Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Implied Historical Federal Reserve Bank Behavior Under Uncertainty

Muhittin Yilmaz*, Selahattin Ozcelik† and Nuri Yilmazer*

*Frank H. Dotterweich College of Engineering
Department of Electrical Engineering and Computer Science
Texas A&M University-Kingsville, Kingsville, Texas 78363-8202
Emails: muhittin.yilmaz@tamuk.edu and kfny000@tamuk.edu

†Frank H. Dotterweich College of Engineering
Department of Mechanical and Industrial Engineering
Texas A&M University-Kingsville, Kingsville, Texas 78363-8202
Email: kfso000@tamuk.edu

***Abstract*—This paper presents the implied United States Federal Reserve Bank (FED) policy behavior under multiplicative model and shock uncertainty, defined through performance objectives, cases by using historical data. Robust system theory frameworks are used to empirically study the characteristics of the FED short-term interest rate-inflation dynamics under different circumstances by using a single-input single-output model. The main result of this study demonstrates that the historical FED actions was conservative under model and shock uncertainty.**

***Index Terms*—Inflation dynamics, uncertainty, US FED**

I. Introduction

Economic dynamics and optimization have been a central concern for the economic policy makers, especially for central bankers and governments, to stabilize the overall behavior of the economy under uncertainty. It is pointed out that although policy makers acknowledge model uncertainty, the type of uncertainty can not be characterized by subjective or objective probability distributions, which the policy makers have limited information [1], [2]. Moreover, the term "risk" is used to refer to the outcomes for known probabilities and the term "uncertainty" is used for the outcomes for unknown probabilities [3]. Furthermore, it has been proposed that the policy makers should act conservatively, i.e., make smaller changes than the ones suggested by the nominal model, during the economic stabilization problems involving uncertainty [4]. Mainly, there are two major frameworks to address model uncertainty in economic policy analysis; Bayesian analysis and robust systems framework.

Bayesian analysis has widely been used to optimize the monetary policies. This approach uses an assigned a-priori probability distribution for each model parameter and optimizes the monetary policy by minimizing the expected loss against these probability distributions. However, the prior probability distribution assignment to each model parameter is difficult and becomes intractable for systems with large number of parameters. Moreover, the parametric nature of this approach is prone to more difficult cases under different regimes of the economic dynamics. Advanced setups such as varying parameters and learning to update posteriors attempt to mitigate the shortcomings of this approach [5], [6]. The Bayesian analysis for macroeconomic stabilization has historically shown that the monetary policy should be less aggressive when model uncertainty exists.

Robust systems framework presents a systematic approach to handle uncertainty in a worst-case scenario, offering an alternative to Bayesian analysis. Economic policy analysis has been concurrently developed with available mathematical resources. When fixed-model optimal control framework proved to be inefficient to explain real-life paradigms due to its deficiencies related to model variations and uncertainty, robust systems framework including model uncertainty and performance objectives appeared to be the solution starting in the early 80s, with Zames [7], [8], [9]. However, economic dynamics analysis by using robust systems framework have recently become the focus of attention. The robustness of the monetary decision dynamics under model uncertainty has been studied in different forms. Since the fixed models did not account for the uncertainty, there was a practice of averaging the policy rules for a number of plausible misspecified models [10]. Although this study did not include formal robust system techniques, it clearly emphasized the concern for robustness under model uncertainty. Robust systems framework has been utilized by a number of studies [11], [12], [13], [14], [15]. One of the studies [14] used the the structured model uncertainty in robust systems H_∞ and ℓ_1 frameworks but used the explicit Taylor rule[1] as the feedback rule to analyze the characteristics of the corresponding policy rules. .Their results showed that the robust system optimization yields more aggressive policy rules

[1] The generalized Taylor rule is an estimation of the policy rule over years and is given as $i_t = g_\pi \, \overline{\pi_t} + g_y y_t$, where i_t is the nominal interest rate, $\overline{\pi_t}$ is the average of last four quarter inflation rates, y_t is the output gap, and constants $g_\pi = 1.5$ and $g_y = 0.5$

978-1-4244-2794-9/09/$25.00

than Linear Quadratic Gaussian framework and that the various uncertainty descriptions significantly altered the corresponding robust policy rules. The robust systems optimization, identification, and model (in)validation frameworks can identify and optimize different economic problems involving model uncertainty [16], can quantify the uncertainty in a systematic way, and can test the model variations with respect to the uncertainty bounds for the future outputs.

This paper extends the robust systems model identification and (in)validation concepts [17] to analyze the implied and historical FED behavior under model and shock uncertainty. Section 2 covers the robust systems framework, Section 3 follows with the macroeconomic stabilization problem description and analysis in detail and Section 4 concludes.

II. Robust Systems Framework

Robust systems framework deals with uncertainty in a typical linear fractional transformation (LFT) framework [18], combining the system nominal model, a bounded uncertainty, and a controller. Nominal performance implies the achieved performance objectives for the nominal plant only and it can be checked by the norm of the appropriate performance objective functions. Robust stability ensures the system stability for all models of the plant family, a linear time-invariant model with a bounded uncertainty. The small-gain theorem tests the robust stability by performing an appropriate transfer function norm test. Robust performance means that the LFT system is both stable and achieves the desired objectives for all possible plant models simultaneously. Conveniently, the robust performance of a system can be determined by using the structured singular value test for the structured uncertainty case.

Assume that a system is described by

$$\begin{bmatrix} w \\ e \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix}}_{G} . \begin{bmatrix} z \\ d \\ u \end{bmatrix} \quad (1)$$

$$z = \Delta.w \quad (2)$$

$$u = K.y \quad (3)$$

where the transfer function matrix G denotes the augmented plant model including the performance and uncertainty weighting functions, Δ denotes the norm-bounded uncertainty such that $\Delta \in B_\Delta$, where $B_\Delta \doteq \{\Delta : ||\Delta||_\infty \leq 1\}$, and K denotes the controller. While z, w, u and y signals are self-explanatory, d and e signals represent the input and output signals, respectively, to measure the performance specifications. Then, the LFT framework, shown in Fig. 1, controller loop can be closed to achieve the lower LFT representation $F_l(G,K) = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} + [G_{13}\ G_{23}]\, K(I - G_{33}K)^{-1}[G_{31}\ G_{32}]$ such that $\begin{bmatrix} w \\ e \end{bmatrix} = F_l(G,K). \begin{bmatrix} z \\ d \end{bmatrix}$.

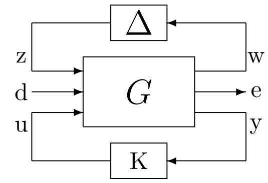

Fig. 1. The Robust Systems Framework.

Robust performance problems[2] are typically handled by using one block for the model uncertainties (Δ_U) and another (fictitious) block (Δ_P) for the performance objectives, yielding the augmented uncertainty structure (Δ) as $\Delta = \begin{bmatrix} \Delta_U & 0 \\ 0 & \Delta_P \end{bmatrix}$, where $\Delta_U \in B_\Delta$, $\Delta_P \in \mathbb{C}^{m_d \times m_e}$, where m_d and m_e denote the length of the d and e variables, the performance objective variables, as shown in Fig. 1.

The robust performance problems with structured uncertainty is solved by using the μ-synthesis framework [18]. This framework defines a structured singular value, $\mu(.)$,

$$\mu_\Delta(F_l(G,K)) \doteq \frac{1}{\min\{\overline{\sigma}(\Delta) : \Delta \in B_\Delta, \det(I - F_l(G,K)\Delta) = 0\}} \quad (4)$$

and states that the system is stable and has $\overline{\sigma}(T_{de}) < 1$, where $\overline{\sigma}(.)$ denotes the maximum singular value of the associated transfer function matrix, if and only if $F_l(G,K)$ is stable and

$$\max_{\omega} \quad \mu_\Delta(F_l(G,K)(j\omega)) < 1$$

Therefore, it solves

$$\min_{K \text{ stabilizing}} \quad \max_{\omega} \quad \mu_\Delta(F_l(G,K)(j\omega)) \quad (5)$$

Since the exact calculation of $\mu_\Delta(.)$ is not known yet, it can be approximated by

$$\min_{K \text{ stabilizing}} \quad \max_{\omega} \quad \overbrace{\min_{D \in \mathbf{D}} \quad \overline{\sigma}(DF_l(G,K)(j\omega)D^{-1})}^{\simeq \mu_\Delta(F_l(G,K)(j\omega))} \quad (6)$$

after a proper D-scaling[3] and this approximation is performed by D-K iteration[4]:

$$\min_{K \text{ stabilizing}} \quad \min_{D \in \mathbf{D}} \quad ||DF_l(G,K)D^{-1}||_\infty \quad (7)$$

[2]Economic sub-dynamics may involve unstructured or structured uncertainty. If there is no relationship assumed among the uncertainty variables, i.e., Δ contains all nonzero elements, it yields a well-known H_∞ problem [19], [20], easier to solve but yields more conservative results. On the other hand, if the uncertainty block is assumed to have some structure, i.e., Δ has some internal structure, it yields a μ-synthesis problem [18], harder to solve but yields better results.

[3]The elements of D-scaling matrix can take nonzero complex values without changing the upper bound and the D matrix can be any real, rational, stable and minimum-phase transfer function.

[4]The drawbacks of D-K iteration are that the $\mu_\Delta(.)$ is *approximated* by its upper bound and that the iteration may get stuck at a local minimum.

III. THE MACROECONOMIC STABILIZATION PROBLEM

The United States FED has been mandated by law to conduct the monetary policy to maintain low inflation levels while ensuring the growth of the economy by utilizing its various policy control inputs, the short-term interest-rate being the most effective one. Although there are many economic variables to optimize, the main objectives of the FED are the inflation and growth parameters. This study assumes that the overall economic dynamics between the FED short-term interest rates and the annual inflation can be represented by a single-input single-output (SISO) and a deterministic model and compares the historical and implied FED behavior during the inflation stabilization problems under model uncertainty. Furthermore, the proposed framework also treats all other economic variables as internal dynamic parameters whose effects are reflected in this SISO model.

The inflation stabilization problem utilizes the nominal system model [17] transfer function, which was already analyzed and validated for the respective historical data:

$$\frac{Y[z]}{U[z]} = \frac{0.522z^4 + 0.208z^3 + 0.184z^2 + 0.276z - 0.024}{z^4 - 0.526z^3 - 0.074z^2 + 0.13z - 0.506} \tag{8}$$

where $Y[z]$ is the annual inflation output variable and $U[z]$ is the FED short-term interest rate change. Since the ∞-norm is transferable in continuous and discrete-times, the 'bilinear' transformation, $z = e^{sT} \approx \frac{1+s\frac{T}{2}}{1-s\frac{T}{2}}$ or $s \approx \frac{2(z-1)}{T(z+1)}$, is used to obtain $G_N(s)$, the continuous-time equivalent system model of $G[z]$, as

$$G_N(s) = \frac{0.2426s^4 + 5.686s^3 + 12.84s^2 + 20.08s + 22.86}{s^4 + 17.98s^3 + 15.25s^2 + 46.2s + 0.4706} \tag{9}$$

and it can easily be treated as $G(s) = G_N(s)(1+\Delta(s))$, where $\Delta(s) \in B_\Delta$ to utilize the multiplicative uncertainty case. It should be noted that the comparison simulations are done in the discrete-time after the continuous-time optimization problem is solved.

Once the underlying economic model has been determined, the corresponding nominal performance, robust stability and robust performance issues can be presented by using the robust systems theory as defined in Sec. II to analyze the implied nature of the FED policy. A number of different uncertainty relationships and properties, different weighting functions, and various exogenous inputs can be utilized to interpret the FED policy choices. For example, the disturbance rejection problem block diagram including the nominal plant ($G_N(s)$), a controller ($K(s)$), a multiplicative uncertainty term (Δ_G), an uncertainty weighting function ($W_G(s)$), a shock (disturbance) weighting function ($W_d(s)$) and an error weighting function, ($W_e(s)$) is shown in Fig. 2. The disturbance or shocks, $d(s)$, can be anything to affect the inflation rates, such as a sudden jump in oil prices or natural disasters in large areas.

The robust performance objectives should be determined to correctly place the appropriate weighting functions. In general, many performance objectives are defined by using weighted

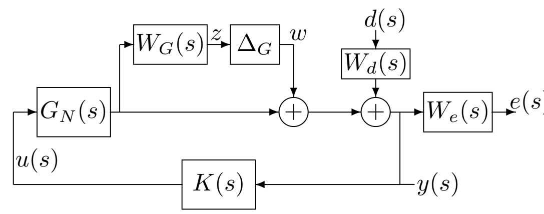

Fig. 2. The control system interconnection structure.

transfer functions which, in turn, are incorporated to the controller design. Weighting functions can be seen as frequency-dependent scaling of the corresponding transfer functions or signals. However, there is no exact established relationships other than some guidelines between the minimum steady-state error and the weighting function values, the exact weighting function selection for different performance objectives is still an open research area. Therefore, the weighting function selection process should be iterated to reflect the real-life situations. Since the FED projects a monetary policy aiming low inflation levels and sustainable Gross Domestic Product (GDP) growth rates, the corresponding inflation targeting problems are supposed to optimize the inflation dynamics under possible disturbances while maximizing the GDP output. However, the current model focuses solely on the inflation dynamics, i.e., the dynamics between the FED short-term interest rate and the annual inflation, the minimum tracking error due to the disturbance input (exogenous shocks) at the inflation output is taken one of the performance objective and the error weighting function is chosen to be

$$W_e(s) = 10\frac{\frac{s}{1}+1}{\frac{s}{0.01}+1} \tag{10}$$

a low-pass filter, to emphasize the steady-state error, i.e., low frequency behavior, which is assumed to be the main concern and is placed on the path of the weighted error output, $e(s)$.

The disturbance weighting function $W_d(s)$ can be placed on the disturbance input path to define the nature of disturbance. For example, if the noise type disturbance, i.e., highly volatile changes, is the main concern, a high-pass filter transfer function should be used. On the other hand, if sustained low frequency shocks, i.e., steady changes, are of the main concern, the $W_d(s)$ should be adjusted properly to reflect the disturbance spectrum. Once the disturbance weighting function is determined, the disturbance bound $||d(s)||_2 \leq 1$ can typically be taken. For simplicity and brevity of the FED policy analysis, $W_d(s) = 1$ is taken for this section, i.e., all possible disturbance variations are penalized equally.

The uncertainty weighting function is determined according to the model uncertainty characteristics such that low and high frequency model uncertainty components can be reflected. Since the low frequency behavior is much better understood than the high frequency behavior, this example assumes that the model uncertainty at low frequencies is 10 times smaller with respect to the one at high frequencies, i.e., the uncertainty

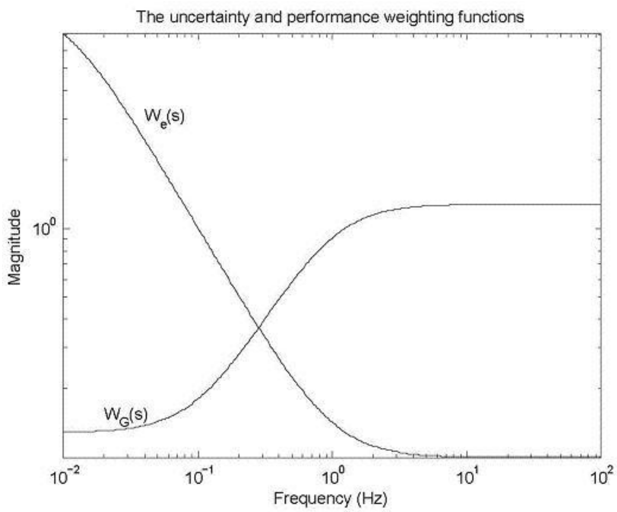

Fig. 3. The uncertainty and performance weighting functions.

weighting function reflects the 10% uncertainty at the low frequencies and 100% uncertainty for the high frequencies as given in (11). The error and uncertainty weighting functions are also shown in Fig. 3.

$$W_G(s) = 1.28\frac{s+0.1}{s+1} \tag{11}$$

Once the weighting functions are properly defined, the control system in Fig. 2 can be converted into an LFT framework as shown in Fig. 4.

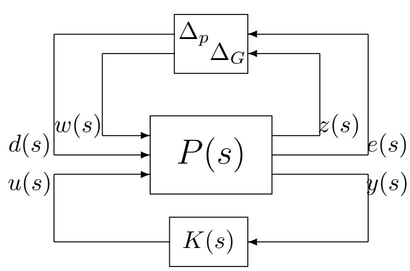

Fig. 4. The macroeconomic stabilization problem in robust systems framework.

The augmented transfer function matrix ($P(s)$) in Fig. 4 can be easily found by finding the transfer functions between each input-output pair;

$$P(s) = \begin{bmatrix} 0 & 0 & G_N(s)W_G(s) \\ W_e(s) & W_d(s)W_e(s) & G_N(s)W_e(s) \\ -1 & -W_d(s) & -G_N(s) \end{bmatrix} \tag{12}$$

The LFT structure in Fig. 4 can be easily solved by using the Matlab *robust control* toolbox [21], designing H_∞ and μ controllers, for the unstructured and structured uncertainty cases, respectively. The corresponding bounds of selected weighting functions for the H_∞ and μ-synthesis problems are shown in Fig. 5.

As seen from Fig. 5, the controller designs for the same system with different uncertainty structures yielded a lower μ bound than H_∞ bound. This is an expected result since the μ-synthesis framework exploits the uncertainty structure and always finds a lower or equal value of H_∞ bound. For this simulation, the worst-case μ bound is equal to 0.4576. Since $\mu = 0.4576 < 1$, it implies that the system satisfies the robust performance and that there is still some room for performance improvement.

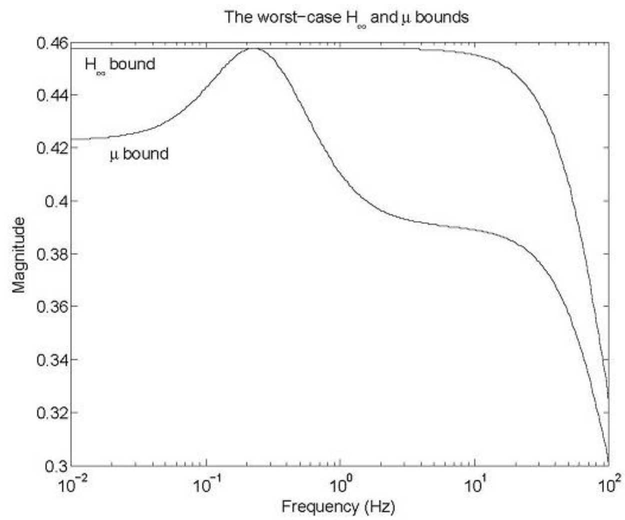

Fig. 5. The H_∞ and μ bounds.

The weighting functions specify the a-priori known nature of each variable and do not exist in the actual closed-loop system. Therefore, when the overall closed-loop system is simulated for reference or disturbance inputs, all weighting functions are removed as shown in Fig. 6.

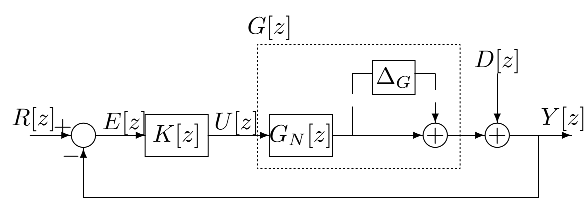

Fig. 6. The macroeconomic stabilization closed loop system.

The closed loop system is proposed as an inflation tracking system to analyze the historical and implied FED behavior and is simulated assuming that the historical inflation is the reference input, $R[z]$, and the estimate of the inflation is the output variable, $Y[z]$, and the FED fund rate change is the control action, $U[z]$. The inflation tracking problem performance can be studied by using the input-output inflation dynamics, $\frac{Y[z]}{R[z]}$, the top plot in Fig. 7. The inflation-FED fund rate dynamics can be studied by using the desired inflation input, i.e., the historical inflation rates, and the corresponding FED control action, $\frac{U[z]}{R[z]}$, the bottom plot in Fig. 7. As seen from Fig. 7, the input-output inflation response has some tracking error due to loosely chosen weighting functions, especially the error weighting function. When the corresponding implied and historical FED control actions are compared, the result suggests that the implied FED action is more aggressive. It should be noted that these simulations are closely dependent on chosen weighting functions.

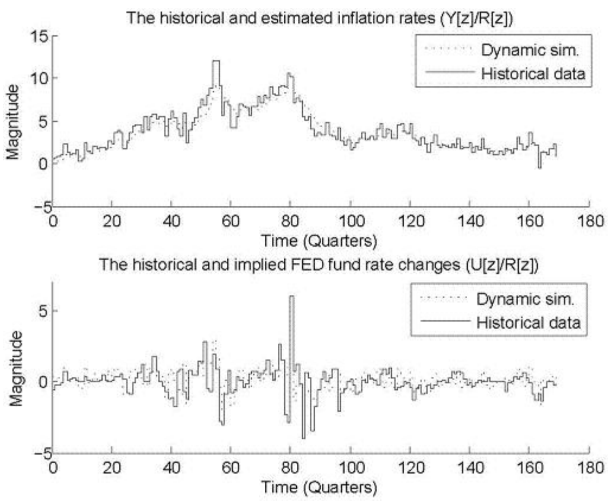

Fig. 7. The closed loop system simulations.

The same control system can be used to analyze the implied FED fund rate behavior against model uncertainty. Since the system reference signal is historical inflation rates, the same inflation data is expected to appear at the system output if the FED fund rate policy was optimal with respect to the uncertainty and performance objectives chosen for the underlying problem. This case can be analyzed by adjusting the error weighting function, $W_e(s)$, so that perfect inflation tracking, i.e., zero steady-state error, occurs. Then, the implied FED fund rate changes versus historical FED fund rate changes can be compared. Since the tracking error can be adjusted by changing the error weighting function, the new weighting function is adjusted to

$$W_e(s) = 70\frac{\frac{s}{1}+1}{\frac{s}{0.01}+1} \tag{13}$$

The corresponding worst-case μ bound is found to be 0.9984, shown in Fig. 8, and the corresponding simulations for the inflation tracking and FED fund rate changes are given in Fig. 9. As seen from Fig. 9, the inflation tracking problem achieved its goal, i.e., the error between the desired and historical inflation is almost negligible. The corresponding actual historical FED fund rate changes are much more smoother and smaller in magnitude than the optimally implied FED fund rate changes, clearly implying that the very conservative central bank policy. Thus, it can be concluded that, under the case of model uncertainty and chosen weighting functions, the FED policy tends to be more cautious and less reactionary, as most economists believe, than the policies suggested by a nominal model.

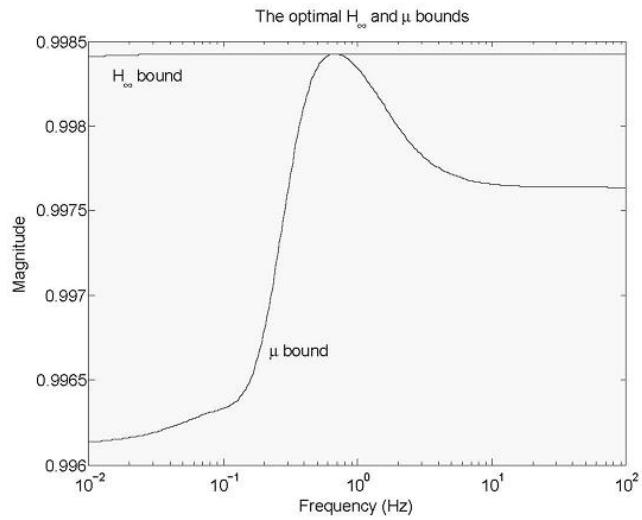

Fig. 8. The H_∞ and μ bounds for the (implied) optimal FED behavior.

IV. Conclusion

The main results of this paper are that the formal robust controller framework offers another avenue to investigate the uncertainty issues in macroeconomic stabilization problems, in particular, inflation tracking problems, and another supporting evidence of the FED behavior under model uncertainty when its actions are limited to inflation output only, which is more conservative.

The current analysis can be enhanced by utilizing a multi-input multi-output model to compare the FED behavior in different frameworks such as the robust systems framework, Bayesian analysis and rule-based policy results and in different stability and performance objective norms, i.e., a multi-objective problem.

Although the uncertainty is modeled as a model uncertainty, the individual model parameters can be assumed varying from the nominal values by some standard deviation ranges such as 1-standard deviation or 2-standard deviation. These deviations can be treated as parameter uncertainties and the corresponding optimization problem can be presented in statistical accuracy ranges, which is currently preferred by the policy makers.

References

[1] M. Friedman, "The effects of a Full-Employment Policy on Economic Stability: A formal Analysis", *In Friedman, Milton(ed.), Essays in Positive Economics*, University of Chicago Press, Chicago, IL, 1953.

[2] M. Friedman, *A Program for Monetary Stability*, Fordham University Press, 1959.

[3] F.H. Knight, *Risk, Uncertainty and Profit*, Houghton Mifflin Company, 1921.

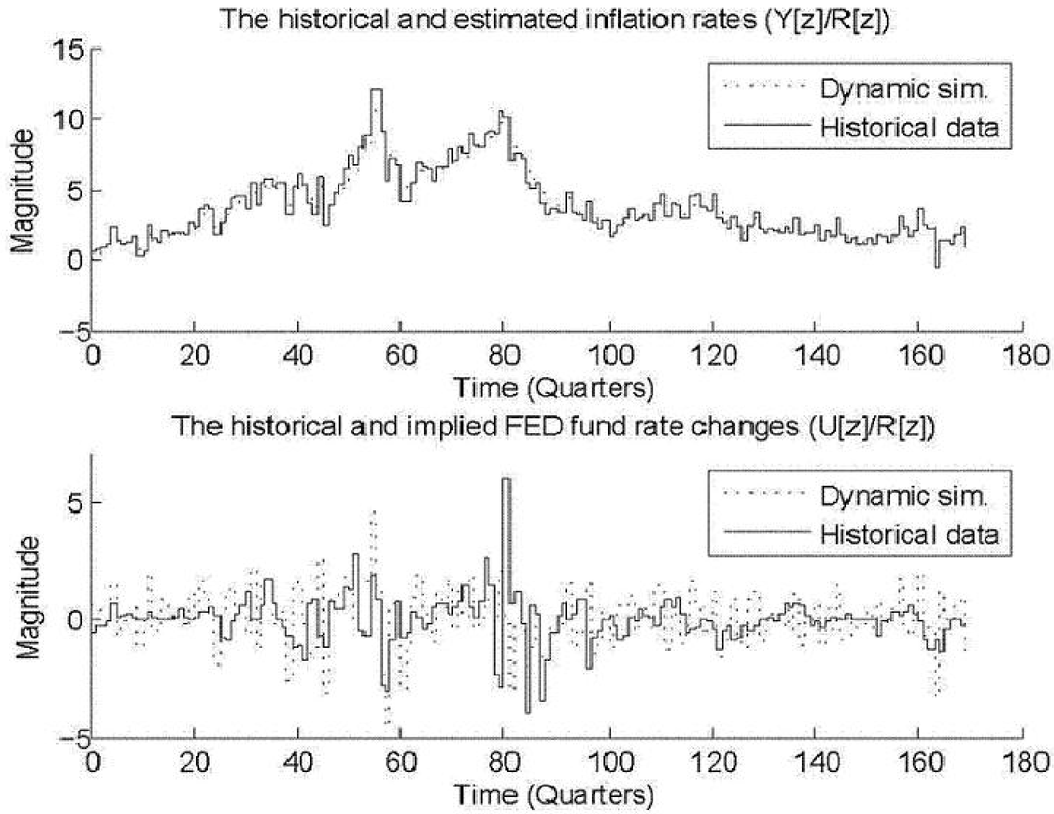

Fig. 9. The optimal inflation tracking and FED fund rate changes.

[4] W. Brainard, "Uncertainty and the effectiveness of policy", *American Economic Review*, vol 57(2), 1967, pp 411-425.

[5] G.D. Rudebusch, "Is the Fed too Timid? Monetary Policy in an Uncertain World", *The Review of Economics and Statistics*, MIT Press, vol. 83(2), 2001, pp 203-217.

[6] V. Wieland, "Monetary Policy, Parameter Uncertainty and Optimal Learning", *Journal of Monetary Economics*, vol. 46(1), 2000, pp 199-228.

[7] G. Zames, "Feedback and Optimal Sensitivity: Model Reference Transformations, Multiplicative Seminorms, and Approximate Inverses", *IEEE Transactions on Automatic Control*, vol. 26, pp 301-320.

[8] T. Basar, P. Bernhard, H_∞*- Optimal Control and Related Minimax Design Problems*, Birkhäuser, 1991.

[9] K. Zhou, J. Doyle, K. Glover, *Robust and Optimal Control*, Prentice-Hall, 1996.

[10] B. T. McCallum, "Robustness properties of a rule for monetary policy", *Carnegie-Rochester Conference Series on Public Policy*, vol. 29, 1998, pp 173-204.

[11] J. T. Sargent, *Comment, In John B. Taylor (ed.), Monetary Policy Rules*, University of Chicago Press, 1999, pp 144-154.

[12] J. T. Sargent, "Wanting Robustness in Macroeconomics, Robust control and filtering of forward-looking models", *Working papers*, http://www.stanford.edu/ sargent/research.html

[13] Tornell, Aaron., "Robust-H_∞ Forecasting and Asset Pricing Anomalies", *Working paper 7753*, NBER, 2000.

[14] A. Onatski, J. Stock, "Robust monetary policy under model uncertainty in a small model of the US economy", *Macroeconomic Dynamics*, vol. 6, 2002, pp 85-110.

[15] R. J. Tetlow, P. Von zur Muehlen, "Robust Monetary Policy with Misspecified Models: Does Model Uncertainty Always Call for Attenuated Policy?", *FEDS Working Paper No. 2000-28*, May 2000.

[16] M. Yilmaz, "Robust System Theory Applications to Macroeconomic Stabilization Problems", *PhD Dissertation*, Penn State University, 2005.

[17] W. Ma, M. Yilmaz, M. Sznaier, "Semi-Blind Robust Identification/Model (In)Validation with Applications to Macro-Economic Modeling", 16th IFAC World Congress, Prague, July, 2005.

[18] G. Balas, I. Fialho, A. Packard, W. Tan, , G. Wolodkin, *Theory and Application of Linear Parameter Varying Control Techniques*, NASA, Langley, VA, September 15-16, 1997.

[19] J. Doyle, A. Packard, P. Khargonekar, B. Francis, "State-Space Solutions to $\mathbb{H}_2$ and $\mathbb{H}_\infty$ Control Problems", *IEEE Journal of Automatic Control*, vol. 34, 1989, pp 831-847.

[20] Gahinet, P., Apkarian, P., "A Linear Matrix Inequality Approach to $\mathbb{H}_\infty$ Control", *Int. J. Robust and Nonlinear Control: Special Issue on* $\mathbb{H}_\infty$ *Control*, vol. 4, 1994, pp 421-438.

[21] MATLAB, "Robust Control Toolbox", MathWorks, Inc., 2005.