ON THE PRACTICAL APPLICABILITY OF KIRCHHOFF APPROACH FOR SCATTERING FROM FRACTAL SURFACES

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Preferred topic: Microwave Scattering and Propagation

It is well-known that natural surfaces can be very accurately described in terms of fractal geometry. In fact, natural surfaces exhibit statistical scale-invariance properties that are very well modeled by using the 2D fractional Brownian motion (fBm) stochastic process. Therefore, for all of the applications in which it is of interest to compute the electromagnetic scattering from natural surfaces (i.e., remote sensing, wireless telecommunications, etc.), it is certainly useful to devise methods to evaluate the field scattered by a surface whose roughness is an fBm process. In the last two decades, several attempts have been made in this direction [1-4]. In particular, the Kirchhoff approach for scattering from dielectric fBm surfaces was introduced in [3] and better detailed in [4]. It was verified that use of the fBm surface model leads to scattering results that are in better agreement with measurements than those obtained by using classical, non-fractal, surface models.

By using the Kirchhoff approach in conjunction with an fBm description of the scattering surface, it turns out that the scattered power density is analytically expressed by a scattering integral that can be expanded in terms of two different series, which are, to some extent, complementary: in fact, one is convergent for surface fractal dimension $D \leq 2.5$, the other for $D \geq 2.5$; in addition, one is an asymptotic expansion of the scattering integral for small incidence angle/high frequency, the other for large incidence angle/low frequency. In practice, this approach is useful if, for at least one of the two series, a good approximation of the scattering integral is obtained with a sufficiently small number of terms. However, experiments show that numerical evaluation of these series is sometimes problematic.

In this paper, a rigorous analysis of the behavior of the two series is performed, in order to derive their "region of practical applicability". In particular, to this aim we properly reformulate the series expressions, so that the key parameters on which the series behavior depends are clearly identified. Then, we theoretically analyze the series convergence properties and, by making use of the Leibniz and Stieltjes criteria [5-6], we obtain a criterion which, given the surface and illumination parameters, and given the required accuracy and the computer floating-point format, allows us to choose which of the two series (if any) can be used. We finally verify that for values of surface parameters of practical interest (i.e., values actually encountered in real natural surfaces and for which the Kirchhoff approach can be used), for reasonable values of the required accuracy, and if the IEEE standard floating-point double-precision numbering format is used, then there is always at least one of the two series that provides an approximation of the scattering integral with the required accuracy. In other words, the union of the "regions of practical applicability" of the two series includes all the natural surfaces of practical interest.

We finally underline that the obtained criterion for the choice of the series can be easily implemented in a software code for the efficient evaluation of scattering from natural surfaces.

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