BAYESIAN RESTORATION OF INTERFEROMETRIC PHASE THROUGH BIASED ANISOTROPIC DIFFUSION

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1. INTRODUCTION

In this paper a new a bayesian algorithm for interferometric phase restoration is introduced. It is based on a nonlinear anisotropic diffusion of orientation fields. A statistical term is introduced into the formulation to account for the specific statistics of InSAR. φ denotes the acquired interferometric phase and ϕ the restored.

2. METHODOLOGY

2.1. Anisotropic Orientation Diffusion

Perona introduced a discrete diffusion process for orientation fields [1]. It efficiently regularizes orientations which are encoded as phases. Nevertheless, it is isotropic. Inspired from [2], the formulation is extended by the introduction of a 2π periodic robust error norm $\tilde{\rho}$. Given $\mathcal{S}$ the set of points in the sampling grid and $\mathcal{N}_s$ the first order neighbourhood of s

$$\mathcal{E} = \sum_{s\in\mathcal{S}} \left[\sum_{p\in\mathcal{N}_s} \tilde{\rho}(\phi_p - \phi_s) \right] = \sum_{s\in\mathcal{S}} \left[\sum_{p\in\mathcal{N}_s} \rho(1-\cos(\phi_p-\phi_s)) \right] \quad \Rightarrow \quad \frac{\partial \phi_s}{\partial t} = \lambda \sum_{p\in\mathcal{N}_s} \rho'(1-\cos(\phi_p-\phi_s))\sin(\phi_p-\phi_s).$$

Concretely in this paper we propose $\rho(x) = 1 - erfc(x/k)$ and $\rho' = \exp(-x^2/k^2)$, where k is a scale parameter. The diffusion becomes anisotropic. Moreover, $\mathcal{E}$ is the potential function of a GRF on $\mathcal{S}$ and the first order clique system [3].

2.2. Statistical Bias

Perona also proposed adding a bias term to the energetic formulation. We propose its extension to robust error $\widetilde{C}_{\rho,\gamma_s}$ and to bias the diffusion towards the original phase value, as a likelihood or fitting term. γ_s is the coherence at point s and pdf_{γ_s} the associated phase statistic. An adapted conditional likelihood $f_{\varphi_s|\phi_s}(\varphi_s|\phi_s)$ between the observed phase value φ and the restored ϕ, the one generated by diffusion, is introduced.

$$\widetilde{C}_{\rho,\gamma_s}(\phi_s - \phi_{s,d}) = \int_{-\pi}^{\pi} \rho\Big(1-\cos\left((\phi_s-\phi_{s,d})-z\right)\Big) pdf_{\gamma_s}(z)dz \quad \underset{\phi_{s,d}=\varphi_s}{\Longrightarrow} \quad f_{\varphi_s|\phi_s}(\varphi_s|\phi_s) = \frac{1}{Z_{(\rho,\gamma_s)}} \exp\Big(-\widetilde{C}_{\rho,\gamma_s}(\varphi_s-\phi_s)\Big).$$

2.3. Maximum a Posteriori Estimation

Bayesian estimation and GRF can be used for signal restoration, as shown in [4]. Given the conditional likelihood from section 2.2 and the GRF prior from 2.1, the maximum a posteriori restoration problem is stated as

$$\begin{aligned} \hat{\Phi} &= arg \max_{\Phi\in[-\pi,\pi[^{S}} \left\{ \frac{1}{Z_\rho} \exp\left[\ln\left(\prod_{s\in\mathcal{S}} f_{\varphi_s|\phi_s}(\varphi_s|\phi_s) \right) - \frac{1}{T} U\Big(\{\phi\}_{s\in\mathcal{S}}\Big) \right] \right\} \\ &= arg \min_{\Phi\in[-\pi,\pi[^{S}} \left\{ \sum_{(\phi_p,\phi_s)\in\mathcal{C}_1} \rho(1-\cos(\phi_p-\phi_s)) + T\sum_{s\in\mathcal{S}} \widetilde{C}_{\rho,\gamma_s}(\phi_s-\varphi_s) \right\}. \end{aligned}$$

Its gradient descent minimization is equivalent to anisotropic orientation diffusion when biased towards the initial phase. The approach is thus justified as a bayesian restoration algorithm

$$\frac{\partial \phi_s}{\partial t} = \lambda \Big[\sum_{p \in \mathcal{N}_s} \rho'(1 - \cos(\phi_p - \phi_s)) \sin(\phi_p - \phi_s) + \frac{T}{2} q_{\rho,\gamma_s}(\phi_s - \varphi_s) \Big], \qquad q_{\rho,\gamma_s}(\phi_s - \varphi_s) = -\frac{\partial C_{\rho,\gamma_s}}{\partial \phi_s}(\phi_s - \varphi_s).$$

3. RESULTS

In figure 1 the effect of the anisotropy and likelihood are illustrated. The number of iterations is kept high in order to show clearly the differences. The isotropic orientation diffusion regularises the phase without taking into account the fringe structure of the interferogram *(b)*. Hence, fringes are merged and broken. The anisotropic diffusion tends to preserve such structures as a result of the predominant diffusion in the local tangential direction *(c)*. Nevertheless, the smoothing is excessive. Once the likelihood term is introduced *(d)* the diffusion remains anisotropic for fringe preservation, while reconstructive for higher accuracy. Finally, figure 2 shows that the patterns in the restored phase fit those in the original one.

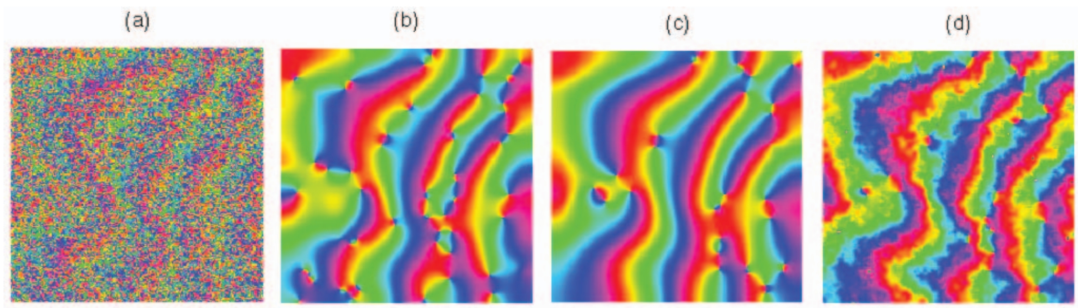

Fig. 1. Diffusion and bayesian restoration of the interferometric phase. (a) 200×200 section from a HH High Resolution Spotlight interferogram of the Stromboli volcano acquired by TerraSAR-X; (b) isotropic orientation diffusion of *(a)* (param. $\lambda = 0.1$); (c) anisotropic orientation diffusion of *(a)* (param. $\lambda = 0.1/0.7476$, $k = 1.3$); (d) statistically biased anisotropic orientation diffusion *(a)* (param. $\lambda = 0.1/0.7476$, $k = 1.3$, $T = 4$). The number of iterations is 800.

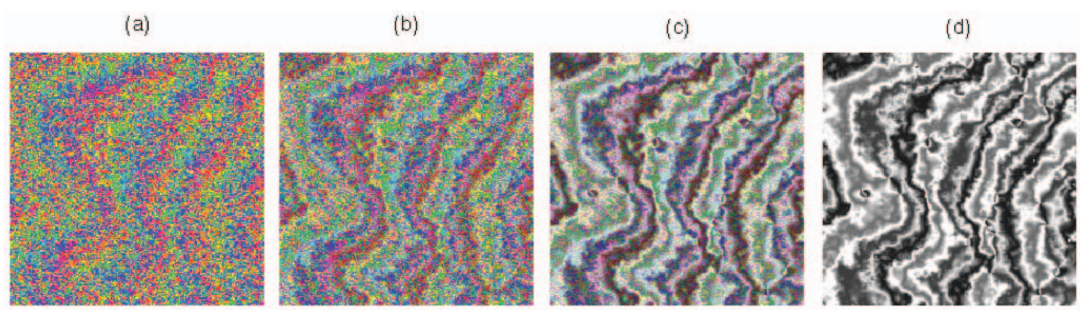

Fig. 2. Visualization of the restored phase. Transparency of the diffused phase (fig. 1 *(d)*) in grey scale over the color wheel representation from the original phase (fig. 1 *(a)*). (a) 0% opacity; (b) 33% opacity; (c) 66% opacity; (d) 100% opacity.

4. REFERENCES

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