

# ENHANCING COMPLEX INTERFEROGRAMS BY ANISOTROPIC DIFFUSION

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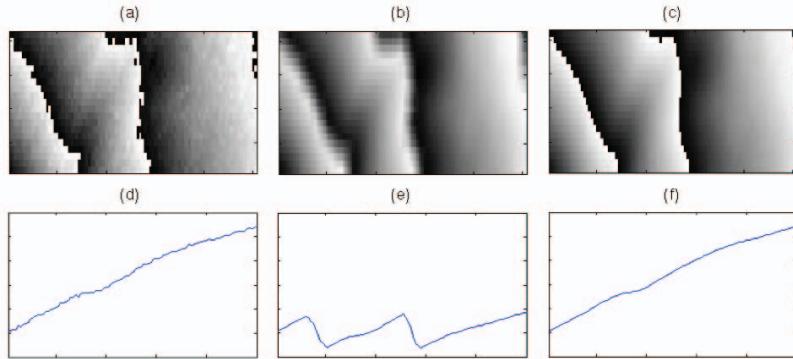
## 1. INTRODUCTION

Across-track interferometric SAR has proven to be a useful tool for the generation of a global DEM of the Earth surface [1]. The information available is a sampling of the wrapped phase  $\psi$  in slant range and azimuth  $\psi = \mathcal{W}\{\phi\}$ , where  $\phi$  is the absolute phase. This phase  $\phi$  is needed in order to derive the DEM. Thus phase unwrapping is needed in order to solve the ambiguity. Even if the signal is sampled at a high enough frequency, the noise introduces residuals due to the non-linearity of  $\mathcal{W}$ . As a consequence, enhancement of the interferometric phase prior to phase unwrapping is needed in order to accurately reconstruct the DEM. In this paper we propose a anisotropic diffusion algorithm for the restoration of the interferometric phase which accounts for the topology of  $\psi$ .

## 2. METHODOLOGY

### 2.1. Processing in $S^1$

The angular representation  $\psi \in [-\pi, \pi]$  is a parametrization of  $S^1$ , the unit circle when embedded in  $\mathbb{R}^2$ . Since it is a one dimensional manifold, one parameter is enough for the representation of each angle. Nevertheless, any algorithm defined over  $\psi$  must account for such fact. For instance  $\psi = \pi - \epsilon$  and  $\psi = -\pi + \epsilon$  with  $0 < \epsilon \ll 1$  are far in terms of  $\psi$  but extremely close in  $S^1$ . For instance, given the interferogram in figure 1(a) a moving averaging window  $5 \times 5$  over the unwrapped phase  $\psi$  leads to (b). The artificial boundaries of the representation have been blurred because of not accounting for such metric fact. If the phase of the central row of both of them is unwrapped, resp. (d) and (e), we see that the DEM has been destroyed.



**Fig. 1.** Low pass filtering on different representations of  $S^1$ . By rows: (top) wrapped phase; (down) one dimensional unwrap of the central line of (up). By columns: (left)  $20 \times 100$  section of a VV High Resolution Spotlight interferogram from Montana acquired by TerraSAR-X; (center)  $5 \times 5$  moving average over  $\psi$ ; (right)  $5 \times 5$  moving average over  $\exp(j\psi)$ .

Nevertheless, if the averaging is performed on the complex representation  $(I_1, I_2) = (\cos \psi, \sin \psi) = (\cos \phi, \sin \phi)$  and then re-projected to unit norm, the result accounts for the real metric in  $S^1$ , as it can be verified in (c) and (f). Thus, our diffusion algorithm is defined over  $S^1$ , directly on  $(I_1, I_2)$ , which is a parametrization of  $S^1$  once embedded in  $\mathbb{R}^2$ .

## 2.2. Proposed anisotropic diffusion

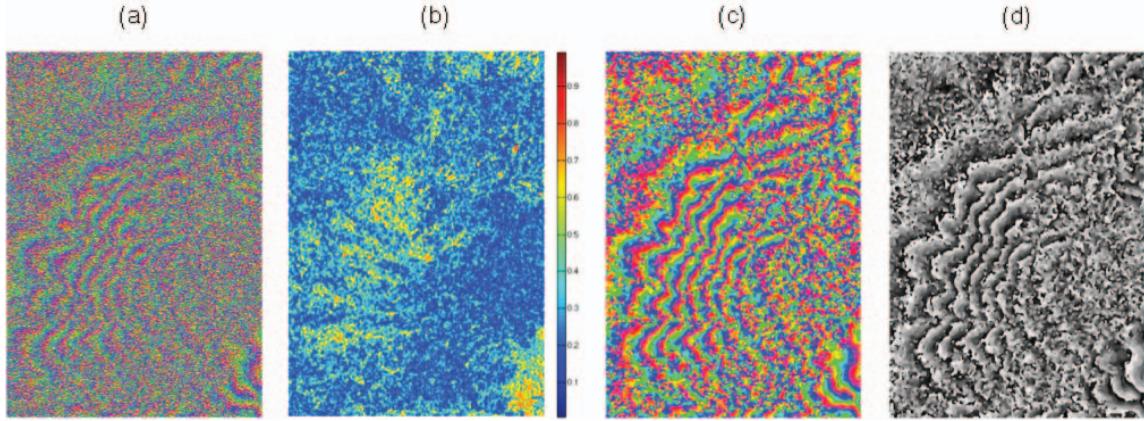
In [2] Weickert proposes Coherence Enhancing Diffusion (CED), a type of anisotropic diffusion nonlinear filter which enhances elongated structures. The formulation is based on defining a diffusion tensor  $\bar{\bar{D}}$  which controls the diffusion process. Its definition depends on the local geometry, which is described by the structure tensor. Since fringes have an elongated structure, its application for interferogram enhancing can be envisioned. Such principle was applied in [3]. Nevertheless, no derivation of Partial Differential Equations (PDEs) for  $S^1$  was derived and the experimental results show the same effect as in 1 (b). It follows that the metric of  $S^1$  was not embedded in the PDEs and a direct implementation of Weickert's on  $\psi$  was applied.

Through variational calculus and inspired on the Directions Diffusion [4], an equivalent formulation in  $S^1$  is given. Moreover, to account for outliers in the defined metric, robust estimation is introduced [5]. Finally, the eigenvalues of Weickert's  $\bar{\bar{D}}$  are normalized in order to enable isotropic diffusion in flat areas, been the new tensor  $\bar{\bar{D}}'$ . The resulting pair of coupled PDEs is

$$\frac{\partial I_i}{\partial t} = \operatorname{div} \left( g(\nabla I_1 \bar{\bar{D}}' \nabla I_1 + \nabla I_2 \bar{\bar{D}}' \nabla I_2) \bar{\bar{D}}' \nabla I_i \right) + (\nabla I_1 \bar{\bar{D}}' \nabla I_1 + \nabla I_2 \bar{\bar{D}}' \nabla I_2) I_i \quad i \in \{1, 2\}.$$

## 3. RESULTS

Results over the Stromboli volcano are given in figure 2. It can be verified that the diffusion follows the geometry of the fringes. The estimation of the structure tensor over a wide are thanks to its double kernel structure allows the robust extraction of tangential and normal directions. In slopes the diffusion tensor increases the diffusion predominantly in the tangential direction. In flat areas, the diffusion becomes isotropic. Finally, in (d) the strong discontinuities can be observed, proving that the diffusion took place according to a  $S^1$ -based metric.



**Fig. 2.** Phase enhancing in Stromboli. (a)  $1057 \times 743$  section from a HH High Resolution Spotlight interferogram of the Stromboli volcano acquired by TerraSAR-X; (b) terrain corrected coherence map; (c) diffused phase (color wheel); (d) diffused phase (grey scale). Diffusion parameters:  $\alpha = 0.005$ ,  $C = 10^{-5}$ ,  $\nu = 4 * \sigma_{\mathcal{J}}$ ,  $\sigma_{\mathcal{J}} = 4 * \sigma_{Stat}(\gamma = 0.4)$ ,  $\sigma_{ROB} = 0.7$ ,  $\lambda = 0.05$ ,  $blocks = 10$ ,  $iterPerBlock = 15$ .

## 4. REFERENCES

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