THE MATHEMATIC MODEL OF MULTIPATH ERROR IN AIRBORNE INTERFEROMETRIC SAR SYSTEM

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1. INTRODUCTION

Generation of digital elevation models (DEM) from interferometric synthetic aperture radar (SAR) data is a well established technique [1]. But there are still some imperfections, such as multipath effect, in airborne interferometric SAR systems. The interferometric derivations assumes that a single path with the signal propagating from the transmit antenna, to the target, and back to the receive antenna. In the practical system, however, there will be reflections by parts of the aircraft platform which will also be received [2]. This is called multipath effect and it will cause oscillating phase errors and hence height errors that are generally quasiperiodic sinusoid [3]. To achieve a high geodetic fidelity, multipath error must be corrected. This paper presents a theoretical model describing the impact of multipath error in airborne two-antenna interferometric SAR system. On the basis of the model, a method and processing procedure can be used to correct multipath error effectively, and this paper illustrates its successful application to interferometric SAR data collected by Institute of Electronics, Chinese Academy of Sciences.

2. MULTIPATH ERROR MODEL

In single-pass interferometric SAR system, the platform has two antennas as shown in Fig. 1. An object near the antennas is reflecting the returned pulse into the antennas [4]. The coherent addition of the direct return and the reflected return could cause multipath effect. As shown in Fig. 1, we can then model the signals at each antenna as

$$S_a = ae^{j\phi_{obj}}e^{-j\frac{4\pi}{\lambda}\rho_a} + \sum_{k=1}^N a\varepsilon_{ak}e^{j\phi_{obj}}e^{-j\frac{2\pi}{\lambda}(\rho_a + \rho_{mk} + b_{amk})}$$
(1)

$$S_{b} = ae^{j\phi_{obj}}e^{-j\frac{2\pi}{\lambda}\left[2\rho_{a}+Q\cdot(\rho_{b}-\rho_{a})\right]} + \sum_{k=1}^{N}a\varepsilon_{bk}e^{j\phi_{obj}}e^{-j\frac{2\pi}{\lambda}(\rho_{a}+\rho_{mk}+b_{bmk})}$$
(2)

where *N* is the number of reflection points; ε_{ak} , ε_{bk} are the reflection coefficients; ρ_a , ρ_b , ρ_{mk} , b_{amk} , b_{bmk} are the path lengths; $ae^{j\phi_{obj}}$ is the backscatter coefficient of the target; and Q = 1 if the transmit antenna is common for



Fig.1 Geometry of single-pass interferometric SAR and multipath propagation

the two channels, Q = 2 if each channel uses only one antenna to both transmit and receive. The interferometric signal is

$$S_{a}S_{b}^{*} \approx a^{2}e^{-\frac{2Q\pi i}{\lambda}(\rho_{a}-\rho_{b})} \cdot e^{-i\sum_{k=1}^{N}\varepsilon_{ak}\sin\left[\frac{2\pi}{\lambda}(\rho_{mk}+b_{amk}-\rho_{a})\right]} \cdot e^{-i\sum_{k=1}^{N}\varepsilon_{bk}\sin\left[\frac{2\pi}{\lambda}(\rho_{b}-\rho_{mk}-b_{bmk})\right]}$$
(3)

Because $-\frac{2Q\pi}{\lambda}(\rho_a - \rho_b)$ is the ideal interferometric phase, the multipath phase error is

$$\phi_{error} = -\sum_{k=1}^{N} \varepsilon_{ak} \sin\left[\frac{2\pi}{\lambda} (\rho_{mk} + b_{amk} - \rho_{a})\right] - \sum_{k=1}^{N} \varepsilon_{bk} \sin\left[\frac{2\pi}{\lambda} (\rho_{b} - \rho_{mk} - b_{bmk})\right]$$
(4)

Since the variables ρ_{mk} , ρ_a , ρ_b are correlative and all depend on the look angle, the multipath phase error can be expressed as a function of the look angle, which is a more clearly and concisely form. To get this form, we need some geometric analysis about the positions of antennas and reflected points. As shown in Fig. 2, the reflection points can be divided into three different classes: below horizontal line (point F_1), above horizontal line and below baseline (point F_2), above baseline (point F_3). For example, the geometry of the first class points is shown in Fig. 3. From the geometry of multipath problem for each class of the reflection points, we can get the equations below.

$$\rho_a - \rho_{mk} = b_{amk} \sin(\theta - \alpha_{amk}), \quad \rho_b - \rho_{mk} = b_{bmk} \sin(\theta - \alpha_{bmk})$$
(5)

Inserting (5) in (4), the multipath phase error model becomes

$$\phi_{error} = -\sum_{k=1}^{N} \varepsilon_{ak} \sin\left[\frac{2\pi}{\lambda} b_{amk} \left(1 - \sin\left(\theta - \alpha_{amk}\right)\right)\right] + \sum_{k=1}^{N} \varepsilon_{bk} \sin\left[\frac{2\pi}{\lambda} b_{bmk} \left(1 - \sin\left(\theta - \alpha_{bmk}\right)\right)\right]$$
(6)

Now the model is a function of look angle θ . However, the parameters b_{amk} , b_{bmk} , α_{amk} and α_{bmk} are still correlative, and can be replaced by the position coordinates of the reflection points. Assuming the origin locates



Fig.2 Different positions of reflection points

Fig.3 Geometry of multipath problem

the phase center of antenna A, and the coordinates of point F_k is (x_k, y_k) , we can get the equations below.

$$\sin \alpha_{amk} = \frac{y_k}{b_{amk}}, \quad \sin \alpha_{bmk} = \frac{y_k - b \sin \alpha}{b_{bmk}}, \quad \cos \alpha_{amk} = \frac{x_k}{b_{amk}}, \quad \cos \alpha_{bmk} = \frac{x_k - b \cos \alpha}{b_{bmk}}$$

$$b_{amk} = \sqrt{x_k^2 + y_k^2}, \quad b_{bmk} = \sqrt{(x_k - b \cos \alpha)^2 + (y_k - b \sin \alpha)^2}$$

$$(7)$$

By inserting (7) in (6), the multipath phase error model can be expressed as

$$\phi_{error} = \sum_{k=1}^{N} \varepsilon_{ak} \sin\left[\frac{2\pi}{\lambda} \left(x_{k} \sin\theta - y_{k} \cos\theta - \sqrt{x_{k}^{2} + y_{k}^{2}}\right)\right] - \sum_{k=1}^{N} \varepsilon_{bk} \sin\left\{\frac{2\pi}{\lambda} \left[\left(x_{k} - b\cos\alpha\right)\sin\theta - \left(y_{k} - b\sin\alpha\right)\cos\theta - \sqrt{\left(x_{k} - b\cos\alpha\right)^{2} + \left(y_{k} - b\sin\alpha\right)^{2}}\right]\right\}$$

$$(8)$$

The final model of multipath phase error is a function of look angle θ , and all the other parameters are not correlative and can be estimated from distributed targets with known elevation.

3. PROCESS PROCEDURE

Before using the model to correct multipath phase error, the system parameters including the channel delay, the absolute phase, the baseline length and angel should be already calibrated and the unknown parameters in the model such as N, ε_{ak} , ε_{bk} , x_k and y_k should be estimated from distributed targets with known elevation. Then the look angle θ can be solved from the measured phase ϕ_m , and the multipath error ϕ_{error} , as a function of θ according to (8), can be easily calculated. In fact, as the look angle θ should be solved from the phase without multipath error, this processing technique is iterative, but usually two or three iterations suffice.

The relation between ideal phase ϕ and multipath error ϕ_{error} can be given by

$$\phi_m = \phi + \phi_{error} \left(\theta \right) \tag{9}$$

As the look angle θ is a function of ideal phase ϕ , the equation becomes

$$\phi_m = \phi + \phi_{error} \left(\phi \right) \tag{10}$$

where ideal phase ϕ is the only unknown parameter. In other words, the multipath error can be removed by solving equation (10).

4. EXPERIMENTAL RESULTS

The mathematic model of multipath error has been applied to interferometric SAR data collected by Institute of Electronics, Chinese Academy of Sciences. By using the scheme described above, the multipath error has been effectively reduced. The results are shown in Fig. 4-5 where the surface of the chosen area is nearly flat actually. The profile in Fig. 5 is by cutting one range line of the area in Fig. 4.



Fig.4 DEM with multipath error (left) and after multipath error removed (right)



Fig.5 Profile with multipath error (upper) and after multipath error removed (lower)

5. CONCLUSION

In airborne two-antenna interferometric SAR system the multipath phase error can be modeled as a function of look angle or ideal phase. After estimating its parameters from distributed targets with known elevation, the model can be applied to deal with the multipath errors to achieve a high geodetic fidelity.

6. REFERENCES

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