Effects of Discrete Design-variable Precision on Real-Coded Genetic Algorithm

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***Abstract*—In this paper, we describe how discretizing design variables on real-coded genetic algorithms (RCGAs) can influence the convergence and the diversity of Pareto optimal solutions. We use Non-dominated Sorting Genetic Algorithm II (NSGA-II) as an RCGA based on Pareto dominance, changing the number of significant digits after the decimal point for each design variable. Test problems and engineering problems are investigated. Computational results show that the use of a smaller number of significant figures instead of larger ones achieves better convergence that a larger number in many cases. In the DTLZ3 test problem, low applied precision avoids dominance-resistant solutions (DRSs) and improves both the generational distance (GD) and the inverted generational distance (IGD). On the other hand, in the DTLZ4 test problem, low digit precision improves GD, whereas it worsens IGD. This indicates that a minimum digit precision is required to maintain the diversity of Pareto optimal solutions in some problems. When we use RCGAs, it is critical to set the number of significant digits after the decimal point to realistically represent actual engineering problems.**

I. Introduction

In recent years, many kinds of RCGAs have been proposed and successfully applied to actual multi-objective optimization problems. Real-world optimization problems often have not only many objective functions but also many constraints. Therefore, most of the RCGAs mainly focus primarily on handling solutions efficiently toward optimum directions[1], [2], [3] and finding feasible solutions[4]. In addition, the discretization techniques of the objective space such as the epsilon dominance can be also used to improve the convergence of Pareto optimal solutions.

When we use RCGAs, we do not usually think about significant figures in each design variable because computers can handle a great digit precision. Design variables are considered to be continuous variables in most of the test problems. On the other hand, in practical optimization problems, the significant figures in each design variable are often limited to the small or realistic numbers.

In the following assumption[5], it is known that the low-resolution representation can improve the convergence of Pareto optimal solutions in binary-coding genetic algorithms (BCGAs) because the search space is smaller as the resolution decreases. However, in RCGAs, the highest-resolution representation is always employed. The effect of changing significant figures in RCGAs has not been investigated well. In particular, it may be important to set an appropriate number of significant figures in practical applications.

The objective of this study was to assess the search ability in RCGAs by changing the number of significant figures. Here, we use Non-dominated Sorting Genetic Algorithm-II (NSGA-II)[6] as an RCGA, which is based on the Pareto dominance, and a set of test problems, including some simple engineering problems. This study focused on using 2, 4, 6, 8, or 16 significant digits.

II. Related works

This section presents a brief review of the discretization in an evolutionary multiobjective optimization (EMO) algorithm.

A. Discretization in objective function space

Studies about discretization of an objective function space have been focused on in recent years. The idea of ϵ dominance, which was proposed by Laumanns[7], is one of the most popular discretization techniques of the objective function space. It makes all points within a small distance (the ϵ distance) from a set of Pareto-optimal points dominated. This technique can reduce the number of Pareto-optimal solutions so that the selection pressure is increased over the Pareto front by using this technique[8].

Ishibuchi, et.al. examined the effects of discrete objective functions with various granularities[9]. They focused on the granularity difference between discrete objective functions in combinatorial optimization problems and showed that a distinct objective function with coarse granularity (low resolution) slows down the search ability of EMO algorithms along that objective.

B. Discretization in design-variable space

The studies related to discrete design variables have been conducted in binary-coded genetic algorithms(BCGAs).

Jaimes, et. al.[5] proposed the island parallel Multi-Objective Evolutionary Algorithm (pMOEA) in which each island had a different resolution (different length of binary strings). During optimization, the resolution in each island

TABLE I
THE PROPERTIES OF BENCHMARK PROBLEMS.

Problem	Obj.	Var.	Con.	Separability	Modality	Bias	Geometry
DTLZ2	M	N	-	separable	uni	-	concave
DTLZ3	M	N	-	separable	multi	-	concave
DTLZ4	M	N	-	separable	uni	✓	concave
WFG2	M	N	-	non-separable	multi	-	convex,disconnected
WFG4	M	N	-	separable	multi	-	concave
WFG6	M	N	-	non-separable	uni	-	concave
WFG7	M	N	-	separable	uni	✓	concave
UF2	2	N	-	separable	multi	-	convex
UF9	3	N	-	separable	multi	-	linear, disconnected
CF2	2	N	1	separable	multi	-	convex, disconnected
CF7	2	N	2	separable	multi	-	convex
C1DTLZ3	M	N	1	separable	multi	-	concave
C2DTLZ2convex	M	N	1	separable	uni	-	convex, disconnected
C3DTLZ1	M	N	M	separable	multi	-	convex(feasible surface is PF)
Car Side Impact	3	7	10			uncertain	
Welded Beam	2	4	4			uncertain	

Obj. = the number of objectives; Var. = the number of variables; Con. = the number of constraints.
M = the user predefined number of objectives; N = the user predefined number of variables.
- = no value or no characteristic; ✓= that the problem has characteristic.
Car Side Impact and Welded Beam Design problems have uncertain properties because of engineering problems.

started low and gradually increased so that it could achieve rapid convergence, while maintaining diversity.

Kim, et.al.[10] proposed a variable chromosome length genetic algorithm for topology optimization problems. This algorithm also starts with a low-resolution (a short chromosome) and increases the number of bits after finding an optimum solution at the current resolution.

Dynamic resolution techniques of [5], [10] are based on the assumption described above, showing better search ability than with the fixed resolution. However, to the best of our knowledge, the studies investigating discretization of the variable space in RCGAs has not been explored well.

III. Experimental Study

A. Test Problems

In this study, 16 benchmark test problems were considered: three DTLZ problems[11], four WFG problems[12], four CEC2009 problems[13], three CDTLZ problems[14], and two engineering problems[14], [15]. The properties of the adopted benchmark problems are summarized in Table I, where each problem is classified under four characteristics[12]: separability, modality, bias and geometry. Separability indicates whether a correlation relationship among parameters exists. Separable problems are characterized by parameter independence, whereas non-separable problems are characterized by parameter dependencies, and are more difficult to analyze than those characterized by parameter independence. Modality shows the characteristics of fitness landscapes. Multimodal problems have many local optimal points, whereas unimodal problems only have one global optimal point. Bias is another characteristic of the fitness landscape and indicates bias of distribution in objective space. Geometry refers to the shape of the Pareto-front surface.

The car side impact problem[14] and the welded beam problem[15] are the engineering problems considered.

The car side impact problem is a multi-objective minimization problem. Three objective functions are considered: minimizing of the weight of car, minimizing of the public force experienced by a passenger, and minimizing of the average velocity of the V-Pillar responsible for withstanding the impact load. These objectives conflicting with one another; therefore, a three-dimensional trade-off front is expected.In addition, this problem has ten constraints and seven real-parameter variables.

The welded beam design problem is also a multi-objective minimization problem. Two objective functions are considered here: minimizing the cost of fabrication and minimizing of the end deflection of the welded beam. These objectives also conflict; therefore, a two-dimensional trade-off front is expected.Additionally, this problem has four constraints and four real-parameter variables.

B. Computational Conditions

Non-dominated Sorting Genetic Algorithm-II (NSGA-II) is used in this study. The common parameter settings are shown in Table II, where η_c and η_m are indices of simulated binary crossover (SBX) and of polynomial mutation, respectively. The other parameters adopted in each test problem are shown in Table III. The population size, generation number, the number of objective functions, and the number of design variables are different in each benchmark problem. The number of significant figures after the decimal points is either to 2, 4, 6, 8, or 16. A value with 16-digit precision is almost equal to a continuous value. Design variables are rounded to a specified decimal place after SBX and polynomial mutation in NSGA-II.

C. Results and Discussions

We have chosen two types of performance metrics: the first is the generational distance (GD) for the convergence and the second is inverted generational distance (IGD) for the diversity. The average GD / IGD is computed using feasible

TABLE II
VALUES OF THE COMMON PARAMETERS USED IN THE EXPERIMENTAL STUDY.

parameter	value
Crossover rate	1.0
Mutation rate	1/n
η_c	30
η_m	20
Trial	10

non-dominated solutions at each generation. In engineering problems, approximate Pareto-optimal solutions are created by merging non-dominated solutions of all trials, and this is used it to evaluate GD / IGD.

Table III shows the averaged GD in each digit and benchmark problem. The averaged GD is evaluated at the 10th, half, and final generation because the convergence rate is important in MOEA as well as the final converged value. As shown, in almost all benchmark problems except for WFG2, WFG4, and the Welded beam problem, the final GD can improve as the number of digits drops. This improvement means that lower resolution, namely a more discretized design variable, establishes convergence. In the DTLZ test problems, the convergence rate can also be improved. In the WFG2, WFG4, and the welded beam problem, using the lower digits does not produce both better-converged values and convergence rates, because these problems have large search ranges, and the influence of changing the significant figures after the decimal point becomes tiny. To make sure statistically differences of GD trends between the lower digit (2-digit) and the higher digit (16-digit), we applied the Wilcoxon rank-sum test in each problem. According to the results of the test, it was confirmed that there were significant differences between two digit precisions in DTLZ2-4, UF9, CF2, CF7, C1DTLZ3, C2DTLZ2 convex, C3DTLZ1 and welded beam design problem.

Table IV shows the averaged IGD in each digit and benchmark problem. The averaged IGD is evaluated at the 10th, half, and final generation because it is important to maintain diversity during optimization in MOEA as well as to obtain better-distributed Pareto-optimal solutions. As shown, in DTLZ2 and DTLZ3, the final IGD can improve as the number of digits drops. In addition, diversity is maintained during optimization in these problems. In the UF, CDTLZ, and Welded Beam test problems, the IGD can improve as the number of digits drops. However, in the DTLZ4 test problem, using a lower number of figures does not improve final IGD. DTLZ4 has the almost the same structure as DTLZ2. The major difference is that some variables carry the exponent $\alpha(= 100)$ in the objective functions in DTLZ4 so that small differences in the values of these variables have large effects on the values of the objective functions. Therefore, to maintain solution multiplicity, it is necessary to perform searches with high precision. In the WFG test problems, except for WFG7, using a lower number of figures does not improve the IGD.

The DTLZ test problems allow examination of the effect of the significant figures in detail.

Figure 1 shows the histories of GD in DTLZ2, DTLZ3, and DTLZ4. As shown in Fig. 1, lower resolution can improve both the final convergence and convergence speed. However, in the DTLZ3 test problem, though the difference in convergence speeds among applied resolutions was largest, the convergence was not sufficient in either case

Figure 2 shows the histories of IGD in DTLZ2, DTLZ3, and DTLZ4. As shown in Fig. 2, lower resolution in DTLZ2 and DTLZ3 can improve both the final distribution and distribution during optimization. On the other hand, in the DTLZ4 test problem, lower resolution worsens diversity.

Figures 3, 4, and 5 show the distribution of 2-and 16-digit resolutions in each DTLZ problem. As shown in Fig. 3, both the convergence and the distribution of the 2-digit resolution are same as those of the 16-digit resolution in the DTLZ2 test problem. As shown in Fig. 4, the convergence of 16-digit resolution is not enough compared to that of 2-digit resolution in the DTLZ3 test problem. We can find dominance-resistant solutions (DRSs)[16] from the distribution of 16-digit resolution. As shown in Fig. 5, the distribution of 2-digit resolution is entirely lost. Lower resolution was found to have a large effect on the search performance characteristics, particularly on convergence.

Both GD and IGD are not fully converged in the DTLZ3 test problem. Therefore, to compare progress to the final convergence, computations were made until sufficient convergence was attained. Figure 6 shows the computational result up to the 1500th generation in the DTLZ3 test problem. As shown, the difference in final convergence between all levels of precision was not large. However, at various stages of convergence, lower precision yielded better results than higher precision. In real-world optimization problems, turn-around times for optimization must be shortened, so it is significantly important to set the appropriate precision, depending on the problem.

Finally, further computations were preformed to consider this effect of handling significant figures. One possible reason is a difference of distribution in crossovers (SBXs) and polynomial mutation by digit precision. However, as seen by the results of a Wilcoxon rank-sum test, we found no result indicating any statistically significant difference between the distribution of low digits and the distribution of high digits. Crossover and polynomial mutation are not significantly affected by differences in digit precision.

To search further for the cause of evolutionary differences, we focused our investigation once again on the DTLZ3, where the largest differences in convergence resulting from differing applied precisions were found. As shown in Fig. 1(b), a large difference between the GD obtained with 2- and 16-digit precisions had already become apparent by generation 20.

Figures. 7(a) and (b) show the cumulative frequency histograms of design variable x_1 and x_2, obtained using 2- and 16-digits at the 20th generation. Design variables in DTLZ test problems are separated into those that determine the position in the objective function space and those that determine the distance from the origin in that space. Both x_1 and x_2 are for-

TABLE III

AVERAGED GD AT THE 10TH, HALF AND FINAL GENERATION. BEST CASE IS HIGHLIGHTED IN BOLD.

problem	MG	PS	Obj.	Var.	Gen	2-digit	4-digit	6-digit	8-digit	16-digit
						GD	GD	GD	GD	GD
DTLZ2	100	100	3	38	10	**1.647**	1.756	1.766	1.759	1.759
					50	$\mathbf{1.576 \times 10^{-1}}$	2.247×10^{-1}	2.414×10^{-1}	2.595×10^{-1}	2.565×10^{-1}
					100	$\mathbf{5.525 \times 10^{-2}}$	1.576×10^{-1}	9.490×10^{-2}	1.003×10^{-1}	1.011×10^{-1}
DTLZ3	200	500	3	38	10	$\mathbf{2.191 \times 10^{3}}$	2.508×10^{3}	2.518×10^{3}	2.512×10^{3}	2.508×10^{3}
					100	$\mathbf{9.935 \times 10}$	2.230×10^{2}	2.878×10^{2}	3.154×10^{2}	4.039×10^{2}
					200	**6.521**	4.580×10	5.622×10	6.196×10	6.669×10
DTLZ4	100	300	3	38	10	$\mathbf{8.213 \times 10^{-1}}$	1.286	1.216	1.223	1.223
					50	$\mathbf{1.910 \times 10^{-2}}$	5.531×10^{-2}	6.576×10^{-2}	6.889×10^{-2}	7.147×10^{-2}
					100	$\mathbf{1.101 \times 10^{-2}}$	1.819×10^{-2}	2.320×10^{-2}	2.363×10^{-2}	2.522×10^{-2}
UF2	200	200	2	20	10	$\mathbf{2.978 \times 10^{-1}}$	3.106×10^{-1}	3.149×10^{-1}	3.001×10^{-1}	3.001×10^{-1}
					100	2.639×10^{-2}	$\mathbf{2.312 \times 10^{-2}}$	2.362×10^{-2}	2.451×10^{-2}	2.640×10^{-2}
					200	1.573×10^{-2}	$\mathbf{1.349 \times 10^{-2}}$	1.573×10^{-2}	1.532×10^{-2}	1.701×10^{-2}
UF9	300	200	3	20	10	3.063	3.048	2.695	**2.684**	**2.684**
					150	6.940×10^{-1}	$\mathbf{5.745 \times 10^{-1}}$	6.584×10^{-1}	6.003×10^{-1}	6.425×10^{-1}
					300	$\mathbf{4.206 \times 10^{-1}}$	4.518×10^{-1}	4.892×10^{-1}	4.470×10^{-1}	4.389×10^{-1}
WFG2	100	100	3	10	10	2.350×10^{-1}	$\mathbf{2.218 \times 10^{-1}}$	2.428×10^{-1}	2.428×10^{-1}	2.428×10^{-1}
				$(k = 6)$	50	$\mathbf{6.324 \times 10^{-2}}$	7.273×10^{-2}	6.490×10^{-2}	7.088×10^{-2}	7.141×10^{-2}
				$(l = 4)$	100	4.277×10^{-2}	4.745×10^{-2}	4.516×10^{-2}	4.530×10^{-2}	$\mathbf{4.220 \times 10^{-2}}$
WFG4	100	100	3	10	10	2.249×10^{-1}	2.178×10^{-1}	2.143×10^{-1}	$\mathbf{2.134 \times 10^{-1}}$	$\mathbf{2.134 \times 10^{-1}}$
				$(k = 6)$	50	9.867×10^{-2}	9.530×10^{-2}	9.935×10^{-2}	9.563×10^{-2}	$\mathbf{9.510 \times 10^{-2}}$
				$(l = 4)$	100	6.772×10^{-2}	6.628×10^{-2}	6.678×10^{-2}	$\mathbf{6.619 \times 10^{-2}}$	6.654×10^{-2}
WFG6	100	100	3	10	10	3.896×10^{-1}	3.611×10^{-1}	3.592×10^{-1}	$\mathbf{3.560 \times 10^{-1}}$	$\mathbf{3.560 \times 10^{-1}}$
				$(k = 6)$	50	1.545×10^{-1}	$\mathbf{1.432 \times 10^{-1}}$	1.590×10^{-1}	1.653×10^{-1}	1.653×10^{-1}
				$(l = 4)$	100	1.201×10^{-1}	$\mathbf{1.114 \times 10^{-1}}$	1.276×10^{-1}	1.215×10^{-1}	1.215×10^{-1}
WFG7	100	100	3	10	10	3.525×10^{-1}	$\mathbf{3.506 \times 10^{-1}}$	3.553×10^{-1}	3.602×10^{-1}	3.602×10^{-1}
				$(k = 6)$	50	1.226×10^{-1}	1.086×10^{-1}	$\mathbf{9.925 \times 10^{-2}}$	1.061×10^{-1}	1.061×10^{-1}
				$(l = 4)$	100	7.178×10^{-2}	$\mathbf{5.886 \times 10^{-2}}$	6.009×10^{-2}	6.561×10^{-2}	6.517×10^{-2}
CF2	200	300	2	20	10	$\mathbf{8.459 \times 10^{-1}}$	9.424×10^{-1}	9.266×10^{-1}	9.583×10^{-1}	9.583×10^{-1}
					100	$\mathbf{2.069 \times 10^{-1}}$	2.539×10^{-1}	2.729×10^{-1}	2.795×10^{-1}	2.851×10^{-1}
					200	1.694×10^{-1}	$\mathbf{1.500 \times 10^{-1}}$	1.987×10^{-1}	2.051×10^{-1}	1.964×10^{-1}
CF7	200	300	2	20	10	2.584×10	2.605×10	$\mathbf{2.359 \times 10}$	2.679×10	2.721×10
					100	**1.044**	1.329	1.242	2.232	1.792
					200	$\mathbf{7.834 \times 10^{-2}}$	4.992×10^{-1}	4.602×10^{-1}	7.675×10^{-1}	6.057×10^{-1}
C1DTLZ3	300	100	3	38	10	$\mathbf{2.661 \times 10^{3}}$	2.829×10^{3}	2.845×10^{3}	2.871×10^{3}	2.871×10^{3}
					150	$\mathbf{1.051 \times 10^{2}}$	2.214×10^{2}	2.601×10^{2}	3.168×10^{2}	3.267×10^{2}
					300	$\mathbf{4.986 \times 10}$	1.012×10^{2}	1.043×10^{2}	1.297×10^{2}	1.243×10^{2}
C2DTLZ2 convex	300	100	3	38	10	$\mathbf{4.041 \times 10}$	5.867×10	7.045×10	7.017×10	7.017×10
					150	$\mathbf{1.091 \times 10^{-1}}$	1.299×10^{-1}	1.160×10^{-1}	1.191×10^{-1}	1.114×10^{-1}
					300	$\mathbf{9.363 \times 10^{-2}}$	9.723×10^{-2}	9.941×10^{-2}	9.688×10^{-2}	9.450×10^{-2}
C3DTLZ1	300	100	3	38	10	1.163×10^{3}	1.166×10^{3}	$\mathbf{1.152 \times 10^{3}}$	$\mathbf{1.152 \times 10^{3}}$	$\mathbf{1.152 \times 10^{3}}$
					150	$\mathbf{2.497 \times 10^{2}}$	2.797×10^{2}	3.001×10^{2}	4.157×10^{2}	3.801×10^{2}
					300	$\mathbf{1.721 \times 10^{2}}$	2.002×10^{2}	2.144×10^{2}	2.304×10^{2}	2.587×10^{2}
Car Side Impact	300	500	3	7	10	7.528×10^{-2}	7.492×10^{-2}	7.618×10^{-2}	7.269×10^{-2}	$\mathbf{6.863 \times 10^{-2}}$
					150	$\mathbf{2.753 \times 10^{-2}}$	3.066×10^{-2}	3.167×10^{-2}	3.173×10^{-2}	3.003×10^{-2}
					300	$\mathbf{2.602 \times 10^{-2}}$	2.720×10^{-2}	2.970×10^{-2}	2.826×10^{-2}	2.881×10^{-2}
Welded Beam	300	500	2	4	10	$\mathbf{1.247 \times 10^{-1}}$	6.547×10^{-1}	5.801×10^{-1}	5.850×10^{-1}	5.850×10^{-1}
					150	2.395×10^{-3}	3.043×10^{-3}	2.330×10^{-3}	2.764×10^{-3}	$\mathbf{2.071 \times 10^{-3}}$
					300	2.094×10^{-3}	1.854×10^{-3}	1.159×10^{-3}	$\mathbf{1.071 \times 10^{-3}}$	2.295×10^{-3}

MG = the number of generation set; PS = the number of population size set.
Obj. = the number of objectives; Var. = the number of variables.
Gen = the generation that results were computed on.
k = position parameter in WFG problems; l = distance parameter in WFG problems.

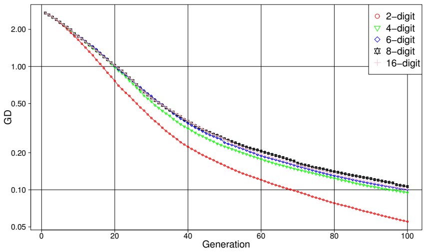
TABLE IV
AVERAGED IGD AT THE 10TH, HALF AND FINAL GENERATION. BEST CASE IS HIGHLIGHTED IN BOLD.

problem	MG	PS	Obj.	Var.	Gen	2-digit IGD	4-digit IGD	6-digit IGD	8-digit IGD	16-digit IGD
					10	$\mathbf{1.261}$	1.269	1.278	1.264	1.264
DTLZ2	100	100	3	38	50	$\mathbf{1.469 \times 10^{-1}}$	1.662×10^{-1}	1.753×10^{-1}	1.782×10^{-1}	1.782×10^{-1}
					100	$\mathbf{5.433 \times 10^{-2}}$	6.017×10^{-2}	6.096×10^{-2}	6.453×10^{-2}	6.213×10^{-2}
					10	$\mathbf{1.657 \times 10^{3}}$	1.694×10^{3}	1.685×10^{3}	1.647×10^{3}	1.666×10^{3}
DTLZ3	200	500	3	38	100	$\mathbf{6.871 \times 10}$	1.545×10^{2}	1.921×10^{2}	2.001×10^{2}	2.195×10^{2}
					200	$\mathbf{5.059}$	3.451×10	4.000×10	4.369×10	4.607×10
					10	$\mathbf{1.174}$	1.257	1.241	1.236	1.236
DTLZ4	100	300	3	38	50	6.164×10^{-1}	3.498×10^{-1}	$\mathbf{3.195 \times 10^{-1}}$	3.503×10^{-1}	3.517×10^{-1}
					100	5.803×10^{-1}	2.773×10^{-1}	2.794×10^{-1}	$\mathbf{2.507 \times 10^{-1}}$	$\mathbf{2.507 \times 10^{-1}}$
					10	2.131×10^{-1}	$\mathbf{2.125 \times 10^{-1}}$	2.256×10^{-1}	2.283×10^{-1}	2.283×10^{-1}
UF2	200	200	2	20	100	$\mathbf{4.521 \times 10^{-2}}$	4.646×10^{-2}	5.174×10^{-2}	5.228×10^{-2}	5.312×10^{-2}
					200	4.147×10^{-2}	$\mathbf{3.917 \times 10^{-2}}$	4.591×10^{-2}	4.411×10^{-2}	4.534×10^{-2}
					10	1.161	1.183	$\mathbf{1.069}$	1.102	1.102
UF9	300	200	3	20	150	3.057×10^{-1}	$\mathbf{2.840 \times 10^{-1}}$	3.077×10^{-1}	3.169×10^{-1}	3.145×10^{-1}
					300	$\mathbf{2.272 \times 10^{-1}}$	2.235×10^{-1}	2.487×10^{-1}	2.543×10^{-1}	2.537×10^{-1}
				10	10	4.981×10^{-1}	$\mathbf{4.963 \times 10^{-1}}$	4.964×10^{-1}	4.964×10^{-1}	4.964×10^{-1}
WFG2	100	100	3	$(k = 6)$	50	3.452×10^{-1}	3.462×10^{-1}	3.307×10^{-1}	3.087×10^{-1}	$\mathbf{3.072 \times 10^{-1}}$
				$(l = 4)$	100	3.008×10^{-1}	2.945×10^{-1}	3.185×10^{-1}	$\mathbf{2.699 \times 10^{-1}}$	2.713×10^{-1}
				10	10	4.941×10^{-1}	$\mathbf{4.402 \times 10^{-1}}$	4.466×10^{-1}	4.638×10^{-1}	4.638×10^{-1}
WFG4	100	100	3	$(k = 6)$	50	1.520×10^{-1}	1.477×10^{-1}	1.546×10^{-1}	1.465×10^{-1}	$\mathbf{1.439 \times 10^{-1}}$
				$(l = 4)$	100	1.003×10^{-1}	9.719×10^{-2}	1.016×10^{-1}	9.837×10^{-2}	$\mathbf{9.700 \times 10^{-2}}$
				10	10	4.857×10^{-1}	4.829×10^{-1}	4.589×10^{-1}	$\mathbf{4.538 \times 10^{-1}}$	$\mathbf{4.538 \times 10^{-1}}$
WFG6	100	100	3	$(k = 6)$	50	1.834×10^{-1}	$\mathbf{1.690 \times 10^{-1}}$	1.810×10^{-1}	1.846×10^{-1}	1.846×10^{-1}
				$(l = 4)$	100	1.345×10^{-1}	$\mathbf{1.247 \times 10^{-1}}$	1.346×10^{-1}	1.359×10^{-1}	1.359×10^{-1}
				10	10	5.090×10^{-1}	5.143×10^{-1}	5.014×10^{-1}	$\mathbf{4.986 \times 10^{-1}}$	$\mathbf{4.986 \times 10^{-1}}$
WFG7	100	100	3	$(k = 6)$	50	2.361×10^{-1}	$\mathbf{2.246 \times 10^{-1}}$	2.375×10^{-1}	2.345×10^{-1}	2.345×10^{-1}
				$(l = 4)$	100	1.779×10^{-1}	1.701×10^{-1}	1.722×10^{-1}	$\mathbf{1.658 \times 10^{-1}}$	1.664×10^{-1}
					10	4.487×10^{-1}	$\mathbf{4.484 \times 10^{-1}}$	4.807×10^{-1}	4.719×10^{-1}	4.715×10^{-1}
CF2	200	300	2	20	100	1.074×10^{-1}	$\mathbf{9.756 \times 10^{-2}}$	9.820×10^{-2}	1.023×10^{-1}	1.015×10^{-1}
					200	1.073×10^{-1}	9.058×10^{-2}	9.864×10^{-2}	9.672×10^{-2}	$\mathbf{8.944 \times 10^{-2}}$
					10	1.747×10	1.751×10	$\mathbf{1.720 \times 10}$	1.742×10	1.765×10
CF7	200	300	2	20	100	$\mathbf{4.949 \times 10^{-1}}$	5.319×10^{-1}	$\mathbf{4.949 \times 10^{-1}}$	6.665×10^{-1}	7.408×10^{-1}
					200	5.217×10^{-1}	3.410×10^{-1}	3.300×10^{-1}	3.072×10^{-1}	$\mathbf{2.853 \times 10^{-1}}$
					10	$\mathbf{1.964 \times 10^{3}}$	2.052×10^{3}	$\mathbf{1.940 \times 10^{3}}$	1.950×10^{3}	1.950×10^{3}
C1DTLZ3	300	100	3	38	150	$\mathbf{9.702 \times 10}$	1.660×10^{2}	1.783×10^{2}	2.002×10^{2}	1.956×10^{2}
					300	$\mathbf{4.338 \times 10}$	7.928×10	7.859×10	9.252×10	8.024×10
					10	$\mathbf{3.799}$	4.373	4.221	4.338	4.338
C2DTLZ2	300	100	3	38	150	$\mathbf{7.416 \times 10^{-2}}$	8.165×10^{-2}	7.831×10^{-2}	7.594×10^{-2}	7.828×10^{-2}
convex					300	$\mathbf{7.111 \times 10^{-2}}$	7.861×10^{-2}	7.477×10^{-2}	7.388×10^{-2}	7.260×10^{-2}
					10	7.123×10^{2}	6.939×10^{2}	$\mathbf{6.765 \times 10^{2}}$	6.938×10^{3}	6.938×10^{3}
C3DTLZ1	300	100	3	38	150	$\mathbf{5.704 \times 10}$	7.554×10	8.131×10	9.100×10	8.776×10^{2}
					300	$\mathbf{1.921 \times 10}$	2.938×10	3.119×10	3.399×10	3.508×10
					10	2.222×10^{-1}	1.836×10^{-1}	1.861×10^{-1}	2.025×10^{-1}	$\mathbf{1.824 \times 10^{-1}}$
Car Side Impact	300	500	3	7	150	2.896×10^{-2}	2.954×10^{-2}	2.934×10^{-2}	2.926×10^{-2}	$\mathbf{2.879 \times 10^{-2}}$
					300	2.289×10^{-2}	2.261×10^{-2}	2.230×10^{-2}	2.229×10^{-2}	$\mathbf{2.258 \times 10^{-2}}$
					10	2.932×10^{-1}	2.986×10^{-1}	$\mathbf{2.495 \times 10^{-1}}$	2.507×10^{-1}	2.507×10^{-1}
Welded Beam	300	500	2	4	150	6.491×10^{-2}	6.480×10^{-2}	5.166×10^{-2}	4.184×10^{-2}	$\mathbf{3.196 \times 10^{-2}}$
					300	3.929×10^{-2}	$\mathbf{1.808 \times 10^{-2}}$	4.589×10^{-2}	3.456×10^{-2}	1.863×10^{-2}

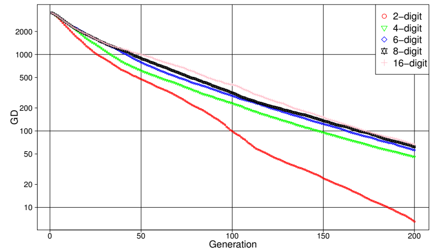
mer variables. Here we call the former position variables. The distribution obtained using either precision indicate a strong tendency for large-frequency concentrations near the poles 0 and 1. The cause of this concentration may be that when the position variables are near 0 and 1, these become polar regions in the objective function space, where non-inferior solutions readily form and are frequently taken as parents. These DRSs reportedly tend to prevent effective optimization.

As shown in Figs. 7(a) and (b), 2-digit precision yields fewer individuals near poles than 16-digit precision does, suggesting that 2-digit precision tends to reduce the occurrence of DRSs, thereby reducing the number of search iterations near these poles. Discretization of the number of significant digits is equivalent to discretization of the objective function space, and a reduction in the number of DRSs that can be generated may be the result of improved convergence. In this hypothesis, observed increases in convergence and diversity by the use of lower applied digit precision results from its prevention of evolution-impeding DRS formation.

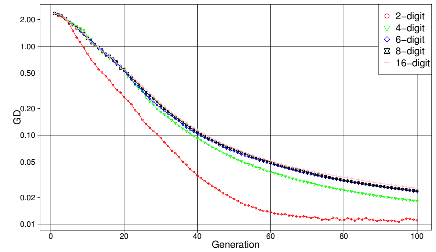
We tested this hypothesis using DTLZ3 with 16-digit applied precision overall, but modified it by reducing the applied precision when solutions approached the poles to determine whether lower precision tended to prevent DRS formation, thus promoting efficient evolution. In DTLZ3, the approach



(a) DTLZ2

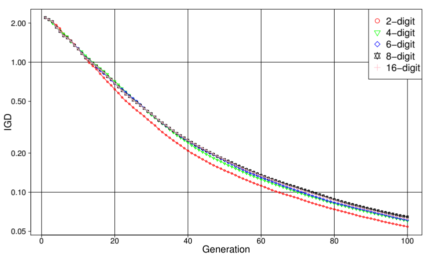


(b) DTLZ3

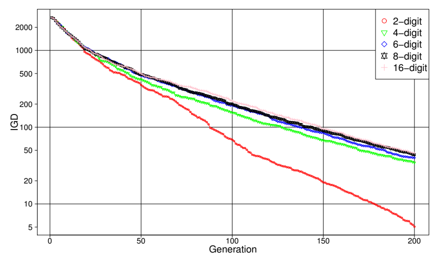


(c) DTLZ4

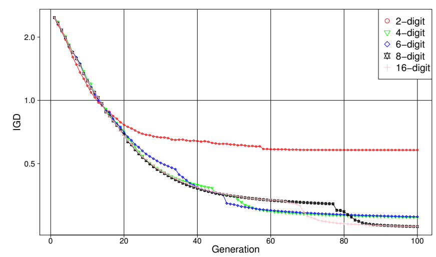
Fig. 1. History of GD



(a) DTLZ2

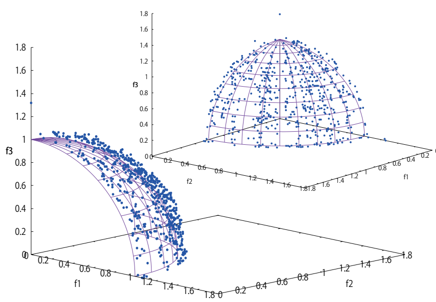


(b) DTLZ3

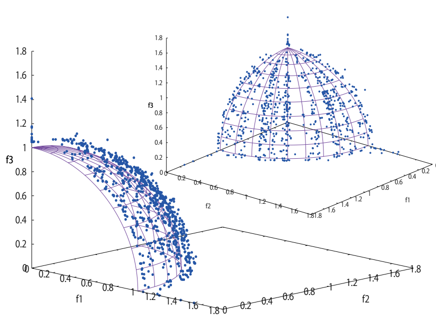


(c) DTLZ4

Fig. 2. History of IGD

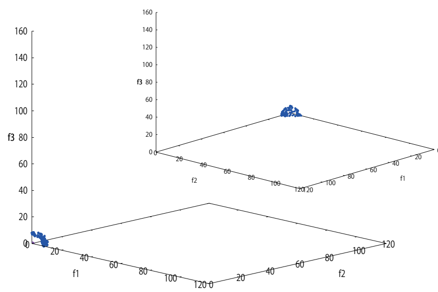


(a) 2-digit

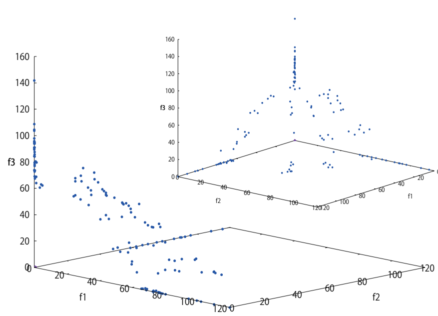


(b) 16-digit

Fig. 3. Distribution of non-dominated solutions (DTLZ2)

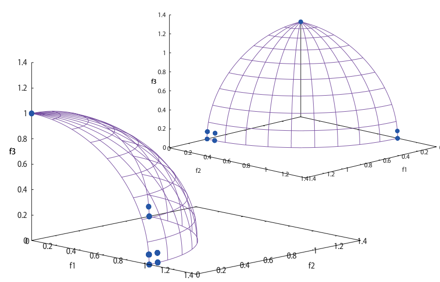


(a) 2-digit

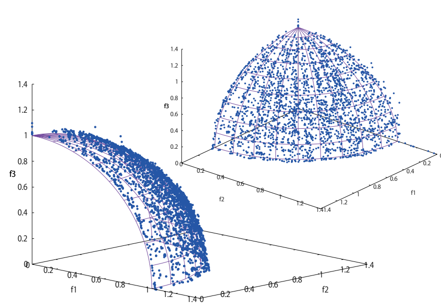


(b) 16-digit

Fig. 4. Distribution of non-dominated solutions (DTLZ3)



(a) 2-digit



(b) 16-digit

Fig. 5. Distribution of non-dominated solutions (DTLZ4)

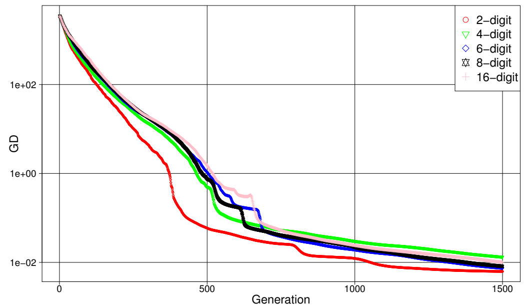


Fig. 6. GD transitions to generation 1500 (DTLZ3).

to the poles occurred when the position variables were near 0 or 1. The applied precision was then lowered to 2 digits whenever the position variables x_1 and x_2 reached 0.1 or less or 0.9 or more. The same computational conditions given in Table II were used.

As shown in Fig. 7(c), the modified 16-digit precision substantially reduced the number of individuals near poles 0 and 1. Table V shows the averaged GD and IGD at the 10th, half, and final generation when using 2-, 16-, and modified 16-digit precisions. Figure 8 shows the histories of GD and IGD, and plots of the final non-dominated solutions. The modified 16-digit precision yielded substantially better convergence than the non-modified 16-digit precision. As shown by the plot of non-dominated solutions in Fig. 8(c), the modified 16-digit precision improved both convergence and diversity. These results clearly indicate that convergence is accelerated in DTLZ3 by locally reducing the applied precision, thereby preventing DRS formation. This tendency is also evident in the distributions found in DTLZ2, and in the smaller number of DRSs using 2-digit precision. This suggests that in DTLZ2 as well as DTLZ3, lowering the applied precision reduces DRSs formation, thereby accelerating convergence.

IV. Conclusion

In this study, we investigated the general effects of convergence and diversity via discrete design variables changed by the number of significant figures using NSGA-II and various types of multi-objective problems. The results show that using low-precision digits quickly creates convergence in many cases, but it does not always produce better distribution. In actual engineering optimization problems, turn-around time for optimization must be shortened, so it is significantly important to set the appropriate precision, depending on the problem.

As seen by the results of the Wilcoxon rank-sum test, SBX, and polynomial mutation are not significantly affected by the number of significant figures in design variables. One factor that allows quick convergence while using low-precision digits is DRSs. Simulated binary crossover and polynomial mutation are not significantly affected by the number of significant figures in design variables. If DRSs are not created using lower-digit precision, using lower-digit precision with NSGA-II may accelerate convergence.

Further investigation will be necessary to find techniques for dynamically selecting the appropriate level of precision for efficient searching, including cases of real-world problems with unknown properties. One possible procedure may consist of first performing rapid low-resolution search and convergence using a low precision and then applying a higher precision for the higher-resolution search.

In addition, we only changed significant figures after the decimal point in this study so that all variables are same degree of discretization by digits precision. Design variables may have different number of granularity even if variables have same digits precision in general. Therefore, future work also includes examining the effects of resolution while using the same number of granularity with design variables.

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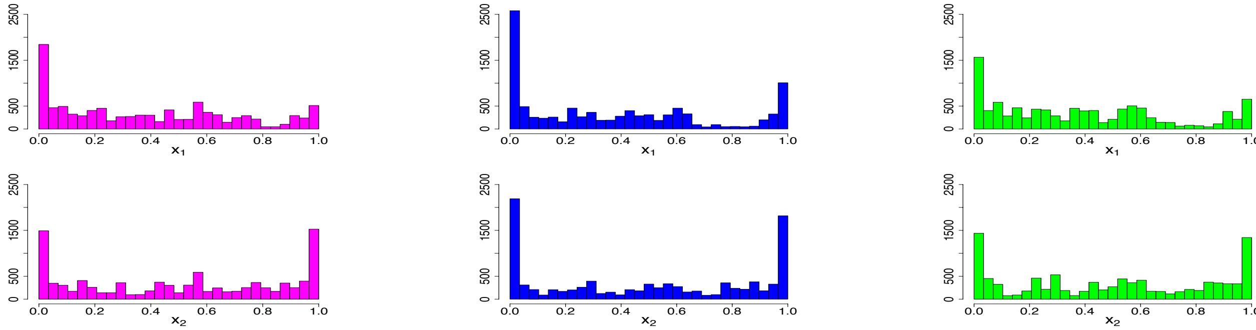


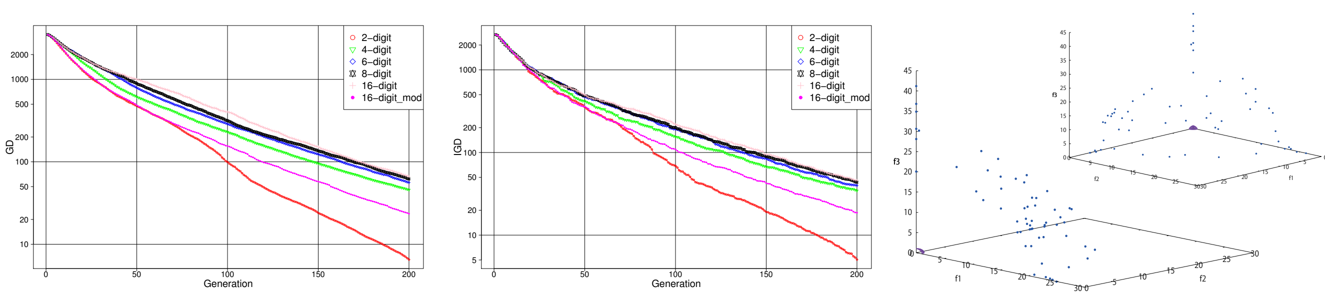
(a) 2-digit (b) 16-digit (c) modified 16-digit

Fig. 7. Cumulative-frequency histograms of position variables x1 and x2 obtained using 2- and 16- and modified 16-digit.

TABLE V

AVERAGED GD AND IGD AT 10TH, HALF AND FINAL GENERATION FOR USING 2-DIGIT, 16-DIGIT, MODIFIED 16-DIGIT ON DTLZ3. BEST PERFORMANCE IS SHOWN IN BOLD.

problem	MG	PS	Obj.	Var.	Gen	2-digit	16-digit	modified 16-digit
						GD	GD	GD
DTLZ3	200	500	3	38	10	$\mathbf{2.191 \times 10^3}$	2.508×10^3	2.210×10^3
					100	$\mathbf{9.935 \times 10}$	4.039×10^2	1.554×10^2
					200	**6.521**	6.669×10	2.345×10
						IGD	IGD	IGD
DTLZ3	200	500	3	38	10	$\mathbf{1.657 \times 10^3}$	1.666×10^3	1.675×10^3
					100	$\mathbf{6.871 \times 10}$	2.195×10^2	1.090×10^2
					200	**5.059**	4.607×10	1.867×10



(a) History of GD (b) History of IGD (c) modified 16-digit

Fig. 8. GD / IGD histories and plots of final non-dominated individuals in the case of using modified 16-digit on DTLZ3.

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