ROBUST CONTROL OF THE C5 PARALLEL ROBOT

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Abstract: This paper deals with the dynamic control of a parallel robot with C5 joints. Computed torque control and robust control have been studied and implemented. For this purpose, we have used the inverse dynamic model whose parameters have been experimentally identified. The closed loop stability has been studied using the Lyapunov principle. The addition of a robustness term based on sliding mode technique ensures good tracking performances. The experimental results show the effectiveness of the robust control.

1 INTRODUCTION

Parallel manipulator is a closed-loop mechanism in which the end-effector (mobile platform) is connected to the base by at least two independent kinematic chains. Compared to the serial ones, the parallel architectures have potential advantages in terms of stiffness, accuracy, high speed and payload. They are widely applied to the following fields, like the Pick and Place operation in food, medicine, electronic industry, etc.

Due to their complex architecture, precise and robust control of parallel machines is a hard and open problem which has been widely addressed in the litterature. When the task requires fast motion of robot and high precision, it is very important to design a controller with good performances in order to match the mechanism. In literature, various control methods are proposed such as proportional, integration, derivative (PID) control, computed torque control (Middleton and Goodwin, 88) adaptive control (Slotine and Li, 88), neural networks control (Miller et al, 87), fuzzy control, fuzzy adaptive control.

Computed torque control has been proposed in the litterature. The latter requires the exact knowledge of inverse dynamic model. In theory, it ensures the decoupling and the linearization of equations of robot motion, resulting in a uniform response for any robot configuration . This technique is more efficient in term of precision for high moving than PD and PID linear control. However it is sensitive to the parameter variations and external disturbances of the system (Zhiong et al, 07). In practice, the dynamic model of robot can not be exactly known. Therefore in order to circumvent the problem of dynamic model uncertainties, an adaptive technique is needed.

In literature, several works have been published in the field of intelligent control methods such as Fuzzy control, neural network control (Miller et al, 87). Thus, a fuzzy neural network hybrid control (FNN) is proposed. In this control technique the hybrid control system, combines the computed torque controller, the FNN uncertainty observer and a compensated controller to control the position of a slider of the motor–toggle servomechanism. Recently, a new approach, combining the computed torque control with fuzzy control has been proposed in literature. The latter is used to approximate lumped uncertainty due to parameters variations. Among other recent work, an on-line updated PID algorithm is proposed (Chen and Huang, 2004).

In this paper, we have addressed the robust control of a parallel robot with C5 joints. This type of control allows us to improve the trajectory tracking for fast motions. This approach is based on sliding mode technique. It consist to add a compensation term in the control law in order to compensate the identification and modeling errors.

This paper is organized in five sections. The first one describes the mechanical architecture of the C5 parallel robot. The second section presents the dynamic model and its properties. Section 3 is devoted to the control law synthesis. Section 4 is dedicated to the presentation and analysis of the experimental results. Finally, a conclusion and some perspectives are given in the last section.

2 DESCRIPTION OF THE C5 PARALLEL ROBOT

The C5 parallel robot consists of a static part and a mobile part connected together by six actuated links. Each segment is embedded to the static part at point A_i and linked to the mobile part through a spherical joint attached to two crossed sliding plates at point B_i (Fig. 1)

Theoretical study concerning this architecture has been presented in the literature. The C5 links parallel robot is equipped with six linear actuators; each of them is driven by a DC motor. Each motor drives a ball and screw arrangement. The position measurements are obtained from six incremental encoders, which are tied to the DC motors.

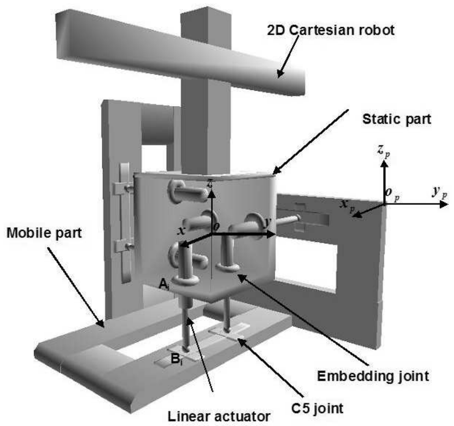

Figure 1: Parallel robot.

3 DYNAMIC MODEL AND PROPERTIES

The dynamic model of the considered parallel robot is given by the following equation :

$$\Gamma = M(q)\ddot{q} + H(q,\dot{q}) \qquad (1)$$

with

- q the (6×1) vector of joint positions
- $\dot{q}$ the (6×1) vector of joint velocities
- $\ddot{q}$ the (6×1) vector of joint accelerations
- $M(q)$ the (6×6) inertia matrix
- $H(q,\dot{q})$ the(6×1) vector of gravitational forces, frictions, Coriolis centripetal forces and other dynamics.
- Γ the vector of the torques.

The robot dynamics (1) have physical properties that can be used in the control law synthesis :

Property 1. The matrix M is Symmetric Positive Definite (*SPD*).

Property 2. The matrix C can be chosen so that $\dot{M} - 2C$ is skew symmetric.

Matrices M and H are identified by using least squares method. We note by $\hat{M}$ and $\hat{H}$ the estimation of M and H respectively. The detail of this identification method is given in (Janot et al, 07).

4 CONTROL LAW SYNTHESIS

Assuming that the dynamic model is exactly identified (case of negligible identification errors), we can use the following control law :

$$\Gamma = \hat{M}(q)\ddot{q}_r + \hat{H}(q,\dot{q}) \qquad (2)$$

with

$$\ddot{q}_r = \ddot{q}^d + k_v\dot{e} + k_p e \qquad (3)$$

$$e = (q^d - q) \qquad (4)$$

$$\dot{e} = (\dot{q}^d - \dot{q}) \qquad (5)$$

where

- e is the trajectory tracking error vector
- $q^d, \dot{q}^d, \ddot{q}^d$ are respectively, desired joint positions, velocities and accelerations
- k_p and k_v are respectively, diagonal positive definite matrices that represent the proportional and derivatives gains.

  The combination of equations (2 and (3) gives the following equation:

$$\ddot{e} + k_v\dot{e} + k_p e = 0 \qquad (6)$$

The solution of equation (6) is globaly exponentially stable. In our case, the functions of the dynamic model (matrices M and H) are estimated by least squares method. The parameters of these functions are fixed. The robot carries out different tasks and generally the identification error is never close to zero. It is then imperative to take into account these identification errors. For this purpose, we introduce in the control law a term of robustness δu based on the sliding mode technique. The control law is then given by:

$$\Gamma = \hat{M}(q)\ddot{q}_r + \hat{H}(q,\dot{q}) + \delta u \quad (7)$$

In closed loop, the equation of the dynamic error is given by :

$$\ddot{e} + k_v\dot{e} + k_p e = \hat{M}^{-1}\left[(M-\hat{M})\ddot{q} + (H-\hat{H}) - \delta u\right] \quad (8)$$

We consider the state form of the equation (9) :

$$\dot{\mathbf{e}} = A\,\mathbf{e} + B\hat{M}^{-1}\left[(M-\hat{M})\ddot{q} + (H-\hat{H}) - \delta u\right] \quad (9)$$

with

- $\mathbf{e} = \begin{bmatrix} e \\ \dot{e} \end{bmatrix}$
- $A = \begin{bmatrix} 0 & I \\ -k_p & -k_v \end{bmatrix}$
- $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$
- 0 and I are the$(n \times n)$ zero and identity matrix respectively.

Consider a new signal error s:

$$s = C\mathbf{e} \quad (10)$$

with

$$C = \begin{bmatrix} \Lambda & I \end{bmatrix} \quad (11)$$

Λ is a positive diagonal matrix, such that the transfer matrix $[C(pI-A)^{-1}B]$ is strictly positive real (SPR). For the purpose of the stability analysis we use the formulation given in (Meddah, 98).

We choose δu inspired from sliding-mode theory as follows :

$$\delta u = \beta sign(s) \quad (12)$$

where β is the sliding gain.

For stability study, we use the following Lyapunov function V :

$$V(t) = \frac{1}{2}\mathbf{e}^T P\mathbf{e} \quad (13)$$

where P is a symmetric positive definite matrix solution of the Lyapunov equation:

$$A^T P + AP = -Q$$
$$PB = C^T$$

Q a symmetric positive definite matrix.

The time derivative of the Lyapunov function (13) is expressed by the following equation:

$$\begin{aligned} \dot{V}(t) &= -\frac{1}{2}e^T Q\mathbf{e} \\ &\quad + s^T\hat{M}^{-1}\left[(M-\hat{M})\ddot{q} + (H-\hat{H}) - \beta sign(s)\right] \end{aligned} \quad (14)$$

Consequently $\dot{V} \leq 0$ when the following inequality is satisfied :

$$\beta \geq \frac{\lambda_{\hat{M}}^{\max}}{\lambda_{\hat{M}}^{\min}}\left\|(M-\hat{M})\ddot{q} + (H-\hat{H})\right\| \quad (15)$$

where $\lambda_{\hat{M}}^{\max}$ and $\lambda_{\hat{M}}^{\min}$ are respectively the greatest eigen value and the smallest eigen value of $\hat{M}^{-1}$.

If we choose the gain β, according to(15) we obtain :

$$\dot{V}(t) \leq -\frac{1}{2}\mathbf{e}^T Q\mathbf{e}$$

for any $t \geq 0$, thus s is bounded.

To prove that $s \to 0$ when $t \to \infty$, we can apply a Barbalat lemma to the following non negative function :

$$V_1(t) = V(t) - \int_0^t \left(\dot{V}(t) + \frac{1}{2}\mathbf{e}^T Q\mathbf{e}\right) d\tau \quad (16)$$

$$\dot{V}_1(t) = -\frac{1}{2}\mathbf{e}^T Q\mathbf{e} \quad (17)$$

$\dot{V}_1(t)$ is uniformly continuous.

According to Barbalat lemma we can conclude that $\dot{V}_1(t) \to 0$ and consequently $\mathbf{e} \to 0$ and $s \to 0$. Therefore the system represented by equation (9) is asymptotically stable.

Even if the value of the gain β is determined from condition (15), it is difficult to find this value as matrices M and H are unknown. Thus it is not possible to obtain the exact value of $((M-\hat{M})\ddot{q} + (H-\hat{H}))$. In practice, β is chosen heuristically. Note that the term *sign* used in (14) produces the chattering phenomenon in the control input. In order to avoid this drawback, Slotine and Li (Slotine and Li, 91) propose to replace the function *sign* by the function *sat*. (saturation)

5 EXPERIMENTAL RESULTS

In this section we present the experimental results of the application of the control laws described in section 4. These control approach is compared to a PID control. A chirp function is used as a desired trajectory, the frequency varies between $0.1hz$ and $0.3hz$. The trajectory tracking error concerning the first axis (filtered by the 4th order Butterworth) is shown in Fig. 2. Concerning other axis, we obtained the same performance as the first one.

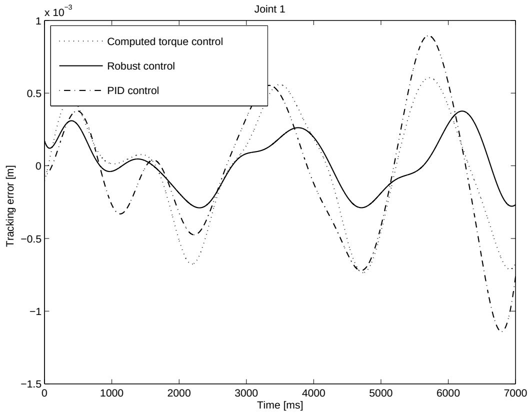

Figure 2: Tracking error of computed torque control, robust control and PID control.

5.1 Discussion

Figure 2 show that computed torque approach clearly improve the tracking performances compared to the PID control, but it remains insufficient because the precise values of M and H are difficult to obtain due to measuring errors, environment and parameters variations. Therefore, we can conclude that computed torque method exhibits good performances only when robot dynamics model is precisely identified.

However, we obtained good tracking performances when the robust control (control law with compensation term δu) is used. Besides, we also note that the errors increase for fast motions due to PID control. For robust control, these errors remain small with respect to PID control and Computed torque control errors.

6 CONCLUSIONS

In this paper, we implemented a sliding mode approach for the robust control of the C5 parallel robot. The stability of the system in closed loop with the control law including a compensation term is guaranteed. Thereafter a comparative study of above approaches show that the control using a robust term ensures a good trajectory tracking compared to the others presented approaches. The experimental results show clearly that the robustness term, based on the sliding mode method, reduces the effect of the identification errors. In our short term project, we propose a new control law such that the identification and modeling errors will be compensated in an adaptive way.

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