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Reduced Order Modeling Using Genetic-Fuzzy algorithm

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***Abstract*—many *high*-order systems have a large state space. Such systems need to additional computation time for complex calculation to find the output response. Traditionally, Iteration methods have been applied to solve this problem. In this paper advantages of stability equation method derived by Parmer, [1], and the error minimization technique used in genetic-fuzzy algorithm have been combined to propose a new method for order reduction of linear dynamic systems described via state-space models. Genetic part has been used in this formulation to find the optimal solution(s) to minimize the objective function "J" that depends on the error term between the original output and the desired or reduced output. Fuzzy sets have been used to determine the step size action (point crossover or multiple crossover) depending upon fuzzy rules based on the current and previous error terms. An example of reduced order modeling from power systems is presented to illustrate the algorithm.**

***Keywords*—linear continuous-time system, genetic algorithm, fuzzy sets, model reduction.**

I. INTRODUCTION

THIS paper presents a novel order reduction approach that combines genetic algorithm(GA) with fuzzy set theory to optimally reduce the high order of the state space and output equations for large-scale systems. The approximation of high order models is an important problem in system theory. The use of reduced order model makes it easier to implement analysis, simulation and control system designs of such systems. In the literature, numerous methods for order-reduction of linear continuous-time systems in time domain as well as frequency domain are available. For example, extension of single-input single-output (SISO) methods to reduce multi-input multi-output (MIMO) system has been proposed in [1]-[4]. We note that each of these methods has both advantages and disadvantages when tried on a particular system model. Further, no approach universally gives the best results for all systems. Genetic algorithms (**GA**s) are optimization algorithms based on principles of natural evolution. There are five factors influencing the performance of a **GA**: 1) the method of representing solutions (how solution are encoded as chromosomes); 2) the initial population of solutions (the group of chromosomes created at the beginning of the evaluation process); 3) the evaluation function (the metric for measuring the fitness of a chromosome); 4) the genetic operators (e.g. the three basic operators, selection, crossover and mutation, for choosing chromosomes for a new generation, creating new chromosomes by combining existing chromosome and producing a new chromosome by altering an existing chromosome); and, 5) the control parameters (e.g. the size of the population of chromosomes and rates of crossover and mutation). The first three factors and, to some extent, the fourth factor, are problem dependent. For instance, in a problem where there is prior knowledge of reasonably fit solutions, they can be used to form the initial population to speed up the optimization process. The fifth factor, the **GA** control parameters, tends to be much less problem dependent and there is more scope for genetic work aimed to improve the performance of a **GA** by manipulating its control parameters [5] - [7]. We note that optimal mutation and crossover rates can be selected to provide superior model reduction compared to traditional iteration approaches.

Fuzzy Logic was introduced in 1965, by Lotfi Zadeh, as a mathematical tool for dealing with uncertainty. It offers a soft computing paradigm that realizes an important concept of computing with words'. Further, it provides a technique to deal with imprecision and information granularity. The fuzzy set theory provides a mechanism for representing linguistic constructs such as "many," "low," "medium," "often," "few." In general, the fuzzy logic offers an inference structure that emulates appropriate human reasoning capabilities. On the contrary, the traditional binary set theory describes crisp events, events that either do or do not occur. It uses probability theory to present the like hood if an event will occur, measuring the chance with which a given event is expected to occur. The theory of fuzzy logic is based upon the notion of relative graded membership and similar to the functions of cognitive processes in the brain [8, 9].

II. PROBLEM STATEMENT

We considered a general MIMO linear system of order 'n' with state equation given as:

$$\dot{x}(t) = A\, x(t) + B\, u(t) \qquad (1)$$

where *A, B are* represented dynamics and input matrix. The corresponding output equation (assume free output response):

$$y(t)=C\,x(t) \tag{2}$$

where C is the output matrix. This system is assumed to be stable with minimal realization given by *(A, B, C)*. The system transfer function given as:

$$G(s) = \frac{a_{n-1}s^{n-1}+\cdots+a_1s^1+a_0}{s^n+b_{n-1}s^{n-1}+\cdots+b_1s^1+b_0} \tag{3}$$

where's' is the Laplace transform variable, and a's and b's are constant polynomial coefficients. Genetic-fuzzy algorithm aims to find a stable reduced order model of dimension r (where $r < n$) with minimal realization matrix denoted by *(F, G, H)*.Therefore, the reduced order model linear system can be described as:

$$\dot{z}(t) = F\,z(t) + G\,u(t) \tag{4}$$

where F, and G are the reduced dynamics and input matrix. Therefore, the reduced output state space linear system equation is given as:

$$y_r(t) = H\,z(t) \tag{5}$$

where H is the reduced order linear system model output matrix. The reduced linear system model transfer function is given as:

$$G_r(s) = \frac{\alpha_{n-1}s^{r-1}+\cdots+\alpha_1s^1+\alpha_0}{s^r+d_{r-1}s^{r-1}+\cdots+d_1s^1+d_0} \tag{6}$$

where all α's, and d's coefficients are constants. Such that the error between the estimated and actual outputs is minimized using the following equation:

$$e\,(t) = y(t)-y_r(t) \tag{7}$$

where $y\,(t)$ is the actual output, and $y_r(t)$ is desired output for the reduced model. There are three methods to calculate the error norm, the first methods is given as:

$$J_1 = \int_0^\infty e^T(\tau)\,Qe(\tau)d\tau \tag{8}$$

where Q is positive semi-definite matrix. The second method is given as:

$$J_2 = \int_0^\infty \parallel w^T e(\tau) \parallel d\tau \tag{9}$$

where W is column vector assigning relative importance to the outputs. The third method defined as Integrate Square Error (ISE) objective index is given as:

$$J_{ISE} = \int_0^\infty [y(t) - yr(t)]^2 dt \tag{10}$$

Let *(F,G,H)* be the realization of the optimal reduced order model in (4) and (5). By differentiating the J_1 with respect to *F, G,* and H and setting the results equal to zero, the optimal reduced-order model is found by Parmer [1]:

$$\mathrm{F = \Gamma A \Omega} \tag{11}$$

$$\mathrm{G = \Gamma B} \tag{12}$$

$$\mathrm{H = C\,\Omega} \tag{13}$$

where the following matrices are defined:

$$\Gamma = -\mathrm{P}_2^{-1}\ \ \mathrm{P}_{12}^{\mathrm{T}} \tag{14.a}$$

$$\mathrm{P} = \begin{bmatrix} \mathrm{P}_1 & \mathrm{P}_{12} \\ \mathrm{P}_{12} & \mathrm{P}_2 \end{bmatrix} \tag{14.b}$$

$$\Omega = \mathrm{R}_{12}\mathrm{R}_2^{-1} \tag{15}$$

where P is the Lyapunov stability matrix. Further, P matrix is always asymmetric (i.e. $P = P^T$).The matrices P_2^{-1}, $P_{12}^T, R_{12}, R_2^{-1}$ satisfy the following matrix equations, which are called the *projection equations* [1], and [5]:

$$\mathrm{A^T P_{12} F + P_{12} F - C^T Q H = 0} \tag{16}$$

$$\mathrm{F^T P_2 + P_2 F + H^T Q H = 0} \tag{17}$$

$$\mathrm{A R_{12} + R_{12} F^T + B V G^T = 0} \tag{18}$$

$$\mathrm{F R_2 + R_2 F^T + G V G^T = 0} \tag{19}$$

In general, because of their nonlinearity these matrix equations must be solved iteratively. The resulting difficulties in solving the projection equations provide motivation for the use of genetic algorithm to optimize the reduce order model.

III. GENETIC ALGORITHM

The parameters, needed to be searched by GA, are the range of values of two matrices P, and R. The initial values of the individual elements are generated using a uniform random number generator in the range [-R, R]. we have used real number gene code. The GA is proposed as searching process based on the laws of natural selection and genetics. The usual genetic algorithm steps are described as follows:

1. Randomly generate an initial population $\mathbf{P^0} = (a_1^0, \ldots a_\lambda^0)$.
2. Compute the fitness $f(a_i^t)$ of each chromosome a_i^t in the current population P^t. depend on eq.10.
3. Create new chromosome P'^t from mating of current chromosomes, by applying mutation and recombination as the parent chromosomes mate. Change type of cross over rate technique and the values of genetic parameters depend on fuzzy rules.

4. Delete appropriate numbers from the existing population to make room for new chromosomes. Compute the fitness of f ($a_i^{`t}$), and insert these into population.
5. Increment current number of generation, if not (end-test) go to step 3, or else stop and return the best chromosome.

The GA cycle is graphically shown in Figure (1). In the figure, the phenotype represents the coding which is used to represent the chromosomes. The mating pool represents the summing of chromosomes that represents the solution of the problem. Offspring represents new chromosomes produced by mutation and crossover operation.

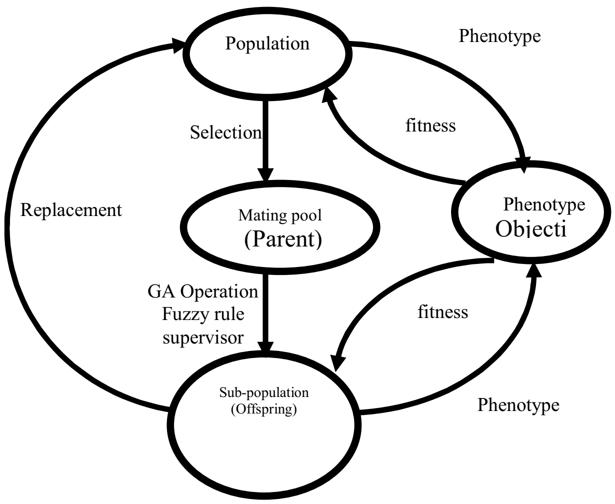

Figure 1. Implementation cycle of the GA

I. FUZZY RULES

A well-known problem with the GA is that the search ability of the ordinary GA is not always optimal especially in the early and final search stages, because genetic parameters (crossover rate, mutation rate and so on) are fixed [9]. In order to overcome this problem, we propose Fuzzy Adaptive Search method for Genetic Algorithm (FASGA) as the modified search methods. The proposed method is able to realize an efficient search by describing fuzzy rules to tune genetic parameters according to the search stage. We claim that FASGA with point crossover and multiple crossovers can reach the optimal point within the smallest time and the lowest error term. This method has the ability to search high-quality solutions through tuning the crossover, and mutation rate by fuzzy reasoning. Figures (2) and (3) show the fuzzy rules which have been considered to determine the value of crossover rate and mutation rate along the generation process.

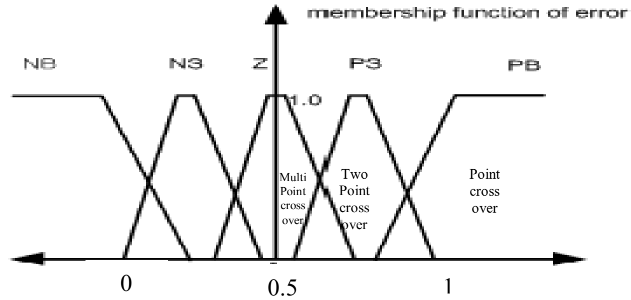

Figure 2. A trapezoidal type membership functions of error.

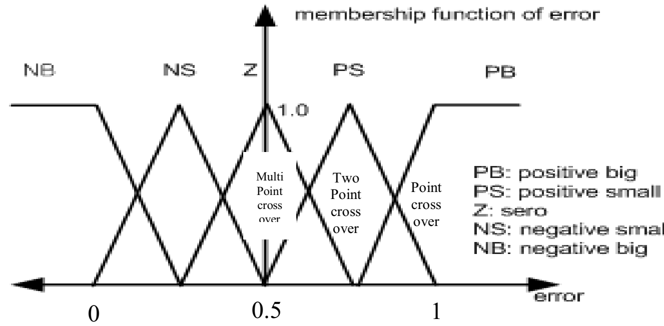

Figure 3. A triangular type membership functions of error with type of crossover rate techniques.

II. NUMERICAL EXAMPLE

A numerical example is chosen from literature for comparison of the resulting low order system (LOS) with the original high order system (HOS). An ISE error index between the transient parts of original HOS response and the proposed LOSresponse is calculated to measure the goodness of the LOS (i.e. the smaller the ISE, the closer is (Gr (s) to Gn(s)). The ISE criterion is given as:

$$J_{ISE} = \int_0^{\infty} [y(t) - y_r(t)]^2 dt \tag{20}$$

where, $y(t)$ and $y_r(t)$ are the unit step responses of original ($G_n(s)$) and reduced ($G_r(s)$) order systems. A table of twenty five fuzzy rules is used which was described in table I.

TABLE I. Fuzzy Rules for error minimization

Previous error/ Current error	NB	NS	Z	PS	PB
NB	PB	NS	Z	Ns	NB
NS	NS	NS	Z	Ns	NB
Z	Z	Z	Z	Z	Z
PS	NS	Ns	Z	PS	PB
PB	NB	NB	Z	PB	PB

Example 1. Consider a fourth-order system described by the following transfer function, [10]-[13]:

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}$$

By using genetic algorithm to minimize the objective function 'J', after 27 generations, the second order model linear system transfer function $G_r(s)$ was found as:

$$G_2(s) = \frac{0744257s + 0699157}{s^2 + 1.4577s + 0.6997}$$

Then, by using fuzzy genetic algorithm to optimize the index objective function depend on the fuzzy rules. If the value of previous error is greater than the current error value, SO, the error will be increased. The algorithm is changed its direction to another new direction with multiple crossover and mutation technique. Otherwise, (i.e. the current error with respect to previous error decrease) the algorithm is decreased its step size with point cross over and mutation techniques. As a result, the algorithm was aimed to reduced order model linear system by 12 generation, the reduced order model transfer function is given as:

$$G_{F2}(s) = \frac{0.73245s + 0.68897}{s^2 + 1.4674s + 0.6997}$$

Figures (4) presents convergence diagrams of the objective function '*J*' without and with fuzzy rules. Figure (5) presents step response of the original transfer function of the linear system ($G_4(s)$), reduced order model system transfer function ($G_2(s)$) ,which is found by using only genetic algorithm ,Finally, reduced order model linear system transfer function ($G_{F2}(s)$),which is found by using genetic fuzzy rules algorithm, respectively. Table II presents the results of our algorithm versus the previous algorithms results that are applied for the same problem.

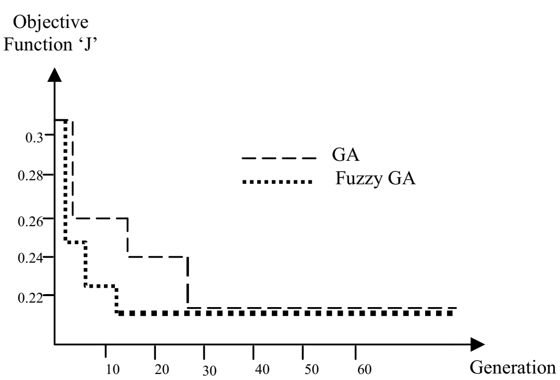

Figure. 4. Convergence of objective function 'J' with GA, and Fuzzy GA.

TABLE II. Comparison reduced order system models results

Method of Order reduction	Reduced model	ISE
Proposed algorithm	$\frac{0.73245S + 0.68897}{S^2 + 1.4677S + 0.6997}$	1.44454×10^{-3}
Parmar[1]	$\frac{0.74425S + 0.66991}{S^2 + 1.4577S + 0.6997}$	1.64454×10^{-3}
Chen et at.[10]	$\frac{0.6997(S + 1)}{S^2 + 1.46771S + 0.6997}$	2.66553×10^{-3}
Davison[11]	$\frac{-S^2 + 2}{S^2 + 3S + 2}$	220.237×10^{-3}
Prasad and Pal[12]	$\frac{S + 34.2465}{S^2 + 239.8082S + 34.2465}$	1.534272
Safonov and Chiang[13]	$\frac{S + 5.403}{S^2 + 8.431S + 4.513}$	4.51613×10^{-2}

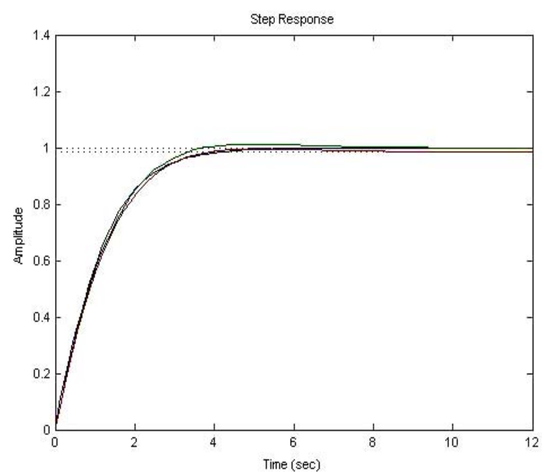

Figure. 5. Step responses of $G_4(s)$, $G_2(s)$, and $G_{F2}(s)$.

CONCLUSIONS

As algorithm which combines the advantages of the stability equation method and the error minimization by fuzzy genetic algorithm to derive stable reduced order models for linear time invariant dynamic system was presented in this paper. The fuzzy rules used to determine the crossover rate and the mutation rate of genetic algorithm were presented in Table I. We caution that although the chances of GA giving local optimal solution are very low, but sometimes a suboptimal solution to the problem may be arrived. Further, for different problems, it is possible that the same probability of crossover and mutation do not generate the best solution (as shown in Figures 4 and 5); however, these can always be changed according to the situation.

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