigated with appearance of friction force. The 2-D model described in this paper is part of our ongoing research to perform complete model of sliding fingertip in manipulation and suggest control strategies. By dividing fingertip into virtual cantilevers, we were able to dig into the nature of slide of the fingertip via assessing orders of movement of distributed contact zones on the contact surface prior to the gross slide. Simulation results and experimental results showed that *center* zone is not the last one moved before gross slide of contact surface. There is a delay. As a result, if we can perceive the slide of the *center* zone by some means, decision of incipient slip of the fingertip will be able to assessed prior the actual one. This delay time, stated as  $t_d$ , differs when the sliding velocity of the fingertip changes. The higher the velocity is, the smaller the delay time is; and vice versa. It is 0.63 s, 0.45 s when velocity is 3.0 mm/s, 4.0 mm/s, respectively. In practical cases, such as grasping an object, the object is accelerated by gravity causing high relative velocity of sliding out of the fingertips. Therefore, it is required to observe the sliding at high velocity. Actually, although the exact detection time for robotics application is hard to perceive, it is sufficient for the sensors which can emulate measured response time in humans (65 - 85 ms [8]).

In this paper, this delayed time, however, was slightly different among simulation results, force/moment results, and visual results. In simulation, it happened earlier compared to experimental results. It is because of the fact that objective soft fingertip used in experiment has specific thickness compared to 2-D simulation model. This will be prevented in the future 3-D model. Besides, moment at which the *center* zone supposed to move is not similar between the micro sensor based result ( $t_A = 1.6$  s) and the visual method based result ( $t_A = 1.9$  s).

The results in our primary submission show that the central zone of the contact surface is not the last contact point right before the gross slip of the fingertip. This model, which is line-to-line contact, and its experimental results showed that the onset of sliding does take place firstly in the periphery. The difference comes from the fact that it does not happen firstly on the entire periphery, but the far parts of the contact line opposite to the direction of slide. It is also different to other theoretical hypotheses in the point that the adjacent area of the central zone of the contact surface is the last one adhering to the rigid surface right before the overt slide of the fingertip. This model actually is supported partly by some previous experiments working on sliding motion of a human fingertip with epidermal ridges ([9], or [10]). In those papers, if we take a look at how contact surfaces of human fingertips change during its sliding motion from stick to slip states (Fig. 10(a) in [9], and Fig. 6 in [10]), there are similarities to our results. We can assess that, along the line crossing contact area and parallel to direction of sliding, the far area is the first part to move; and this movement propagates toward central area. Finally, the last part to move before the overt slide is the one adjacent to the central area. This result is almost coincident with our conclusion in the paper, in which the sliding part originates from (-N)-th cantilever, spreading to (N)-*th* one; and the 5-*th* cantilever is the last one to slide (Fig. 6). As a result, our model is sufficient to interpret sliding motion of a fingertip with epidermal ridges.

#### VI. CONCLUSIONS AND FUTURE WORK

Consequently, this paper has reported our initial approach in modeling the dynamic sliding motion of an artificial soft fingertip. Experimental results validated partly the properness of the simulated model. In the future, a 3-dimension model with multiple states of contact of the soft fingertip will be conducted to fulfill the purpose of the research. In this model, role of friction torque acting around normal axis on the contact surface will be modeled by interactions among neighboring longitudinal and lateral linkage springs. At that time, decision of slip will be based not only on friction force, but also on friction torque. Moreover, in the 3-D model, appearance of material internal damping will be taken into account, thanks to potential of dynamic method. Control laws will be proposed as well.

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