Adaptive H_∞ Consensus Control of Euler-Lagrange Systems on Directed Network Graph

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***Abstract*—A design method of adaptive consensus control of multi-agent systems composed of fully actuated mobile robots which are described as a class of Euler-Lagrange systems on directed network graphs, is presented in this paper based on the notion of inverse optimal H_∞ control criterion. The proposed control scheme is deduced as a solution of certain H_∞ control problem, where estimation errors of tuning parameters are considered as external disturbances to the process. The resulting control system is shown to be robust to uncertain system parameters and the approximate consensus tracking is achieved via adaptation schemes and L_2-gain design parameters.**

I. Introduction

Cooperative control problems of multi-agent systems have been active research fields, and many control strategies were developed in those areas, such as formation control, task assignment, traffic control, and scheduling *et al.* (for example, [1]-[10]). Among those, distributed consensus tracking of multi-agent systems with limited communication networks, has been a basic and important issue, and various research results have been proposed for various processes and under various conditions such as [11]-[20]. In those works, adaptive control methodologies were also investigated in order to deal with uncertainties of agents, and stability of control systems was checked via Lyapunov function analysis. Furthermore, robustness properties of the control schemes were also examined. However, so much attention does not have been paid on control performance such as optimal property or transient performance in those research works, and especially, the case of multi-agent systems composed of processes with unknown and different system parameters on directed information network graphs, does not have been investigated in detail in the previous works.

The purpose of the paper is to propose a design method of adaptive consensus control of multi-agent systems composed of fully actuated mobile robots which are described as a class of Euler-Lagrange systems [21] on directed network graphs based on the notion of inverse optimal H_∞ control criterion [22], [23]. This is an extension of our previous work [24], [25], where the first-order or second-order linear or nonlinear regression models on directed network graphs were considered. The proposed control scheme is derived as a solutios of certain H_∞ control problem, where estimation errors of tuning parameters are considered as external disturbances to the process. The resulting control system is shown to be robust to uncertain system parameters and the approximate consensus tracking is achieved via adaptation schemes and L_2-gain design parameters. Effectiveness of the proposed control scheme is also confirmed by simulation studies.

The present work provides basic schemes for coordinate control of multiple robotic manipulators or formation control of vehicles on highways, and other useful examples.

II. Multi-Agent Systems and Network Graphs

First, mathematical preliminaries on information network graph of multi-agent systems are briefly surveyed [16], [19], [20]. As a model of interaction among agents, a weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is considered, where $\mathcal{V} = \{1, \cdots, N\}$ is a node set corresponding to a set of agents, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set. An edge $(i, j) \in \mathcal{E}$ indicates that agent j can obtain information from i, but not necessarily vice versa. In the edge (i, j), i is denoted as a parent node and j is denoted as a child node, and the in-degree of the node i is the number of its parents, and the out-degree of i is the number of its children. Especially, an agent having no parent (or with the in-degree 0), is called as a root. A directed path is a sequence of edges in the form (i_1, i_2), (i_2, i_3), $\cdots$ $(\in \mathcal{E})$, where $i_j \in \mathcal{V}$. The directed graph is called strongly connected, if there is always a directed path between every pair of distinct nodes. A directed tree is a directed graph where every node has exactly one parent except for a unique root, and the root has directed paths to all other node. An directed spanning tree $\mathcal{G}_S = (\mathcal{V}_S, \mathcal{E}_S)$ of the directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a subgraph of $\mathcal{G}$ such that $\mathcal{G}_S$ is a directed tree and $\mathcal{V}_S = \mathcal{V}$.

Concerned with the edge set $\mathcal{E}$, a weighted adjacency matrix $A = [a_{ij}] \in \mathbf{R}^{N\times N}$ is introduced, and the entry a_{ij} of it is defined by

$$a_{ij} = \begin{cases} > 0 & : \quad (j, i) \in \mathcal{E}, \\ 0 & : \quad \text{otherwise}. \end{cases}$$

For the adjacency matrix $A = [a_{ij}]$, the Laplacian matrix $L = [l_{ij}] \in \mathbf{R}^{N\times N}$ is defined by

$$l_{ii} = \sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij},$$
$$l_{ij} = -a_{ij}, \quad (i \neq j).$$

Laplacian matrix has at least one zero eigenvalue and all nonzero eigenvalues have positive real parts. Especially, it is known that the Laplacian matrix has a simple 0 eigenvalue with the associated eigenvector $\mathbf{1} = [1 \cdots 1]^T$, and that all other eigenvalues have positive real parts, if and only if the corresponding directed graph has a directed spanning tree.

In this manuscript, a consensus control problem of leader-follower type is considered, where y_0 is a leader which each agent $i \in \mathcal{V}$ should follow (i is called a follower). Associated with the leader, a_{i0} is defined such as

$$a_{i0} = \begin{cases} > 0 & : \text{ leader's information is available} \\ & \quad \text{to follower } i, \\ 0 & : \text{ otherwise,} \end{cases} \tag{1}$$

and from a_{i0} and L, the matrix $M \in \mathbf{R}^{N \times N}$ is defined by

$$M = L + \mathrm{diag}\,(a_{10} \cdots a_{N0}). \tag{2}$$

It is shown that $-M$ is a Hurwitz matrix, if and only if 1. at least one a_{i0} $(1 \leq i \leq N)$ is positive, and 2. the graph $\mathcal{G}$ has a directed spanning tree with the root $i = 0$.

Hereafter, we assume that

1) The graph has a directed spanning tree with the root $i = 0$.
2) At least one a_{i0} $(1 \leq i \leq N)$ is positive, that is, the information of the leader y_0 ($\dot{y}_0$, $\ddot{y}_0$), is available to at least one follower i.
3) For the leader, y_0, $\dot{y}_0$, $\ddot{y}_0$ are uniformly bounded.

Hereafter, two adjacency matrices $A = [a_{ij}]$, $C = [c_{ij}] \in \mathbf{R}^{N \times N}$ are introduced for a directed graph $\mathcal{G}$, and the corresponding matrices are denoted as L_a, L_c (Laplacian matrices), and M_a, M_c, respectively.

III. Problem Statement

We consider a multi-agent system composed of N fully actuated mobile robots which are described as a class of Euler-Lagrange systems [8], [9] written as follows:

$$M_i(y_i)\ddot{y}_i + C_i(y_i, \dot{y}_i)\dot{y}_i = \tau_i, \quad (i = 1, \cdots, N), \tag{3}$$

where $y_i \in \mathbf{R}^n$ is an output (a generalized coordinate), $\tau_i \in \mathbf{R}^n$ is a control input (a force vector), $M_i(y_i) \in \mathbf{R}^{n \times n}$ is an inertia matrix, and $C_i(y_i, \dot{y}_i) \in \mathbf{R}^{n \times n}$ is a matrix of Coriolis and centripetal forces. Each component has the following properties as a Euler-Lagrange system.

Properties of Euler-Lagrange Systems [21]

1) $M_i(y_i)$ is a bounded, positive definite, and symmetric matrix.
2) $\dot{M}_i(y_i) - 2C_i(y_i, \dot{y}_i)$ is a skew symmetric matrix.
3) The left-hand side of (3) can be written into

$$M_i(y_i)a_i + C(y_i, \dot{y}_i)b_i = Y_i(y, \dot{y}_i, a_i, b_i)\theta_i, \tag{4}$$

where $Y_i(y_i, \dot{y}_i, a_i, b_i)$ is a known function of y_i, $\dot{y}_i$, a_i, b_i (a regressor matrix), and θ_i is an unknown system parameter vector.

The control objective is to achieve consensus tracking of the leader-follower type for the unknown multi-agent system (unknown θ_i) such as

$$y_i \to y_j, \quad (i, j = 1, \cdots, N), \tag{5}$$
$$y_i \to y_0, \quad (i = 1, \cdots, N), \tag{6}$$

under the limited communication structure $\mathcal{G}$ among agents.

Remark 1 More generalized Euler-Lagrange systems which include damping terms and gravitational forces, can be also considered in the present framework, since those are written in the similar form to (4). However, for simplicity of notations, the description (3) is to be adopted hereafter.

IV. Control Law and Error Equation

As the first step of the controller design, an estimator of $\dot{y}_0$ (the leader's information) is developed via available data from the follower i. A similar estimation procedure was presented in [15].

$$\dot{\hat{z}}_i(t) = -\beta \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij}\{\hat{z}_i(t) - \hat{z}_j(t)\}$$
$$-\beta c_{i0}\{\hat{z}_i(t) - \dot{y}_0(t)\} + n_{i0}\ddot{y}_0(t), \tag{7}$$

where $\hat{z}_i$ is a current estimate of $\dot{y}_0$, and is synthesized from the data available to the follower i. c_{ij} $(1 \leq i \leq N, 0 \leq j \leq N)$ is defined as the entry of the adjacency matrix C and (1) deduced from the directed graph $\mathcal{G}$, and $\beta > 0$ is a design parameter. Associated with c_{i0}, n_{i0} is defined such as

$$n_{i0} = \begin{cases} 1 & : \ c_{i0} > 0, \\ 0 & : \ \text{otherwise.} \end{cases} \tag{8}$$

By employing the estimate $\hat{z}_i$, the control scheme is constructed as follows:

$$\dot{y}_{ri}(t) = \hat{z}_i(t) - \alpha \sum_{\substack{j=0 \\ j \neq i}}^{N} a_{ij}\{y_i(t) - y_j(t)\}, \tag{9}$$
$$s_i(t) = \dot{y}_i(t) - \dot{y}_{ri}(t), \tag{10}$$
$$\tau_i(t) = Y_i(t)\hat{\theta}_i(t) + v_i(t), \tag{11}$$
$$Y_i(t) \equiv Y_i(y, \dot{y}_i, \ddot{y}_{ri}, \dot{y}_{ri}), \tag{12}$$

where a_{ij} $(1 \leq i \leq N, 0 \leq j \leq N)$ is defined similarly from the entry of the adjacency matrix A and (1) deduced from $\mathcal{G}$, and $\alpha > 0$ is a design parameter. $\hat{\theta}_i$ is denoted as a current estimate of unknown θ_i, and v_i is a stabilizing signal which is to be determined later based on the notion of inverse optimal H_∞ control criterion. An estimation error between the leader $\dot{y}_0$ and the estimate $\hat{z}_i$ is defined by

$$\tilde{z}_i(t) \equiv \hat{z}_i(t) - \dot{y}_0(t), \tag{13}$$

and the following relations are deduced for s_i and $\tilde{z}_i$.

$$\dot{\tilde{z}}_i(t) = -\beta \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij}\{\tilde{z}_i(t) - \tilde{z}_j(t)\} -\beta c_{i0}\tilde{z}_i(t) + (n_{i0}-1)\ddot{y}_0(t), \tag{14}$$

$$M_i(y_i)\dot{s}_i(t) + C_i(y_i, \dot{y}_i)s_i(t) = v_i(t) + Y_i(t)\{\hat{\theta}_i(t) - \theta_i\}. \tag{15}$$

Then, the overall representations of the multi-agent system are given as follows:

$$\dot{\tilde{z}}(t) = -\beta\,(M_c \otimes I)\,\tilde{z}(t) + \{(N_0 - \mathbf{1})\otimes I\}\,\ddot{y}_0(t), \tag{16}$$

$$M\dot{s}(t) + Cs(t) = Y(t)\{\hat{\theta} - \theta(t)\} + v(t), \tag{17}$$

where

$$\tilde{z} = [\tilde{z}_1^{\mathsf{T}}, \cdots, \tilde{z}_N^{\mathsf{T}}]^{\mathsf{T}}, \tag{18}$$

$$s = [s_1^{\mathsf{T}}, \cdots, s_N^{\mathsf{T}}]^{\mathsf{T}}, \tag{19}$$

$$M = \text{block diag}\,(M_1, \cdots, M_N), \ \ (M_i \equiv M_i(y_i)), \tag{20}$$

$$C = \text{block diag}\,(C_1, \cdots, C_N), \ \ (C_i \equiv C_i(y_i, \dot{y}_i)), \tag{21}$$

$$Y = \text{block diag}\,(Y_1, \cdots, Y_N), \tag{22}$$

$$\theta = [\theta_1^{\mathsf{T}}, \cdots, \theta_N^{\mathsf{T}}]^{\mathsf{T}}, \tag{23}$$

$$N_0 = [n_{10}, \cdots, n_{N0}]^{\mathsf{T}}, \tag{24}$$

$$\mathbf{1} = [1, \cdots, 1]^{\mathsf{T}}, \tag{25}$$

$$v = [v_1^{\mathsf{T}}, \cdots, v_N^{\mathsf{T}}]^{\mathsf{T}}, \tag{26}$$

and $\otimes$ denotes Kronecker product.

V. Adaptive H_∞ Consensus Control for Euler-Lagrange Systems

Stability analysis of the overall control system is composed of four steps. First, for stability analysis of s and the related terms, a positive function W_0 is defined such as

$$W_0(t) \ = \ \frac{1}{2}s(t)^{\mathsf{T}}Ms(t). \tag{27}$$

Then, the time derivative of W_0 along its trajectory is derived as follows:

$$\dot{W}_0(t) \ = \ s(t)^{\mathsf{T}}[Y(t)\{\hat{\theta} - \theta(t)\} + v(t)]. \tag{28}$$

By considering the evaluation of $\dot{W}_0$ (28), the next virtual system is introduced.

$$\dot{s} = f + g_1 d + g_2 v, \tag{29}$$

$$f = 0, \tag{30}$$

$$g_1 = Y, \ g_2 = I, \tag{31}$$

$$d = (\hat{\theta} - \theta). \tag{32}$$

The virtual system is to be stabilized via a control input v based on H_∞ criterion, where d is considered as an external disturbance to the process. For that purpose, the following Hamilton-Jacobi-Isaacs (HJI) equation and its solution V_0 are introduced.

$$\mathcal{L}_f V_0 + \frac{1}{4}\left\{\frac{\|\mathcal{L}_{g_1}V_0\|^2}{\gamma^2} - (\mathcal{L}_{g_2}V_0)R^{-1}(\mathcal{L}_{g_2}V_0)^{\mathsf{T}}\right\} + q = 0, \tag{33}$$

$$V_0 = \frac{1}{2}s^{\mathsf{T}}s, \tag{34}$$

where q and R are a positive function and a positive definite matrix respectively, and those are deduced from HJI equation based on the notion of inverse optimality for the given solution V_0 and the positive constants γ. The substitution of the solution V_0 (34) into HJI equation (33) yields

$$\frac{1}{4}s^{\mathsf{T}}\left\{\frac{YY^{\mathsf{T}}}{\gamma^2} - R^{-1}\right\}s + q = 0. \tag{35}$$

From (35), R and q are obtained such as

$$R \ = \ \left(\frac{YY^{\mathsf{T}}}{\gamma^2} + K\right)^{-1}, \tag{36}$$

$$q \ = \ \frac{1}{4}s^{\mathsf{T}}Ks, \tag{37}$$

where K is a diagonal positive definite matrix (a design parameter), From R, v is derived as a solution of the corresponding H_∞ control problem as follows:

$$\begin{aligned} v \ &= \ -\frac{1}{2}R^{-1}(\mathcal{L}_{g_2}V_0)^{\mathsf{T}} = -\frac{1}{2}R^{-1}s \\ &= \ -\frac{1}{2}\left(\frac{YY^{\mathsf{T}}}{\gamma^2} + K\right)\hat{B}^{\mathsf{T}}s. \end{aligned} \tag{38}$$

Then, via HJI equation, the time derivative of W_0 (28) is evaluated as follows:

$$\begin{aligned} \dot{W}_0 \ &= \ -q - v^{\mathsf{T}}Rv \\ &\quad + \left(v + \frac{1}{2}R^{-1}s\right)^{\mathsf{T}} R \left(v + \frac{1}{2}R^{-1}s\right) \\ &\quad + \gamma^2\|d\|^2 - \gamma^2\left\|d - \frac{Y^{\mathsf{T}}s}{2\gamma^2}\right\|^2, \end{aligned} \tag{39}$$

and it follows that s is bounded for bounded $\hat{\theta}$ and for the stabilizing signal v (38).

Next, for stablity analysis of the estimation error $\tilde{z}$, a positive function V_1 is introduced such as

$$V_1 = \tilde{z}^{\mathsf{T}}(P_c \otimes I)\tilde{z}, \tag{40}$$

$$P_c M_c + M_c^{\mathsf{T}}P_c = I, \ \ (P_c = P_c^{\mathsf{T}} > 0). \tag{41}$$

There exists a positive definite and symmetric matrix P_c satisfying (41), since $-M_c$ is Hurwitz. Then, the time derivative of V_1 along its trajectory is evaluated as follows:

$$\begin{aligned} \dot{V}_1 \ &= \ -\beta\|\tilde{z}\|^2 - 2\tilde{z}^{\mathsf{T}}(P_c \otimes I)\{(N_0 - \mathbf{1})\otimes I\}\ddot{y}_0 \\ &\leq \ -\frac{\beta}{2}\|\tilde{z}\|^2 + \frac{2}{\beta}\|P_c \otimes I\|^2\|\{(N_0 - \mathbf{1})\otimes I\}\ddot{y}_0\|^2, \end{aligned} \tag{42}$$

and it is shown that $\tilde{z}$ is bounded for bounded $\ddot{y}_0$.

Thirdly, for stability analysis of the control error $y_i - y_0$ and the related terms, $\tilde{y}_i$, $\tilde{y}$ are defined by

$$\tilde{y}_i = y_i - y_0, \tag{43}$$

$$\tilde{y} = [\tilde{y}_1^{\mathsf{T}}, \cdots, \tilde{y}_N^{\mathsf{T}}]^{\mathsf{T}}. \tag{44}$$

Then, the following relation holds

$$\dot{\tilde{y}} = s + \tilde{z} - \alpha(M_a \otimes I)\tilde{y}, \tag{45}$$

and $-M_a$ is shown to be Hurwitz because of the assumption of the network graph $\mathcal{G}$. From that property, a positive function V_2 is defined by

$$V_2 = \tilde{y}^\mathsf{T}(P_a \otimes I)\tilde{y}, \tag{46}$$
$$P_a M_a + M_a^\mathsf{T} P_a = I, \;\; (P_a = P_a^\mathsf{T} > 0). \tag{47}$$

Similarly to the previous case (M_c), there exists a positive definite and symmetric matrix P_a satisfying (47), since $-M_a$ is Hurwitz. Then, the time derivative of V_2 along its trajectory is evaluated as follows:

$$\begin{aligned}\dot{V}_2 \;&=\; -\alpha\|\tilde{y}\|^2 + 2\tilde{y}^\mathsf{T}(P_a \otimes I)(s + \tilde{z}) \\ &\le\; -\frac{\alpha}{2}\|\tilde{y}\|^2 + \frac{4}{\alpha}\|P_a \otimes I\|^2(\|s\|^2 + \|\tilde{z}\|^2).\end{aligned} \tag{48}$$

From the three stages of stability analysis (the evaluations of $\dot{W}_0$, $\dot{V}_1$, $\dot{V}_2$), the next theorem is obtained.

Theorem 1 *The nonlinear control system composed of the control laws (7), (9), (10), (11), (12), (38) is uniformly bounded for an arbitrary bounded design parameter $\hat{\theta}_i$, and bounded y_0, $\dot{y}_0$, $\ddot{y}_0$, and v is an optimal control input which minimizes the following cost functional J.*

$$\begin{aligned}J(t) \;\equiv\; \sup_{d_i, d_2, d_3 \in \mathcal{L}_2} \Bigg[&\int_0^t \{q + v^\mathsf{T} R v\} d\tau + W_0(t) \\ &-\gamma^2 \int_0^t \|d\|^2 d\tau \Bigg].\end{aligned} \tag{49}$$

Also the next inequality holds.

$$\begin{aligned}&\int_0^t \{q + v^\mathsf{T} R v\} d\tau + W_0(t) \\ &\qquad\le\; \gamma^2 \int_0^t \|d\|^2 d\tau + W_0(0).\end{aligned} \tag{50}$$

Theorem 1 denotes the properties of the proposed nonlinear control system (7), (9), (10), (11), (12), (38), where the tunings of $\hat{\theta}$ is not included (or not necessarily required).

Next, the tuning law of $\hat{\theta}$ is determined as follows:

$$\dot{\hat{\theta}}(t) = \mathrm{Pr}\left\{-\Gamma Y(t)^\mathsf{T} s(t)\right\}, \tag{51}$$

where $\mathrm{Pr}(\cdot)$ is a projection operation in which the tuning parameter $\hat{\theta}$ is constrained to a bounded region deduced from upper-bounds of $\|\theta\|$ [26]. As the fourth step of stability analysis of the overall control system, a positive function W_1 is defined by

$$W_1(t) \;=\; \frac{1}{2}s(t)^\mathsf{T}s(t) + \frac{1}{2}\left\{\hat{\theta}(t) - \theta\right\}^\mathsf{T} \Gamma^{-1} \left\{\hat{\theta}(t) - \theta\right\}, \tag{52}$$

and the time derivative of W_1 along its trajectory is evaluated such as

$$\dot{W}_1(t) \le -\frac{1}{2}s(t)^\mathsf{T} R^{-1} s \le 0. \tag{53}$$

From the four stages of stability analysis (the evaluations of $\dot{W}_0$, $\dot{W}_1$, $\dot{V}_1$, $\dot{V}_2$), the next theorem is obtained.

Theorem 2 *The adaptive control system composed of the control laws (7), (9), (10), (11), (12), (38), and the tuning law of $\hat{\theta}$ (51), is uniformly bounded for bounded y_0, $\dot{y}_0$, $\ddot{y}_0$, and it follows that*

$$\lim_{t\to\infty} s(t) = 0. \tag{54}$$

Especially, if $\ddot{y}_0(t) = 0$ or the information of the leader $\ddot{y}_0$ is available for all followers ($\{(N_0 - \mathbf{1}) \otimes I\}\, \dot{y}_0 = 0$), then it follows that

$$\lim_{t\to\infty} \tilde{y}(t) = \lim_{t\to\infty} \dot{\tilde{y}}(t) = 0, \tag{55}$$

and the asymptotic consensus tracking is achieved. Otherwise, when $\ddot{y}_0(t) \neq 0$ and the information of $\ddot{y}_0$ is not available for all followers ($\{(N_0 - \mathbf{1}) \otimes I\}\, \ddot{y}_0 \neq 0$), then the next relation holds.

$$\|\tilde{y}\| \sim \mathrm{const.} \cdot \frac{1}{\alpha\beta}\|\{(N_0 - \mathbf{1}) \otimes I\}\ddot{y}_0\|, \tag{56}$$
$$\|\dot{\tilde{y}}\| \sim \mathrm{const.} \cdot \frac{1}{\beta}\|\{(N_0 - \mathbf{1}) \otimes I\}\ddot{y}_0\|. \tag{57}$$

Theorem 2 denotes the properties of the proposed adaptive control system (7), (9), (10), (11), (12), (38), (51), and states that the asymptotic consensus tracking is achieved under the specified condition ($\{(N_0 - \mathbf{1}) \otimes I\}\, \dot{y}_0 = 0$), and also shows that the approximate consensus tracking with the ratios of $1/(\alpha\beta)$, $1/\beta\,(> 0)$ is still assured, even if that condition is not satisfied ($\{(N_0 - \mathbf{1}) \otimes I\}\, \ddot{y}_0 \neq 0$).

Remark 2 It should be noted the proposed control scheme and the adaptation scheme are all implemented in a distributed fashion, where availabilities of signals for each agent i are highly restricted and prescribed by the directed graph $\mathcal{G}$.

Remark 3 Of course, J in **Theorem 1** is a fictitious cost functional, since d is not an actual disturbance but an estimation error of the tuning parameter, and since it is not generally included in $L^2[0,\infty)$. Nevertheless, v, which is derived as a solution for that fictitious H_∞ control problem, attain the inequality in **Theorem 1**, stabilize the total system, and it means that the L^2 gain from the disturbance d to the generalized output $\sqrt{q + v^T R v}$ is prescribed by the positive constant γ.

VI. Numerical Example

In order to show the effectiveness of the proposed control scheme, numerical experiments for Euler-Lagrange systems are performed.

A multi-agent system composed of simple Euler-Lagrange systems is considered as follows:

$$\begin{aligned}&m_i \ddot{y}_i(t) = \tau_i(t), \quad (i = 1,\, 2,\, 3), \\ &(y_1(0) = 1, \;\; y_2(0) = 0, \;\; y_3(0) = -1),\end{aligned}$$

where $y_i \in \mathbf{R}$, $\tau_i \in \mathbf{R}$, and $m_i \in \mathbf{R}$ is an unknown system parameter. Associated with the information network structure (Fig.1), the adjacency matrix $A = [a_{ij}]\,(= C)$ and $a_{i0}\,(= c_{i0})$ are chosen such that

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$
$$a_{10} = 1, \;\; a_{20} = a_{30} = 0.$$

The control objective is to achieve consensus tracking

$$
\begin{aligned}
&y_i \to y_j, \quad \dot{y}_i \to \dot{y}_j, \\
&y_i \to y_0, \quad \dot{y}_i \to \dot{y}_0, \\
&(i,\, j = 1,\, 2,\, 3),
\end{aligned}
$$

where the virtual leader y_0 is determined such as

$$\ddot{y}_0 + 2\dot{y}_0(t) + y_0(t) = \sin t.$$

The design parameters are chosen as follows:

$$
\begin{aligned}
&\Gamma = 10I, \quad K = 5I, \quad \alpha = \beta = 10, \\
&\gamma_i = 1.
\end{aligned}
$$

As system parameters m_i $(i = 1 \sim 3)$, we consider both time-invariant and time-varying cases such that

$$
\begin{aligned}
&m_1 = 1, \quad m_2 = 2, \quad m_3 = 3, \quad (\mathrm{time-invariant\ case}), \\
&m_1 = f_m(t), \quad m_2 = 2f_m(t), \quad m_3 = 3f_m(t), \\
&\qquad\qquad\qquad\qquad\qquad (\mathrm{time-varying\ case}),
\end{aligned}
$$

where

$$
f_m(t) = \begin{cases} 2 & 0 \le t < 2.5,\ 5 \le t < 7.5,\ \cdots, \\ 1 & 2.5 < t \le 5,\ 7.5 < t \le 10,\ \cdots. \end{cases}
$$

The simulation results of the proposed design scheme (Theorem 2) are shown in Fig.2 (time-invariant case) and Fig.4 (time-varying case). For comparison, the adaptive control systems which do not contain H_∞ control scheme (that is, $\gamma = \infty$), are also shown for both cases; Fig.3 (time-invariant case) and Fig.5 (time-varying case).

From those results, it is seen that the proposed H_∞ adaptive control strategies achieve better tracking convergence properties together with robustness to abrupt changes of the system parameters compared with the non-H_∞ control scheme, and those are owing to disturbance attenuation properties of the proposed H_∞ controllers.

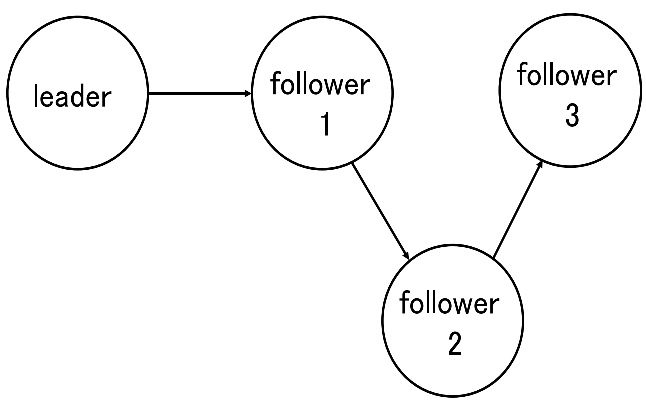

Fig. 1. Information Network Graph

VII. Concluding Remarks

A Design methodology of adaptive H_∞ consensus control of multi-agent systems composed of fully actuated mobile robots which are described as a class of Euler-Lagrange systems on directed network has been presented in this paper as an extension of our previous works. The proposed control scheme is derived as a solution of certain H_∞ control problem based on the notion of inverse optimality, where estimation errors of tuning parameters are considered as external disturbances to the process. The resulting control system is shown to be robust to uncertain system parameters, and the approximate consensus tracking is achieved via adaptation schemes and L_2-gain design parameters. Effectiveness of the proposed design schemes was also confirmed by the simulation studies. In order to deal with additional nonlinear terms with unknown structures in multi-agent systems, three-layered neural networks can be also applied in the proposed control scheme, and that will be shown in the future work.

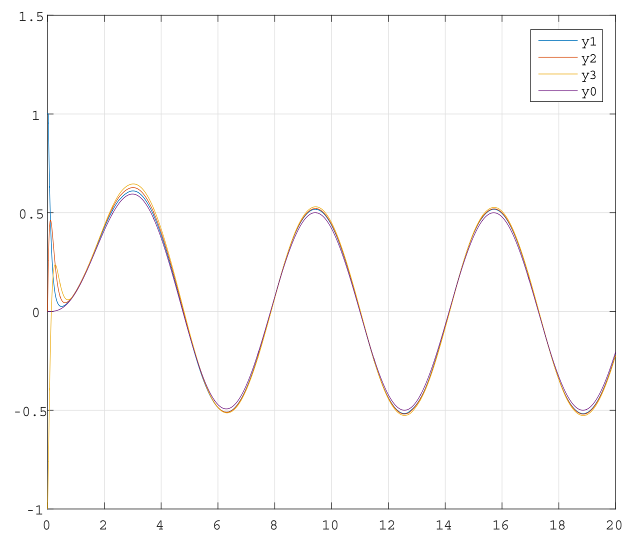

Fig. 2. Simulation Result for Time-Invariant Case with H_∞ Control Scheme

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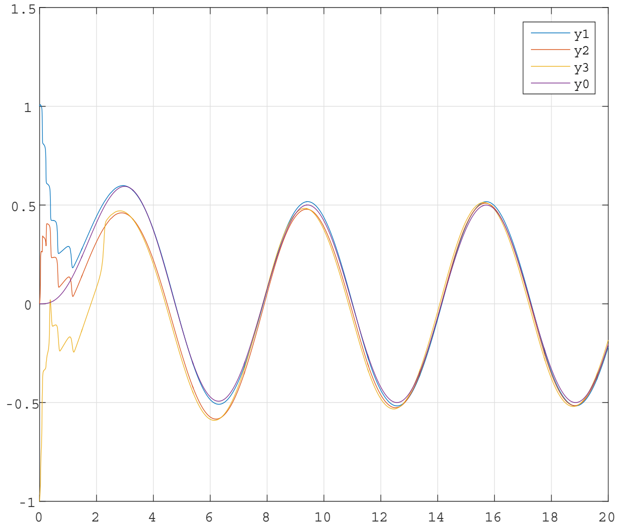

Fig. 3. Simulation Result for Time-Invariant Case without H_∞ Control Scheme

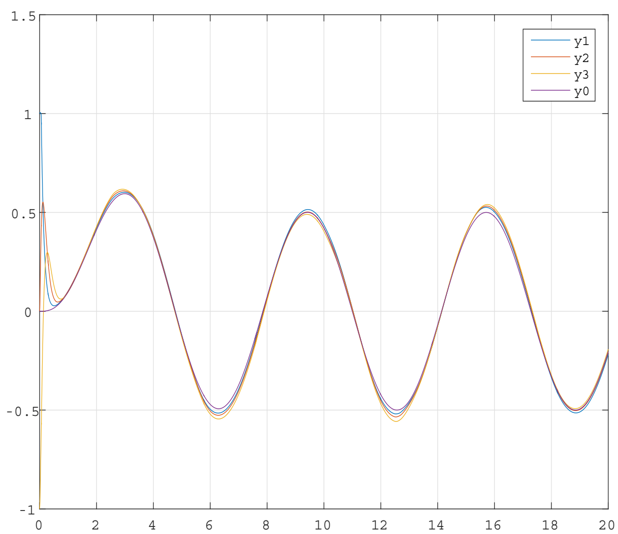

Fig. 4. Simulation Result for Time-Varying Case with H_∞ Control Scheme

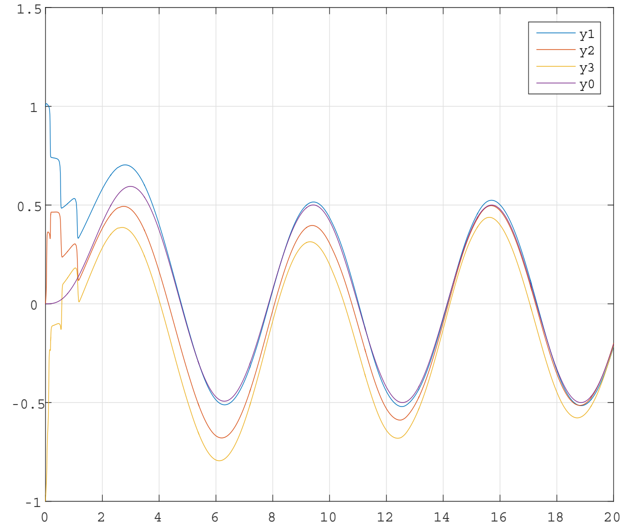

Fig. 5. Simulation Result for Time-Varying Case without H_∞ Control Scheme

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