Load Dependent Lead Times and Sustainability

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Abstract—The reduction of waste is a concept companies adopt in order to enhance their friendliness versus the environment and to contribute to sustainability. Constraints on lifetimes of products force organizations to carefully plan their production in cooperation with their supply chain partners in order to prevent decreased quality or waste of production parts and products. This gains importance because waiting times imply longer lead times charging the production system with work in process inventories. Longer lead times can lead to quality losses due to depreciation, so that parts need to be reworked if possible or discarded. But return flows of products for rework or remanufacturing actions significantly complicate the production planning process. We analyze sustainability options with respect to lead time management by formulating a comprehensive mathematical model. We consider a deterministic, mixed-integer programming model to highlight main characteristics of sustainability options and to derive managerial insights for production planning in the light of sustainability.

I. INTRODUCTION

Lead times of products from the receipt of an order until the delivery of the product at the final customer are among the important key performance indicators of today's business. Only a few models exist for production and lot size planning that take into consideration lead times that depend on the workload, known as load dependent lead times (LDLT); see [1,2]. The same is valid for models linking order releases, planning, and capacity decisions to lead times taking into account the system utilization, batching/lot sizing, and sequencing decisions. For instance, reducing lot sizes reduces inventory holding, but increases setups that consume capacity. On the other hand, increased lot sizes force items to wait for processing which increases work in process (WIP) within the production system. In case that items have a limited lifetime and their quality degrades while waiting, items might pass their valuable lifetime and need to be either discarded or reworked. If rework is possible and performed with regular production resources, i.e., "in-line," resource utilization further raises due to additional setups and processing for rework items causing a downward spiral on lead times and item quality [2].

Models for production and supply chain planning are frequently formulated as *mixed-integer programs* (MIP) where various versions exist; see, e.g., [3] for an overview. One of the most frequently applied models is the *capacitated lot sizing problem* (CLSP) that is also implemented in software systems for enterprise resource planning. It is NP-hard, so that many researchers propose heuristics for solving larger test instances, also being useful to understand mechanisms and relationships of planning issues. In this paper, we present a MIP based on the CLSP including production smoothing for a production system with lifetime restrictions on items, external remanufacturing streams as well as internal rework which pertains to sustainability with the aim of smoothing utilization of production and rework actions. These characteristics can be attributed, e.g., to chip and wafer fabrication where wafers can be recycled and reused multiple times. Perishability occurs due to two issues where the first is the short life cycles of wafers leading to obsolescence of wafer value. The second pertains to in-process waiting time-related deterioration/contamination that requires, e.g., re-polishing. Generally, defective wafers are reworked in the photo-lithography area which is also the system bottleneck. Return flows may consist in "reclaim wafers." These are reworked and used as test wafers for electronic tests and quality assurance; see also [4] for details on wafer types. Moreover, recycling ratios of new and reclaim wafers versus recycled wafers can be assumed with 78% [4]. We use high and low return rates of recycled products for model testing in our numerical study and present our findings as well as conclusions including an outlook for future research.

II. INTEGRATING LDLT AND SUSTAINABILITY

Lead time management and the question "how to do it quicker" has been in the focus of management since the 1990s with considerable interest in the role of lead times in production and supply chain planning. Classical models for production and supply chain planning treat lead times as static input data, but in most situations, the output of a planning model implies capacity utilization which, in turn, influences lead times. This circularity has received much attention in recent years [5]–[8]. Nevertheless, the influence of restricted lifetimes on quality of items and related rework action has not been extensively taken into account [2].

A. LDLT and Production Smoothing

Most models for aggregate planning integrate lead time information using estimates based on experience and rules of thumb as input parameters for planning. But, in most situations, the output of a planning model determines planned capacity utilization which, in turn, influences estimated/planned lead times. For instance, *worst case* lead times might be used in order to have enough "buffer time" to fulfill accepted orders regarding a certain time interval and thus satisfy demand [9]. Orders may be released earlier into the system than necessary to ensure delivery due dates, so that WIP builds up way ahead and resource utilization increases. Items are forced to wait, so that lead times increase, too, compromising the initial idea to assure due dates and service levels. This over-reaction that corrupts lead times leading to high variability becomes a self-fulfilling prophecy and is addressed in the literature as the *lead time syndrome*. It shows the results if the relations between average lead times, WIP, and workload, are ignored; for extensive studies see [10,11].

1) Lead Time Estimates: In the literature, various researchers concentrate on the estimation of planned lead times [12]–[14] by, e.g., testing and iterating lead time estimates regarding their influence on the system using simulation. However, the relationship between system workload, resource utilization, and lead time(s) (distributions) is not taken into account in mathematical models. Besides, not much work is provided that analyzes planned lead times and workload in environments subject to highly varying demands [10]. For instance, the authors of [10] study production smoothing methods with the aim of reducing subcontracting and overtime as options to deal with demand peaks and variability. They propose smoothing strategies on the master planning schedule and planned lead time control at multiple workstations. Other authors incorporate the relation between lead times and production (system) workload in their optimization models linearizing the resulting nonlinear relationship of WIP-related exponentially increasing lead times; see, e.g., [5.8,15].

2) Production Smoothing: Inventory holding is one of the classical tools to smooth production and buffer against demand peaks [16]. If products have limited lifetimes, inventory holding as a smoothing strategy is limited especially regarding quality issues. Smoothing resource utilization of bottleneck machines focuses on workloads that are penalized in the objective function thus preferring lower utilization rates [17,18]. This is contrary to practice aiming at 100% resource utilization avoiding idle times especially in capital intensive industries such as, e.g., wafer fabrication and semiconductor manufacturing. Maximum values smaller or equal 10% idle times are a frequently stated targets [19]. However, the lead time syndrome states that releasing jobs earlier in the production system increases WIP and thus lead times. Thus, production smoothing is likely to have a negative effect on lead times. On the other hand, there is a resource utilization level after which lead times rise significantly and in an exponential manner. As shown by queuing models and also confirmed in practice [20], there is a desirable sector of operation and thus resource utilization (also denoted "operation curve" mainly in the German speaking literature; see [21,22]) where slight reductions of this level lead to great reductions in lead times. This does not say much on WIP, but if workload measured in WIP is another performance indicator of utilization, then lead times can be reduced by holding similar amounts of WIP. Therefore, this operation sector should be targeted by production smoothing actions.

Production smoothing has mainly been taken into account in synchronized assembly line systems; see, e.g., [17,18] and the references therein. Analysis concentrates on both the output rate of finished goods inventory (FGI) at the final assembly stage (output rate variation (ORV)) and on the product rate variation (PRV) which refers to the pull rate of WIP. According to [17], the major part of research is on the PRV, possibly due to raised complexity regarding the settings where ORV is relevant. Setup times are frequently assumed as negligible in these production environments and processing times allow to produce exactly one unit in a time period; see also [17,18]. Accordingly, scheduling of multiple products with the aim to smooth overall workload has not been extensively studied. One example of work in the multiple product case is [23] that considers a PRV approach in a single (final stage) workstation system with stochastic setup and processing times.

Production smoothing is frequently integrated in mathematical models by modifying the objective function according to some criterion(s) as in [24], e.g., defining indicators such as a usage goal that denotes the deviation of actual and ideal resource consumption or a loading goal of an overall production system determining the deviation between planned and actual workload. These can be integrated into an objective function of an optimization model by using, e.g., absolute or *squared* values for the deviation that can take positive or negative values. Other production smoothing criteria taken into account by researchers are the minimization of total production rate variations, WIP, the maximization of system utilization, or responsiveness, where a piecewise linear cost term is used to motivate planned capacity utilization around 70 - 85%. This selection point for the target utilization in the objective function can be implemented using special ordered sets of variables (SOS), e.g., SOS1 (SOS of type 1) as another method to specify the integral condition. SOS1 require that at most one variable takes a strictly positive value while all others be zero. However, there does not seem to be any criterion superior over the other(s) as stated in [18].

3) Our Implementation of Production Smoothing: The model presented in this paper integrates the interval selection according to planned capacity utilization employing binary variables, denoted by $y_{bt} \in \{0, 1\}$ where $b = 1, \ldots, B$ denotes the *B* breakpoints of the piecewise linear capacity constraint. They give the interval borders regarding the utilization sectors where the cost terms are selected depending on the planned processing and setup times, accordingly. In order to correctly model the cost terms, we differentiate between fixed utilization costs when a subsequent interval (or sector) is reached and variable costs according to the planned utilized capacity units. In each time period *t*, an interval/sector needs to be selected according to the planned capacity utilization:

$$\sum_{b=1}^{B} y_{bt} = 1 \qquad \forall t = 1, \dots, T \tag{1}$$

The planned capacity utilization in a time period t is attributed to the load dependent indicator as follows:

$$\sum_{i=1}^{N} \sum_{t=1}^{T} (\xi_i \cdot x_{it} + st_i \cdot \delta_{it}) = \sum_{b=1}^{B} \operatorname{pscu}_{bt} \qquad \forall t \qquad (2)$$

where the first term denotes resource consumption by production and setup time where ξ_i is the resource consumption factor of producing one unit of item i and x_{it} is the decision variable of the amount of items i produced in period t. Moreover, st_i denotes the setup time of item i and δ_{it} is the setup variable. The resource consumption is required to be equal to the sum of the planned smoothed capacity utilization $\sum_{b=1}^{B} pscu_{bt}$ over all breakpoints B. We consider three intervals for the piecewise linear function. The intervals are defined as follows:

 $\forall b = 1, \dots, B, t = 1, \dots, T \quad (3)$ $\forall b = 1, \dots, B, t = 1 \quad T \quad (4)$ $pscu_{bt} \leq y_{bt} \cdot b_B$

$$pscu_{bt} \ge b_b \cdot y_{bt} \qquad \forall \ b = 1, \dots, B, t = 1, \dots, T \quad (4)$$

$$pscu_{bt} \le b_{b+1} \cdot y_{bt} \quad \forall \ b = 1, \dots, B-1, t = 1, \dots, T \quad (5)$$

Inequalitites (3) give the border of the last interval denoted by breakpoint B. Inequalities (4) denote the lower and Inequalities (5) the upper limit of $pscu_{bt}$. The borders of the intervals/sectors b_b are parameters and thus input data.



Fig. 1. Approximation of load dependent resource utilization costs

In order to integrate the load dependent processing and setup consumption into the objective function, we define a fixed cost term pcu_{h}^{f} and a variable cost term pcu_{h}^{v} as well as a cost parameter η to attribute costs to the planned consumed capacity $pscu_{bt}$ that are integrated as follows in the objective function:

$$\sum_{t=1}^{T} \left[\sum_{b=1}^{B} (pcu_b^f \cdot y_{bt} + \eta \cdot pscu_{bt} - pcu_b^v \cdot y_{bt} \cdot b_b) \right]$$
(6)

The load dependent capacity utilization costs are chosen according to the binary variable y_{bt} that becomes 1 if the utilization is within a certain interval; see also Figure 1.

B. Lifetime Restrictions

Deterioration can be defined as the gradual process of decay, damage, or spoilage of items due to various reasons, so that they cannot be used anymore for their original purpose [2]. They go through a change loosing their utility. Perishability can be differentiated from deterioration as a fixed, maximum lifetime like a "best-before" date in practice where the utility of items is all lost after that date due to external factors, laws and regulations etc. that determine their shelf lives [25]. Products including services as a main component incur similar issues, e.g., airline seats or hotel rooms [26,27]. As a result, we can distinguish two subcategories of deterioration: 1) functional deterioration over time. That holds, e.g., for fruit, vegetables, or milk. 2) customer perceived utility where the demand deteriorates although functions and physical condition of the products remain the same, e.g., fashion clothes, high technology products, newspapers [28]. Lifetime restrictions define the time point until which items can or should be used. Different methods are proposed in the literature that can be used to integrate lifetime restrictions into mathematical models for optimization depending on the model type (deterministic/stochastic and time-dependent (finite/infinite)). The interested reader is referred to [28] for these methods.

In this paper, we regard lifetime constraints due to perishability of items that can be reworked which is waitingdependent and integrated by modifying the inventory balance equation of well known production-inventory lot size (I&L) formulations [29] by subtracting items that passed their useful lifetime calculated as follows:

$$I_{it}^{D} \ge \sum_{u=1}^{t-\Theta_{i}^{L}} (x_{iu} + x_{iu}^{r}) - \sum_{u=1}^{t} d_{iu} - \sum_{u=1}^{t-1} I_{iu}^{D}, \qquad (7)$$
$$\forall i, t = 1, \dots, T; \ t \ge \Theta_{i}^{L}$$

where I_{it}^D is the amount of items *i* that perish in a time period t. These are calculated by the sum of produced items x_{iu} and reworked items x_{iu}^r in the time interval $(u, t - \Theta_i^L)$, i.e., the production from the beginning of the planning horizon until period t reduced by the length of the item's lifetime Θ_i^L , less the sum of those items that are used to satisfy demand of period u until period t, i.e. $\sum_{u=1}^{t} d_{iu}$, and items that have already been disposed in previous periods up to t - 1, i.e. $\sum_{u=1}^{t-1} I_{iu}^D$.

Spoilage costs need to be added in the objective function if no production costs are regarded which is mostly the case in classical discrete deterministic lot sizing models. If neither production costs nor spoilage costs are taken into account, the model might produce items and immediately dispose them to prevent inventory costs, because this is not punished by the objective function.

C. Sustainability Options: Remanufacturing and Rework

Sustainability can be widely understood as the "quality that permits to preserve, to keep, to maintain something" [30]. Therefore, if something is able to be kept, it is sustainable"

[31] implying the avoidance of permanently damaging the related item or environment, so that it is available for future generations. For an overview on sustainability in supply chains see [32]. In relation to this, we consider items that pass their lifetime in the production process and are good for rework actions in order to re-use them in the production process. Rework includes all recovery actions required to transform products that do not meet a specific quality (anymore) in a way that they regain quality standards [28,33]. We attribute rework to company-internal problems, e.g., unplanned waiting times or out-of-control production systems leading to defectives [34]. Instead, remanufacturing encompasses industrial processes where worn out products are returned from the customer and restored to "like new" conditions through a series of actions, e.g., dis-assembly, cleaning, refurbishing etc. [35]. In contrast to this, recycling is the recovery of raw materials or parts of used products, e. g., copper, by dis-assembly of products. Moreover, we differentiate between in-line and offline rework: in-line rework uses the same workstations as regular production, so that their utilization raises with rework; see Figure 2.



Fig. 2. In-line production and rework system including external returns

The storage of reworkables is filled with items that have acquired a defect in production or passed their useful lifetime while waiting to be processed together with external returns of used items that might be reworked or finally disposed; ; see also [36] for a Wagner Whitin model with remanufacturing. Assumptions on the point of origin of reworkables raises the complexity of the system, especially when amounts of return flows are unpredictable and/or uncertain. Moreover, return flows increase inventory costs and machine utilization, but decrease the need for replenishment of new parts or materials and, therefore, is both an environmentally friendly strategy and in favor of profit increases due to decreases of procurement costs [2].

In this paper we consider a single-stage production-rework system with a single workstation. A multi-stage setting involving multiple workstations is much more complex due to queuing effects and thus subject to further research.

D. The Optimization Model

The following notation is used to state the optimization model.

Parameters:

- Setup cost factor for product *i*. sc_i Rework setup cost factor for product *i*. rsc_i Inventory holding cost factor of item *i*. h_i h_i^r Rework inventory holding cost factor of item *i*. Disposal cost factor for discarding product *i*. ϕ_i ϕ_i^r Disposal cost factor for discarding reworkable product *i*. Costs factor for planned consumed capacity. η G_{it} Known external return flows of item i in period t. pcu_{h}^{f} Fixed cost term of interval/sector b of utilization. pcu_{h}^{v} Variable cost term of interval/sector b of utilization.
- Overtime cost factor for product *i*. oc_i
- Demand of product i in period t. d_{it}
- Θ_i^L Lifetime of product *i*.
- Available capacity in period t. Cap_t
- Resource consumption factor to produce one entity ξ_i of product *i*.
- Setup time for product *i*. st_i
- Rework setup time for product *i*. rst_i
- Cumulative demand, i. e., $\sum_{t=1}^{T} \sum_{i=1}^{N} d_{it}$. Minimal lot size for product *i*. d^k
- x_i^{\min}

Variables:

 z_i

 z_{it}^r

δ_{it}	Setup variable for product i in period t .
δ^r_{it}	Setup variable for reworking product i in period t .
$I_{it} \ge 0$	Inventory holding of product i in period t .
$I_{it}^r \ge 0$	Rework inventory holding of product i in period t .
$I_{it}^{\tilde{D}} \ge 0$	Amount of spoiled items of product i in period t .
$I_{it}^{Dr} \ge 0$	Amount of spoiled reworkable items of product i
	in period t.
y_{bt}	Binary variable for interval selection regarding
	load dependent utilization costs with breakpoints
	$b = 1, \ldots, B$ for every time period t.
O_t	Overtime used in period t .
$x_{it} \ge 0$	Production amount of product i in period t .
$x_{it}^r \ge 0$	Amount of reworkables of product i in period t .
z_{it}	Setup state for production defined as binary.

Setup state for rework defined as binary.

Indices:

$$i, u \in N$$
 Indices denoting products.
 $t = 1, ..., T$ Time periods.

Set of breakpoints for piecewise linear terms. $b=1,\ldots,B$

We include return flows in relation to demand of items i in the modified CLSP:

$$G_{it} = (1 - \alpha_i^R) \cdot d_{it} \tag{8}$$

where α_i^R is the assumed fraction of items that return from the customer. We add constraints calculating returned perishable items as follows:

$$I_{it}^{Dr} \ge \sum_{u=1}^{t-\Theta_i^L} \left(G_{it} + I_{it}^D \right) - \sum_{u=1}^{t-\Theta_i^L} x_{iu}^r - \sum_{u=1}^{t-1} I_{iu}^{Dr} \qquad \forall \ i,t \ (9)$$

The possibility to rework items is restricted by the returned item lifetime Θ_i^L as shown in the first term of Inequalities (9). We further integrate a rework/remanufacturing inventory balance equation to characterize the flow of remanufacturing stated as follows:

$$I_{it}^{r} = I_{i,t-1}^{r} + G_{it} + I_{it}^{D} - x_{it}^{r} - I_{it}^{Dr} \qquad \forall \ i, t = 1, \dots, T$$
(10)

The returned items/rework inventory balance is given by returned items that were already in inventory since previous periods t - 1, by new external returns G_{it} and internally perished items I_{it}^D less those items that are reworked/remanufactured to serve for demand satisfaction and those that are discarded, because there is no demand in the range of their lifetime restrictions. After remanufacturing of items, they regain full quality standards, so that they are indistinguishable from regular produced products and enter FGI. Rework/remanufacturing of returned items is further restricted by the amount of returns:

$$x_{it}^r \le G_{it} \qquad \forall \ i, t = 1, \dots, T \tag{11}$$

Constraints (11) prevent the system from *phantom* reworked items that come "out of nowhere." These constraints might further serve as a cutting plane to speed up the solution process. A further option is to constrain the perishables (Restriction (13)) from above, so that in case of zero external return flows, reworkables are not forced to be zero as well. Reworked/remanufactured items either serve to satisfy demand d_{it} in period t or they enter regular inventory I_{it} until they are used for demand together with regularly produced items x_{it} stated in the following constraint:

$$I_{it} = I_{i,t-1} + x_{it} + x_{it}^r - d_{it} - I_{it}^D \qquad \forall \ i, t = 1, \dots, T$$
(12)

Disposals at this stage may only be caused by minimal lot size constraints on regular production, because optimization models as the presented one avoid wastage in the first place as this imposes costs; see also [2,28]. As stated elsewhere, we solely constrain regular production to comply to a specific minimal lot size. Disposals of regular and/or reworked items are determined as follows:

$$I_{it}^{D} \ge \sum_{u=1}^{t-\Theta_{i}^{L}} (x_{iu} + x_{it}^{r}) - \sum_{u=1}^{t} d_{iu} - \sum_{u=1}^{t-1} I_{iu}^{D} \qquad \forall i, t > \Theta_{i}^{L}$$
(13)

The modification of the disposal constraint is done in the first term where reworked items that are now as good as new are restricted by the regular lifetime of item i.

The complete model including LDLT via production smoothing, setup-carry overs, minimal lot size constrains, overtime, as well as rework and remanufacturing is stated as follows:

$$\min \sum_{i=1}^{N} \sum_{t=1}^{T} (sc_i \cdot \delta_{it} + rsc_i \cdot \delta_{it}^r)$$

$$+ \sum_{i=1}^{N} \sum_{t=1}^{T} (h_i \cdot I_{it} + h_i^r \cdot I_{it}^r + \phi_i \cdot I_{it}^D + \phi_i^r \cdot I_{it}^{Dr})$$

$$+ \sum_{t=1}^{T} \left[\sum_{b=1}^{B} (pcu_b^f \cdot y_{bt} + \eta \cdot pscu_{bt} - pcu_b^v \cdot y_{bt} \cdot b_b) \right]$$

$$+ oc_i \cdot O_t$$

$$(14)$$

subject to

$$\begin{split} I_{it} &= I_{i,t-1} + x_{it} + x_{it}^r - d_{it} - I_{it}^D \quad \forall \ i,t = 1, \dots, T \\ I_{it}^r &= I_{i,t-1}^r + G_{it} + I_{it}^D - x_{it}^r - I_{it}^{Dr} \quad \forall \ i,t = 1, \dots, T \\ x_{it}^r &\leq G_{it} \qquad \forall \ i,t = 1, \dots, T \end{split}$$

$$I_{it}^{D} \ge \sum_{u=1}^{t-\Theta_{i}^{L}} (x_{iu} + x_{it}^{r}) - \sum_{u=1}^{t} d_{iu} - \sum_{u=1}^{t-1} I_{iu}^{D} \quad \forall i, t$$
$$I_{it}^{Dr} \ge \sum_{u=1}^{t-\Theta_{i}^{L}} (G_{it} + I_{it}^{D}) - \sum_{u=1}^{t-\Theta_{i}^{L}} x_{iu}^{r} - \sum_{u=1}^{t-1} I_{iu}^{Dr} \quad \forall i, t$$

$$\operatorname{Cap}_{t} + O_{t} \ge \sum_{i=1}^{N} \left(\xi_{i} \cdot x_{it} + st_{i} \cdot \delta_{it} \right)$$

$$+ \sum_{i=1}^{N} \left(t_{i} \cdot \xi^{T} - m^{T} + mot_{i} \cdot \delta^{T} \right) \quad \forall t = 1$$

$$(15)$$

$$+\sum_{i=1} (t \cdot \xi'_i \cdot x'_{it} + rst \cdot \delta'_{it}) \quad \forall t = 1, \dots, T$$
$$x_{it} \le (d^k + \operatorname{Cap}_t) \cdot (\delta_{it} + z_{it}) \quad \forall i, t = 1, \dots, T$$
(16)

$$x_{it}^{r} \le \left(d^{k} + \operatorname{Cap}_{t}\right) \cdot \left(\delta_{it}^{r} + z_{it}^{r}\right) \quad \forall \ i, t = 1, \dots, T$$
(17)

$$\sum_{i=1}^{N} (z_{it} + z_{it}^{r}) \le 1 \qquad \forall t = 1, \dots, T$$
(18)

$$z_{it} \le \delta_{i,t-1} + z_{i,t-1} \qquad \forall i,t = 1,\dots,T$$

(19)

$$z_{it}^{r} \leq \delta_{i,t-1}^{r} + z_{i,t-1}^{r}$$
 $\forall i, t = 1, ..., T$
(20)
 $x_{it} \geq x_{i}^{\min} \cdot (\delta_{it} + z_{it})$ $\forall i, t = 1, ..., T$
(21)
 $I_{i0} = I_{iT} = 0$ $\forall z_{it}$
(22)

$$_{i0}^{r} = I_{iT}^{r} = 0 \qquad \qquad \forall i$$
(23)

$$\begin{split} & \mathsf{pscu}_{bt} \leq y_{bt} \cdot b_B & \forall \ b = 1, \dots, B, t = 1, \dots, T \\ & \mathsf{pscu}_{bt} \geq b_b \cdot y_{bt} & \forall \ b = 1, \dots, B, t = 1, \dots, T \\ & \mathsf{pscu}_{bt} \leq b_{b+1} \cdot y_{bt} & \forall \ b = 1, \dots, B - 1, t = 1, \dots, T \end{split}$$

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$$\delta_{it}, \delta_{it}^{r}, z_{it}, z_{it}^{r}, y_{bt} \in \{0, 1\} \quad \forall \ i, t = 1, \dots, T$$
(24)
$$x_{it}, x_{it}^{r}, I_{it}, I_{it}^{r}, I_{it}^{D}, I_{it}^{Dr}, O_{t} \ge 0 \qquad \forall \ i, t = 1, \dots, T$$
(25)

The objective function minimizes total costs of regular production and rework setups, inventory, and rework inventory holding where the latter is less costly, disposal of regularly produced items and returned items, load dependent utilization costs, and overtime. Setup times are taken into account additionally to the unit times of nominal capacity included in the utilization term, so that setup variables are set correctly. Constraints (12) denote the inventory balance equation of regular production with the modification of the rework of returned items *i* that are available for demand satisfaction in period t. Constraints (10) state the returned items inventory balance equation that are composed by returned items *i* that were in inventory in previous periods t - 1, returned items G_{it} and internally perished items I_{it}^D , less those items x_{it}^r that are reworked and disposal of returned items I_{it}^{Dr} . Constraints (11) require that the amount of reworked items can never be greater than returns. Inequalities (13) and (9) determine the items that are disposed due to lifetime restrictions of regular or returned items and according to demand requirements. Returned items can stay in returned inventory until their lifetime expires. Constraints (15) present the regular capacity restrictions including capacity consumption due to waiting time dependent rework and rework setups. These are stated as upper bounds as in the classical CLSP. LDLT are accounted for regarding the planned smoothed utilization. Constraints (16) and (17) give the setups for regular or rework batches that can be carried over from/to the next period in both cases. Nevertheless, the machine state requires to be set up exactly for one product of regular production or rework presented by Constraints (18). Inequalities (19) and (20) determine the setup carry overs of machine states z_{it} . Inequalities (21) present the minimal lot size requirements that are required only for regular production. This can be easily modified for the specific practical case. The remaining Constraints (22), (23), (24), and (25) denote initial and ending inventory levels, binary variables regarding setups, and non-negativity constraints.

E. Numerical Study

The model is implemented in Xpress-IVE calculating test instances on a computer with an Intel Core(TM) 2 Duo CPU processor with 1.60 GHz and 1.6 GB RAM. Large test instances are calculated employing 20 products and 50 time periods. As stated by [37], test instance sizes with less than 10 items and up to 20 periods may already be of practical relevance.

1) Test Instances: Table I states parameters that are equal to all test instances and mainly define relations of parameters of returns. Returns are calculated as in Equation (8) where a fraction of $\alpha_i^R = 0.06$ is assumed, so that the return rates are high with 94%. Return rates for fashion clothes at catalog retailers may be less high, but in comparison to other products

still high [38] with around 60%. The ratio concerning the use of new/reclaimed wafers versus recycled ones can be assumed with 78% [4]. Other authors report return rates from customers to be approximately 6%, but such rates significantly vary by industries [39]. Therefore, we study two cases employing very high return rates of 94% and very low rates of 6%. We assume capacity consumption costs with $\eta = 1$.

TABLE I LIST OF TIME-INFINITE REMANUFACTURING/REWORK PARAMETERS FOR ALL PRODUCTS i

Product	rsc_i	rpc_i	rh_i	ϕ^r_i	ξ_i^r	Θ_i^L
i	$sc_i/10$	$pc_i/100$	h_i	ϕ_i	$\xi_i/10$	4

The intervals/borders and costs for utilization are given in Table II. The borders need to be modified according to capacity and expected use of overtime.

TABLE II Cost parameters of utilization intervals

Cost factor	Interval borders b					
	0	700	850	900	5000	
pcu_b^f	1	42	83	238	523	
pcu_b^v	1	8	17	48	65	

A number of $6 \cdot 4 \cdot 4 \cdot 4 \cdot 5 = 1920$ test instances are generated combining different parameter profiles and variations [2]. Lifetimes are assumed to be equal for all products. A value of $\Theta_i^L = 0$ means that the item cannot be stored, thus it must be consumed in the same period as production takes place, otherwise it must either be reworked or discarded. They are varied assuming different parameter values. Other variations are executed for machine load profiles, *time-between-orders* (TBOs), minimal lot sizes, and demand profiles as follows:

- number of products := 20,
- number of periods := 50,
- five different demand profiles assuming a Gamma distribution,
- lifetimes $\Theta_i^L := 0, 1, 2, 3, 5, 10$,
- machine loads := 0.75, 0.8, 0.9, 1.0,
- TBO profiles := 1, 2, 3, 4,
- minimal lot sizes := 0, 20, 50, 100,

The distribution of demand profiles on products is assumed to be rather low in order to create solvable test instances. Nominal capacities are fixed to 1.000 time units that are used by the instance generator to calculate product coefficients and setup times for all products. Setup times are randomly generated numbers that take into account available capacity that is not exceeded. The same is valid for the generation of production coefficients that checks capacity utilization in order to create multiplicators that emend processing rates, accordingly. The mean TBO is further determined using the classical economic order quantity formula. It expresses the frequency of demands for specific products. For instance, the TBO = 1 denotes that the related product is demanded in every time period.

The maximum solving time for each test instance is fixed to 60 seconds, so that optimality gaps are prevalent. All test instances are feasible and at least one solution is found.

2) Results: Regarding test instances with return rates of 94%, we observe the following: test instances with lowest optimality gaps are those with items having a zero lifetime and zero minimal lot size constraints; see Table III. In fact, this implies a lot-for-lot policy meaning that a plan is created that matches production and demand in each time period, so that no inventory is building up that is restricted by the lifetime constraint and the workstation has to be set up in every time period for each product with demand in that period. This has already been experienced with similar models and small numerical examples [40].

TABLE III TEST INSTANCES WITH LOWEST OPTIMALITY GAPS AND RETURN RATES OF 94%

Nr. of test instance	optimality gap in %	Θ^L_i	TBO	load	x_i^{\min}
1919	0.15	0	4	1	0
1435-1439	0.71	0	3	1	0
1916-1918	0.81	0	4	1	0
475	0.83	0	1	1	0
1915	0.84	0	4	1	0
476-479	0.86	0	1	1	0
955	0.99	0	2	1	0
956-959	1.02	0	2	1	0
355-359	1.45	0	1	0.9	0

On the other hand, test instances with highest optimality gaps are those that have rather high values regarding minimal lot size values while characteristics regarding lifetime loads and TBOs do not provide a clear picture; see Table IV. At least one can conclude from these limited results that incorporating minimum lot sizes seems to influence the solvability of the model more than lifetime constraints.

TABLE IV Test instances with highest optimality gaps and return rates of 94%

Nr. of test instance	optimality gap in %	Θ^L_i	тво	load	x_i^{\min}
222-224	35.28	0	1	0.8	100
220-221	35.29	0	1	0.8	100
1765-1769	35.99	10	4	0.9	50
1540	40.94	0	4	0.75	100
810-814	41.06	10	2	0.9	20
1541-1544	41.32	0	4	0.75	100
1290-1294	41.38	10	3	0.9	20

Test instances with return rates of 6% are mostly solved to optimality in the given time. Highest optimality gaps are reported in Table V. The tables clearly show the influence of the return rate on complexity and the solution process.

TABLE V Test instances with highest optimality gap and return rates of 6%

Nr. of	optimality	ΘL	TDO	laad	min
test instance	gap in %	Θ_i^{\perp}	IBO	load	x_i^{\ldots}
1856-1859	0.53	3	4	1	0
842	0.54	1	2	1	100
1810-1814	0.54	1	4	1	20
840-844	0.56	1	2	1	100
856-859	0.56	1	2	1	0
850-853	0.57	1	2	1	20
855-858	0.57	1	2	1	0
1320-1324	0.58	1	3	1	100

III. CONCLUSION

We present a discrete deterministic production and lot size model for aggregate production planning that takes into account lead times that are dependent on the planned workload of the production resource by smoothing production around the turning point of utilization where lead times significantly grow with small increases of utilization. Up to date, production smoothing is rather accounted for on the operational planning level regarding scheduling decisions mainly in the environment of synchronized assembly lines. No model approaches have been proposed so far for the tactical level. Besides, there are only few model approaches that account for the link between order releases, planning and capacity decisions to lead times taking into account the system workload as well as lot sizing and sequencing decisions. We contribute with our model to fill this gap. Moreover, we take into account sustainability issues and the effects of LDLT on the quality of items and consider lifetime constraints of items as well as their rework in case of exceeding their useful lifetime. Remanufacturing is included as well. Results clearly show that return flows of used items significantly influence the solution process. We will further analyze this issue also with respect to those test instances that had significant optimality gaps.

Further research will be directed to deterioration of products and seasonality thus changing lifetimes if items depending on environmental factors as well as their overall system effects. Moreover, practical case studies should be targeted analyzing and determining practical instances to test real cases and the working of the model. The development of network flow models including the presented issues is interesting as well and their comparison regarding complexity and efficiency of solution finding to the proposed model. According to this, we raise again complexity studying multi-stage systems.

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