The 2009 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 11-15, 2009 St. Louis, USA

Path Planning for Data Assimilation in Mobile Environmental Monitoring Systems

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***Abstract*— By combining a low-order model of forecast errors, the extended Kalman filter, and classical continuous optimization, we develop an integrated methodology for planning mobile sensor paths to sample continuous fields. Agent trajectories are developed that specifically take into account the fact that data collected will be used for near real-time assimilation with large predictive models. This aspect of the problem has significant implications because the trajectories generated are very different from those which do not take the assimilation step into account, and their performance in controlling error is notably better.**

I. INTRODUCTION

Modern atmospheric and oceanic circulation studies depend increasingly on autonomous fixed and mobile sensor agents for continuous sampling. Rapid communication of the resulting data provides a fundamental coupling of large-scale numerical models and experiments: agents that are able to take targeted measurements will position themselves, either individually or as a group, so as to best observe the physical phenomena of interest. Model errors can then be reduced through the data assimilation process, akin to the update step in traditional state estimation. If the model errors can be kept small, predictions will be more accurate.

Intelligent or adaptive mobile sampling toward this end can be posed in a quasi-static sense, and as a dynamic, online data-driven process. For instance, a group of vehicles tracking a gradient or a level contour constitutes a very dynamic system, and the model of the physical process in this case could be quite simple, involving perhaps only a continuity and monotonicity assumption, along with a length scale for the process. Indeed, underwater vehicles and coordinated groups of vehicles have been successfully deployed so as to track ocean fronts [6].

In work employing more detailed, numerically generated forecasts of the field, it is desirable to motivate the trajectories of such agents with the prior predictions; this is the quasi-static scenario. The agents can take measurements where expected future errors are significant, for example at an inflow boundary or in areas of high forecasted gradients. Until recently, the optimal placement of sensors for environmental monitoring was described primarily in a stationary sense, e.g., [1]; in the fields of system identification and control, sensor layout is a discipline in its own right. Work in optimal sensor trajectory planning and sampling in the ocean environment especially, however, has been growing rapidly in recent years. As described in Lermusiaux [13], F. Chavez planned an autonomous vehicle path in Monterey Bay so as to focus the data on gradients. Several additional examples of intelligent sampling in the ocean environment are given also in this reference. Yilmaz [18] has created a path-planning scheme for an autonomous underwater vehicle based on the predictions of a highly-refined ocean dynamics model, and Leonard *et al.* [15] have carried out an extensive set of full-scale experiments in Monterey Bay, demonstrating the effectiveness of gliders flying in synchronized loops. Objective analysis was used to quantify the information gained, based on an unstructured exponential form of the field error covariance. We note that the detailed discussion of practical issues in deployment of large-scale observation systems provided by [15] is extremely valuable to the community. Heaney *et al.* [9] similarly optimized a number of parameters defining fairly simple trajectories for many agents, in a complex ocean field. Such approaches are suitable for open-ocean operations as well as in harbor environments (see Figure 1). Overall, these schemes are computationally tractable for the time scales involved for directing the vehicles, and the focus is on simple trajectories for complex fields.

The objective in the present paper is to *explicitly take the assimilation step into account in path planning*; we develop the problem statement in the next section. In comparison with the above works, our method allows a complex trajectory and vehicle dynamics, with a somewhat simplified field variable model. Such a low-order modal model is appropriate for

Fig. 1. Autonomous marine vehicles are employed in Singapore Harbor for environmental sensing and modeling, through the Singapore-MIT CENSAM Program. Singapore Harbor presents an extremely challenging environment for such operations, with strong, variable currents and winds, and heavy shipping traffic.

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This work was supported by the Singapore-MIT Alliance for Research and Technology (SMART).

global weather systems and open-ocean circulation modeling, but may not be for coastal oceanic processes. The new procedure gives dramatic model error reductions in several examples of sampling a single field variable, while incurring a high but tangible computational cost.

II. Approach and Methods

Our key starting point is the fact that the data assimilation step, for the most part, has been developed independently of the placement or path design of mobile sensor networks. Yet today's sampling objective is often specifically to provide a correction to the state of a large model using the available sensors, in the sense of the Luenberger observer or, more popularly, variants on the Kalman filter. We argue that data assimilation should be an *integral part* of the sensor placement design. This depends closely on the growing ability of agents to communicate their measurements in near real-time, and we make the specific assumption here that the vehicles have unlimited communication bandwidth to a central processing agent who can coordinate the maneuvers through the optimization relative to the model. The assumption is certainly reasonable for slow-moving vehicles and for vehicles that have occasional surface access to wireless/satellite communications.

We note that there is a strong link for this integrated planning idea with the robotic map-based navigation community. Huang *et al.* [11] discuss the use of multi-step optimization in mobile agents, for the main purpose of refining maps of the environment - the explicit minimization of error covariance in the landmark locations mirrors our objective. Huang *et al.* reported only modest gains with respect to a greedy (one-step) optimization, and so the case for multi-step optimization for mapping is not strong.

A. Scaling of the Discretized Integrated Problem

For the purposes of this paper, we will focus on the case of vehicles moving in a plane, for example autonomous surface vehicles, with only one field quantity under consideration, for example sea surface temperature. The scale of this basic integrated data assimilation-planning problem has to include the vehicle states as well as the field variable error states and is roughly as follows:

1) There are at least two vehicle states, Cartesian X and Y say, for a single vehicle moving in the planar domain; dynamic equations enforce the speed and maneuvering constraints of the vehicle. Let the number of different vehicle state variables be M, and the number of agents be Z. Hence the states of all the agents are described with MZ elements.
2) There are typically two control input variables per agent of this type, e.g., forward speed and heading commands. For agents designed to operate at a given speed, a simpler vehicle model can be used, with only one control input (heading).
3) There are N field state variables, typically N amplitudes that are functions of time, each coupled with a known basis function on space $[X, Y]$.

Considering discretization of all the $N + ZM$ aggregate state variables into b quanta, then the discrete state space for the overall optimization problem is of size $O\left(b^{ZM} \times b^{N^2/2}\right)$. Assuming a Kalman approach for assimilation, the $N^2/2$ term here is the number of unique elements in the error covariance matrix of the field variable. Clearly the total is an unreasonable number of states, even for small b and N, exceeding easily the many millions of states that can be solved using optimization procedures today, for example employing value iteration, reinforcement learning, or approximate dynamic programming (e.g., [17]). Indeed, current large finite-element models for atmospheric or oceanographic processes routinely employ $N \approx O(10^7)$ states; spectral methods may have $O(100)$.

B. Rationale for a Continuous, Low-Order Approach

The first scaling difficulty above results from the discretization of states and controls into quanta. The state variables do not have to be discretized in the general case, however. Instead, we may consider a continuous-state approach, so that deterministic optimization tools can be employed; these are based on Pontryagin's Maximum Principle. A fully continuous gradient problem of size $ZM + N^2/2$ results.

With regard to the cost of data assimilation, the extended Kalman filter (EKF) is the optimal result of objective analysis, and hence is a natural choice for the task. The computational burden of the EKF, however, is well-known and problematic, whether it is used only for "open-loop" data assimilation or as part of the integrated optimization problem we describe here. The sheer scale of the $N^2/2$ elements involved in the classical evolution of error covariance has led to the successful ensemble Kalman filter (EnKF, reviewed in [5]), among other treatments. But the EKF remains the "gold standard" in numerical weather prediction [12], and in our developments below we prefer to keep with the classical formulation of error covariance rate $\dot{P}$. This is for the main reason that the gradient optimization method requires us to evolve the optimization adjoint backwards in time - no such operation can be performed if the covariance is evolved using forward-only time simulations as in the EnKF.

An obvious avenue that can connect our approach with real applications is the spectral formulation. Global numerical weather prediction (NWP) models with spectral discretization in the horizontal plane are one good example (e.g., [12]); spectral models are, however, less common in oceanic modeling due to complex boundaries and topographies in coastal regions. Low-order models have been constructed also with the empirical orthogonal function (EOF) method by Zhang [19] for hurricane prediction. The EOF analysis was applied in an ocean setting by Fukumori and Malanotte-Rizzoli [7], to a three-dimensional jet model, to represent the variability with low order. They found that about ninety percent of the variability could be accounted for in five EOF modes, and these modes were used effectively in the construction of an accurate low-dimension filter; more recent work in this area is reported by [4], with $O(100)$ modes. In general, such analysis shows that some of the error modes can be growing while others are decaying, hence providing clear directions for the targeted placement of sensors. Further, although apparent coupling of such experimentally derived modes may or may not exist, they also are strongly driven by white noise processes that encompass environmental inputs as well as modeling errors.

C. The Filtering Aspect

Following the above rationale, we employ the fully continuous-time extended Kalman filter, wherein the observation of field states is determined by the vehicle pose within the environment. The vehicle states are governed by $\dot{x}_v = f_v(x_v, u, t)$, where x_v is the aggregate state vector for all the agents, of size MZ. u is the corresponding aggregate control vector, and t is time. For the purposes of the optimization, the nominal field state variables (whether comprising one or more physical quantities) are forecasted as $\hat{x}_f$, subject to dynamics $\dot{\hat{x}}_f = f_f(\hat{x}_f)$. We assume that the state of the vehicle is fully known, either through competent sensors or through a separate estimation process. The $N \times N$ field variable error covariance propagates according to the standard equations [8]:

$$\begin{aligned} \dot{P} &= F(\hat{x}_f, t) P + PF(\hat{x}_f, t)^T + Q(\hat{x}_f, x_v, t) - \\ & \quad PH^T(x_v, t) R^{-1}(\hat{x}_f, x_v, t) H(x_v, t) P, \end{aligned}$$

where $\hat{x}_f$ is the forecasted field variable as a function of X, Y, and t, F is the gradient of $f_f()$ with $\hat{x}_f$, H is the gradient of the observation with $\hat{x}_f$, encoding the mode shapes, and Q is the process noise. Thus H - a direct function of the agents' states - plays a critical role in modulating the covariance. We allow that the process noise Q and the sensor noise R intensities can depend on any of the agent or field state elements, although in most practical situations they do not. For notational convenience, we assign the content of matrix P to a column vector x_P, that has length $\approx N^2/2$.

Employing the EKF, the field state would normally be updated with the measurements according to

$$\begin{aligned} \dot{\hat{x}}_f &= f(\hat{x}_f, t) + \\ & \quad PH^T(x_v, t) R^{-1}(\hat{x}_f, x_v, t)(z - h(\hat{x}_f, x_v, t)), \end{aligned}$$

but our path-planning method in fact does not require this or any explicit estimate of the actual states. Further, the use of the EKF in the optimization step does <u>not</u> imply that the EKF has to be used for the actual data assimilation.

D. Optimization

Different practitioners of optimization will choose different methods, and there is no particular advantage to our current approach, except that the method is continuous in the state variables and so accommodates nonlinear, continuous behaviors quite naturally. These may be useful, for example, in capturing maneuvering constraints, current effects, and nonlinear interaction of spectral modes. We employ the gradient method [3], augmented with a simulated annealing perturbation [14] to enhance robustness against local minima. Between simulated annealing perturbations to the control, we take advantage of an adaptive gain, e.g., [10]. We focus for the present on the unconstrained problem with fixed terminal time, because of its simplicity. Beginning with initial state $x(0)$ and a trial control trajectory $u(t)$, for the gradient part we iteratively perform the following loop:

- Propagate $\dot{x} = f[x, u, t]$.
- Evaluate $\lambda(T) = \partial\Psi[x(T)]/\partial x$.
- Sweep backward in time: $\dot{\lambda} = -f_x(x, t)\lambda$.
- Modify the control: $\delta u = -K\left[f_u^T[x, t]\lambda + l_u^T\right]$.

Here the aggregate state is $x = [x_v, x_P]^T$, and f is comprised of two parts: agent dynamics $f_v()$, and the state-space equivalent of the $\dot{P}$ error covariance equation above. The total state thus has size $\approx ZM + N^2/2$. $\Psi(x)$ is the cost of the terminal state at fixed time T, λ is the adjoint, f_x and f_u are the derivatives of f with the aggregate state and the control, respectively, and l_x and l_u are the derivatives of the cost integrand $l(x, u, t)$; the total cost is defined as

$$J = \Psi(x(T)) + \int_0^T l(x, u, t)dt.$$

We then aim to minimize the integral trace of P through choosing control action u. This is possible because the optimization explicitly includes both the error covariance evolution and update steps; they are posed in continuous time. Lack of hard constraints in the gradient optimization is a casualty of the continuous-time approach, although there do exist some methods for constrained control action and constrained states (e.g., [3]). In practical terms, optimization that neglects saturation in the control is often still useful, because a feedback system will often be able to account for their effects. Alternatively, modern constrained optimization tools may be applied to this problem [2].

E. Remarks on the Integrated Process

The path is laid out based only on the forecast, without measurements; it is completely deterministic. The state vector comprises a portion that captures the physical state of the agents (speed, heading, position, etc.), and a portion that is the error covariance of the field variable. On the filter side, process noise on the agent states has the interpretation of physical disturbances such as wind or ocean waves, while process noise on the field states provides uncertainty in the evolution of the errors. Sensor noise appears only in the (small) matrix R.

Computational Cost. We envision that the proposed algorithm could be used as part of a continuing simulation and update strategy, in the manner of NWP, and, more recently, oceanographic modeling. Computational cost is therefore an important consideration. Leonard *et al.* [15] used quite simple structures (or collectives) of underwater vehicles, and discuss near-optimal solutions, found by parameterizing collective motions on so-called super-ellipses. In contrast, Yilmaz [18] solves an unstructured problem using mixed integer programming - the costs are significantly higher. The current proposed method is clearly even more expensive. Nonetheless, while an argument can be made that optimization should be straightforward and computationally cheap, at the same time massive resources are regularly brought to bear on the actual forecasts. If a more difficult optimization problem brings additional value from the models and from the field assets, then its effort should be well rewarded.

The computational effort here includes propagating forward and backward the states and co-states for the field variable error modes; for coupled modes with a single field quantity, the cost goes with N^3, because of the matrix operations in the $\dot{P}$ equation. The number of iterations for convergence in the optimization does not depend strongly on the number of agents Z. Instead, it depends on the tuning parameters of both the gradient method (e.g., the adaptive gain dilation and contraction rates), and the simulated annealing (e.g., the maximum probability of a non-improving step, and the schedule for the temperature parameter). Overall, for the example given below, we find practical convergence within several thousand iterations, and often within one hundred.

Physical Agent Model. The continuous-time formulation of system dynamics is well-suited to capturing the physical behavior of dynamic vehicles. This fact is important for vehicles moving at significant speeds, where fluid lift and inertial effects may be pronounced. On the other hand, when the length scales of the mission are very large compared to the vehicle's maneuvering capabilities, one may simply eliminate the higher derivatives from the state equations and increase the time step. We do not take into account uncertain vehicle disturbances (e.g., currents or wind) in our examples; known, time-varying disturbances are easily incorporated into the dynamic equations, however. Needless to say, for slowly moving vehicles, strong disturbances, known or unknown, may make efficient observation of the field impossible.

Energy Budget. All users of marine autonomous systems are aware of the limitations on mission duration that are posed by finite energy storage. Hence any optimization routine has to include some account for efficient usage of these resources. An integral cost is very natural in this regard, because the energy consumed by streamlined vehicles often scales with the integral of the speed cubed - a drag force that scales with the square of speed, times the speed. Cubic functions are smooth enough to be incorporated into continuous-time optimization problems.

Multiple Agents. Multiple vehicles do not substantially change the cost or size of the filter, because the major effort is spent in propagating error covariance of the field. Close proximity of vehicles is penalized through a simple added term in the cost function, for example of the form $[(x_i - x_j)^2 + (y_i - y_j)^2]^{-1}$. Heterogeneous vehicles pose no particular difficulties in the formulation, except that the time step for the overall process has to be fine enough to resolve the fastest elements. These elements include sensors and actuators insofar as they impact the positioning and the sampling. In all cases, we assume the low-level positioning problem to be solved by vehicle controllers and a reliable navigation system, e.g., a long-baseline acoustic net for underwater vehicles, or GPS for surface of flying vehicles. The open-loop trajectories that are created by our proposed algorithm are dynamically constrained by the equations of motion, and form a basis for the linearized control or gain scheduling that is common in many types of vehicles.

Other Costs. Additional components in the construction of the cost function, in the spirit of Heaney *et al.* [9] can be included in our formulation; a notable example would be the variance of the field variable prediction along the path.

III. EXAMPLE

In order to illustrate the technique described here, trajectories for a pair of ocean surface vehicles sampling a wave field are considered, for the case of ten complex modes. This problem scale (232 continuous states) is easily managed on a personal computer, and the number of modes is on a par with EOF analysis. There is assumed to exist a prediction of the wave field, created in accordance with the standard directional spectra [16]. The goal is that the vehicles will maneuver through the wave field in such a way as to optimally track each of the component amplitudes. In the larger context of ocean data assimilation, these waves represent the dominant EOF's or spectral modes in the error of a measurable field.

The results of our optimization with assimilation are compared explicitly with related straight-line trajectories that are also the initial guesses in the gradient iteration, and with trajectories resulting from a cost function that seeks extrema, uniformly weighted over the time window. Many mobile sampling cases in the oceanic and atmospheric sciences use this metric today: the vehicle takes data where the aggregate predicted errors are highest.

In this example, the waves are made up of components having random heading; hence this sea state has no dominant wave direction, and no basis exists up front for choosing the best directions of straight-line paths. We consider three main situations which represent the basic choices one would have in designing good straight-line paths: the two vehicles starting at the same point but with opposite headings (east and west), with right-angle headings (north and east), and with the same heading (east and east). In this last case, the covariance provided by the straight lines is very poor because the two vehicle paths are the same - very little if any information is gained by the second vehicle. They also turn out to be the same paths in the extrema case (although no longer straight), because we did not include in the cost function any penalty for proximity. The cost function integrand has roughly equal parts quadratic penalty on the control channels, and penalty on the field error covariance trace; there is no terminal cost.

We use a time window of fifty seconds, with fourth-order Runge-Kutta integration and time step 0.1 seconds. The ten wave components have a peak frequency of $0.7 rad/s$, so that the time window covers the passage of at least five maxima - this is a very dynamic situation. The vehicle dynamic models are appropriate for a high-speed unmanned surface craft. The sensor noise intensity matrix R is diagonal, and has value $0.01 m^2$ for each of the two craft - the typical aggregate wave elevation is $0.5 m$. Each wave modal amplitude is

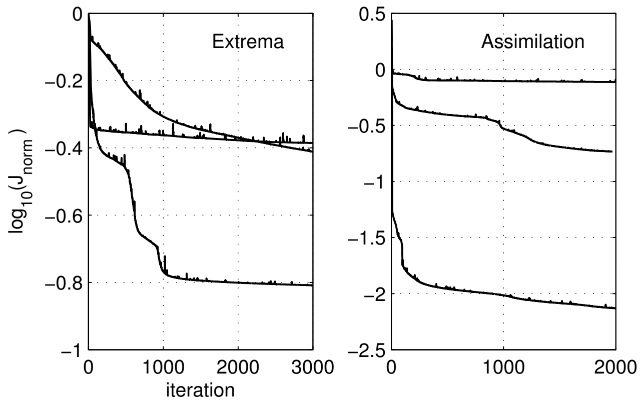

Fig. 2. Convergence behavior for the example, with extrema-seeking (left) and with assimilation (right). The cost shown is normalized with the cost on the initial trial: a pair of straight lines at constant speed. Note that convergence data shown in the extrema-seeking case is inconsistent with the Table 1, because the extrema cost function is not related to $\int tr(P)dt$.

TABLE I
$\int tr(P)dt$ PERFORMANCE, TWO VEHICLES;
MIN DIMENSIONAL VALUE IS $393 \simeq 20^2$.

Path Type	East-West I.C., Relative Value	North-East I.C., Relative Value	East-East I.C., Relative Value
Straight	1.48	5.43	163
Extrema	10.8	3.91	60.6
Assimilation	1.13	1.00	1.06

modeled as an undamped oscillator with natural frequency $0.1 rad/s$, driven by uncorrelated process noise of intensity $0.01 m^2/s^4$. This corresponds to a rapid loss of confidence in the amplitudes, with rather poor measurements. Note that in this example we do not account for variations in frequency, wavenumber, phase, or direction of any given mode, although such extensions are quite reasonable with the method.

Figure 2 shows the convergence behavior of the extrema and assimilation optimizations. The extrema-seeking cases show modest improvement over the straight-line initial trajectories; this means that extrema are already covered to some extent on the initial paths. The best improvement in the extrema cost function, as provided by the optimization, is by about a factor of six. The assimilation cases penalize the error covariance directly and are consistent with Table 1. These curves show improvement over the initial trajectories by about 24% (east-west initial condition), 81% (north-east), and a factor of over one hundred (east-east).

In Table 1, the assimilation runs give better covariance results than the extrema cases for all three initial conditions, by a factor of 3.9 or more. Note that one extrema-seeking case (east-west) controls the covariance worse than its straight-line version; this is not unexpected since the covariance is not part of the cost function when extrema are tracked.

Interestingly, the east-west straight-line course provides error control only slightly worse than the assimilation case. We argue that this success in a straight-line course is merely a fortunate coincidence - the confused wave field gives

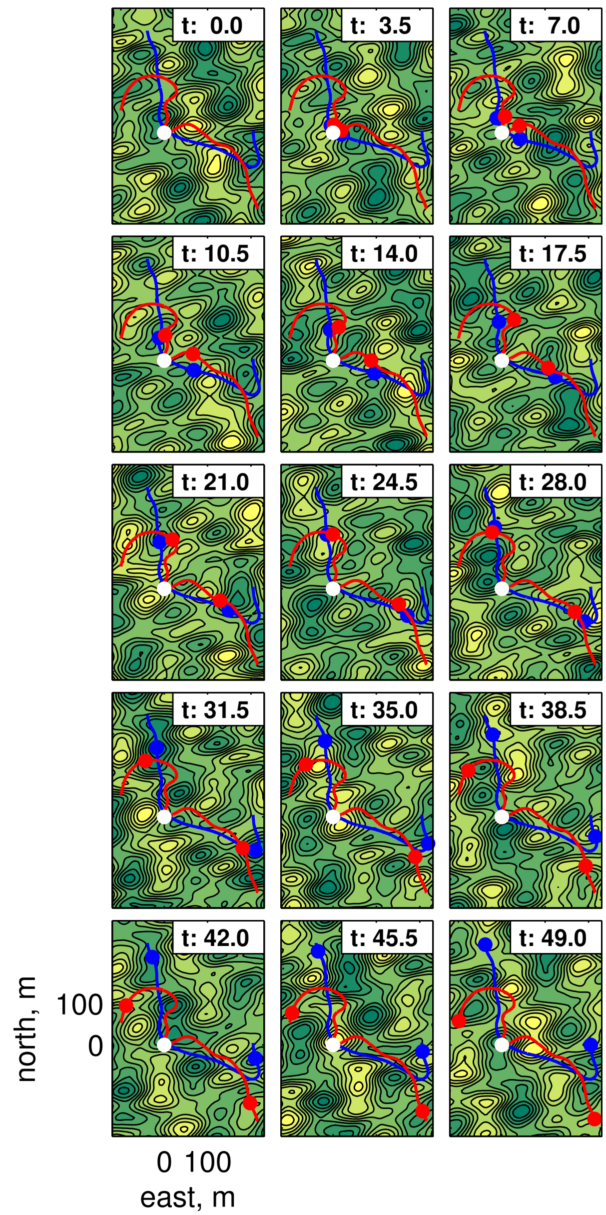

Fig. 3. Optimized trajectories overlaid on the wave field with two vehicles, having initial headings of zero and ninety degrees. Contours give the wave elevation, two red lines give the paths accounting for assimilation, and two blue lines give paths seeking extrema; the starting point at the origin is shown with a white dot.

the user no clear insight on what directions should be pursued to minimize the errors. In additional simulations not shown here, we did find that when the waves are more unidirectional, straight paths normal to the crests are very effective, as expected.

The explicit optimized paths for the north-east initial condition are given in Figure 3, along with the predicted wave field through time. As expected, the extrema paths shown generally pass through the largest peaks and troughs of the predicted waves. The assimilation trajectories take different directions and with an accentuated curvature, possessing no intuitive correlation with the wave field. This apparent

TABLE II
$\int tr(P)dt$ PERFORMANCE, TEN VEHICLES; MIN DIMENSIONAL VALUE IS $13 \simeq 3.6^2$.

Path Type	All-East I.C., Relative Value	Spread I.C., Relative Value
Straight	4635	1.17
Extrema	1120	1.62
Assimilation	1.14	1.00

TABLE III
$\int tr(P)dt$ PERFORMANCE, ONE VEHICLE; MIN DIMENSIONAL VALUE IS $2920 \simeq 54^2$.

Path Type	East I.C., Relative Value
Straight	21.2
Extrema	11.2
Assimilation	1.00

disorder occurs because the path is designed to track each modal amplitude according to the forecast, and this may or may not be connected with the local wave elevation.

Performance summary results from some other runs are given in Tables 2 and 3, for ten vehicles and one vehicle, respectively. In the ten-vehicle case, little is gained by optimization according to any rule, simply because there is redundancy provided by so many sensors. The "spread" initial condition - which has the vehicles starting from a single point but evenly spaced in heading - is particularly effective, as expected. On the other hand, in the single-vehicle case, the optimization brings a full order of magnitude improvement in cost compared with the single-vehicle, extrema solution. The dimensional cost for one vehicle, however, is over two hundred times that of ten vehicles, and more than seven times that of two vehicles. With or without optimization, having more agents is desirable. The computational requirements for all these cases are roughly the same, because the number of states is dominated by the covariance matrix (210 states) - each vehicle adds only six states, and the number of iterations is very similar. On a desktop PC, one such optimization, whether for an extrema or assimilation cost function, takes several hours.

IV. CONCLUSION

We posed and solved a mobile sensor planning problem that takes into account specifically the data assimilation step, so as to minimize error growth in the constituent modes of a distributed field variable. This is a departure from many approaches in use today for such sampling, which target the areas of highest predicted error. While computationally expensive, the new method is practical today for relevant small and mid-sized problems. Given the significant reductions in error that we observed in simple examples, however, a broad argument could be made for committing substantially more resources to optimization and to the creation of accurate low-order, spectral models. This decision is an application-specific tradeoff involving the phenomena under study, the agents available and their capabilities relative to the physical domain, and computing resources.

A number of other questions are raised by this investigation into integrated planning. There are strong connections to be explored between the problems of autonomous mapping, and optimized ocean or atmospheric sampling. Communication and navigation constraints on the agents should be taken into account. Perhaps most importantly, approaches for decoupling error modes should be considered so that the computational costs can be reduced, enabling solution of problems with $> O(100)$ modes.

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