TEST 1

1) CLEAR YOUR DESK TOP OF ALL PAPERS, AND EVERYTHING ELSE EXCEPT
THE TEST PAPER AND A PENCIL.

2) Read through the test completely and work the problems you can, leave the difficult ones
till last.

3) Keep your eyes on your own paper. Cheating will not be tolerated

4) Work problems on the back of the previous page if necessary.

5) SHOW YOUR WORK! ANSWERS WITHOUT WORK WILL NOT BE ACCEPTED!

NAME: Solution Key
Question 1

a) Find the Hexadecimal numbers corresponding to \((23)_4\) and \((12A)_{12}\).

\[
(23)_4 = 2 \times 4^1 + 3 \times 4^0 = 11 = (B)_{16}
\]

\[
(12A)_{12} = 1 \times 12^2 + 2 \times 12^1 + 10 \times 12^0 = 144 + 24 + 10 \\
= (178)_{10}
\]

\[
178 \div 16 = 11 \text{ r } 2
\]

\[
11 \div 16 = 0 \text{ r } 11
\]

\[
(12A)_{12} = (B2)_{16}
\]

b) The Decimal number corresponding to \((1G)_{17}\).

\[
(1G)_{17} = 1 \times 17^1 + 16 \times 17^0 = (33)_{10}
\]
c) Convert \((3417.5)_9\) to base 3, but **without converting to base 10** as an intermediate step.

\[
\begin{array}{c|c}
\text{BASE 3} & \text{BASE 3} \\
0 & 00 \\
1 & 01 \\
2 & 02 \\
3 & 10 \\
4 & 11 \\
5 & 12 \\
6 & 20 \\
7 & 21 \\
8 & 22 \\
\end{array}
\]

\[
(3417.5)_3 = (10110121.12)_3
\]

**NOTE:** \(3^2 = 9\), so you group the digits 2-by-2,

**Just like** \(2^3 = 8\) (group 3-by-3)

\[2^4 = 16\] (group 4-by-4)

d) Convert the number \((53.75)_{10}\) to base 5.

\[
\begin{align*}
53 \div 5 &= 10 \text{ r } 3 \\
10 \div 5 &= 2 \text{ r } 0 \\
2 \div 5 &= 0 \text{ r } 2 \\
0.75 \times 5 &= 3.75 \\
0.75 \times 5 &= 3.75 \\
0.75 \times 5 &= 3.75 \\
\end{align*}
\]

\[\left(203.333\right)_5\]
Question 2

Assume an 8-bit long; two’s complement representation for all parts below.

a) Find the Binary and Hexadecimal numbers corresponding to \((+27)_{10}\) and \((+129)_{10}\). Are both numbers allowed? Give reasons for your answer.

\[ (+27)_{10} = (00011011)_2 = (1B)_{16} \]

\[
\begin{align*}
27 \div 2 &= 13 \text{ r } 1 \\
13 \div 2 &= 6 \text{ r } 1 \\
6 \div 2 &= 3 \text{ r } 0 \\
3 \div 2 &= 1 \text{ r } 1 \\
1 \div 2 &= 0 \text{ r } 1
\end{align*}
\]

\((+129)\) cannot be represented in 8-bit 2's comp.

b) The Binary and Hexadecimal numbers corresponding to \((-27)_{10}\).

\[ (+27)_{10} = (00011011)_2 \quad \text{(from above)} \]

\[ (-27)_{10} = (11100101)_2 = (E5)_{16} \]

Two's complement:

\[ 00011011 \quad \text{NOT} \quad 11100100 +1 \rightarrow 11100101 \]

\[ (+27)_{10} \]

\[ (-27)_{10} \]
c) Calculate \((A7)_{16} - (1B)_{16}\) using only addition of binary numbers.

\[
(A7)_{16} = \begin{array}{c}
1010 \\
\mid 0111
\end{array} _2 \quad \text{(A NEGATIVE \# IN 2's COMPLEMENT)}
\]

\[
(1B)_{16} = \begin{array}{c}
0001 \\
\mid 1011
\end{array} _2 \quad \text{(A POSITIVE \# IN 2's COMP.)}
\]

\[
(-1B)_{16} = \begin{array}{c}
1110 \\
\mid 0101
\end{array} _2
\]

\[
\begin{array}{c}
A7 \\
+ \quad (-1B)
\end{array} = \begin{array}{c}
10100111 \\
\quad + \\
\underline{11100101}
\end{array}
\]

\[
10001100
\]

\[
\text{Answer: } (10001100) _2 \quad \text{or} \quad (8C)_{16}
\]
Question 3

You are to design a combinational logic circuit with three inputs, $B_2, B_1,$ and $B_0,$ and one output, $F$. The output should be:

- 0 when the binary number $B_2B_1B_0$ is zero, it is divisible by two, or it is divisible by three
- 1 for all other inputs

a) Write down the truth table for this function.

<table>
<thead>
<tr>
<th>$B_2$</th>
<th>$B_1$</th>
<th>$B_0$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

b) Write the Boolean expression for the function above in the Sum-of-Products form

$$F = m_1 + m_5 + m_7$$

$$F = \overline{B_2}B_1B_0 + B_2\overline{B_1}B_0 + B_2B_1B_0$$
c) Write the Boolean expression for the function above in the Product-of-Sums form

\[ F = M_0 \cdot M_2 \cdot M_3 \cdot M_4 \cdot M_6 \]
\[ = (B_2 + B_1 + B_0) \cdot (B_2 + \overline{B}_1 + B_0) \cdot (B_2 + \overline{B}_1 + \overline{B}_0) \]
\[ \cdot (\overline{B}_2 + B_1 + B_0) \cdot (\overline{B}_2 + \overline{B}_1 + B_0) \]

\[ d) \text{ draw a circuit for } F \text{ using only NAND gates.} \]

\[ F = m_1 + m_5 + m_7 = \overline{m_1 \cdot m_5 \cdot m_7} = \overline{B_2 \overline{B}_1 B_0} \cdot B_2 \overline{B}_1 B_0 \cdot B_2 B_1 B_0 \]

Test1-W08.doc
Question 4

Use Boolean algebra for both parts a) and b).
Indicate the theorem used (just the name) and write each step on a separate line for credit.

a) Prove that

\[(A \oplus B) = (A \oplus B)\]

You need to remember (or derive) the definition of XOR in terms of the basic operators.

\[
\begin{align*}
A \oplus B &= \overline{AB} + \overline{AB} \\
&= (\overline{A} + B) \cdot (A + \overline{B}) \\
&= \overline{A}A + AB + \overline{A}B + \overline{B}B \\
&= AB + \overline{AB} \\
&= A \oplus B
\end{align*}
\]

**Definition of XOR**

**Note:** if you need to derive the expression for \(\oplus\), then:

\[
\begin{array}{c|cc|c}
A & B & A \oplus B \\
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
\end{array}
\]

\[m_1 = \overline{A}B + A\overline{B} \]

\[m_2 \]

Test1-W08.doc
b) Show that the following two expressions for the $C_{out}$ of a Full Adder are equivalent.

\[(A \oplus B) \cdot C_{in} + A \cdot B = (A + B) \cdot C_{in} + A \cdot B\]

\[(A \oplus B)C_{in} + AB = (A \overline{B} + \overline{A}B)C_{in} + AB\]

\[= A \overline{B}C_{in} + \overline{A}BC_{in} + AB\]

\[= A \overline{B}C_{in} + (\overline{A}C_{in} + A)B\]

\[= A \overline{B}C_{in} + (C_{in} + A)B\]

\[= A \overline{B}C_{in} + BC_{in} + AB\]

\[= A(\overline{B}C_{in} + B) + BC_{in}\]

\[= A(C_{in} + B) + BC_{in}\]

\[= AC_{in} + AB + BC_{in}\]

\[= (A + B)C_{in} + AB\]