

## LAB2: Time responses of first order and second order systems

### And effect of poles and zeros

#### Objectives:

- To design simulink models to investigate the time responses of first order and second order systems.
- The effect of zeros and additional poles on a system.

#### Pre lab: (by Hand)

- What is the transfer function for a standard second order system?(in terms of  $\omega_n$  &  $\zeta$ )
- Determine the location of the poles for each of the following eight scenarios:  
a)  $\omega_n = 2, \zeta = -1$  , b)  $\omega_n = 2, \zeta = 0$  , c)  $\omega_n = 2, \zeta = 1/10$  , d)  $\omega_n = 2, \zeta = 1/4$  ,  
e)  $\omega_n = 2, \zeta = 1/2$  , f)  $\omega_n = 2, \zeta = 5/4$  , g)  $\omega_n = 1, \zeta = 1/4$  , h)  $\omega_n = 4, \zeta = 1/4$
- For a DC motor position control system, with load position denoted by  $\theta_o(t)$  and voltage input by  $e_a(t)$  , the open loop transfer function is given by

$$\frac{\theta_o(s)}{E_a(s)} = \frac{0.2083}{s(s + 1.71)}$$

Let  $e_a = K(\theta_{desired} - \theta_o)$  where  $\theta_{desired}$  is 50 degrees or 0.873 radians. Draw a block diagram for the feedback control system. Try to determine by trial-and-error, suitable values of  $K$  to drive  $\theta_o(t)$  to its desired value in a stable fashion.

#### Background:

##### *Simulation of Prelab problems*

- 1) We will use the MATLAB to generate a simulation of the impulse response for each of the prelab scenarios. (or use plot\_tf.m sent to you via email)
- 2) Use MATLAB's **SYS = tf(NUM,DEN)** to generate each transfer function.
- 3) To generate the impulse response use **impulse(sys)**
- 4) Replot the scenarios:  $\omega_n = 2, \zeta = 1/10$ ;  $\omega_n = 2, \zeta = 1/4$ ; and  $\omega_n = 2, \zeta = 1/2$  on a *single plot*. What is the effect of  $\zeta$ ?
- 5) Use MATLAB's **hold** to overlay the individual plots.
- 6) Replot the scenarios:  $\omega_n = 2, \zeta = 1/4$ ;  $\omega_n = 1, \zeta = 1/4$ ; and  $\omega_n = 4, \zeta = 1/4$  on a single plot. What is the effect of  $\omega_n$ ?

Example of a Second order system (Also check example 2.14 in the text)

A DC motor, whose electric circuit of the armature and the free body diagram of the rotor are shown in Fig. 1

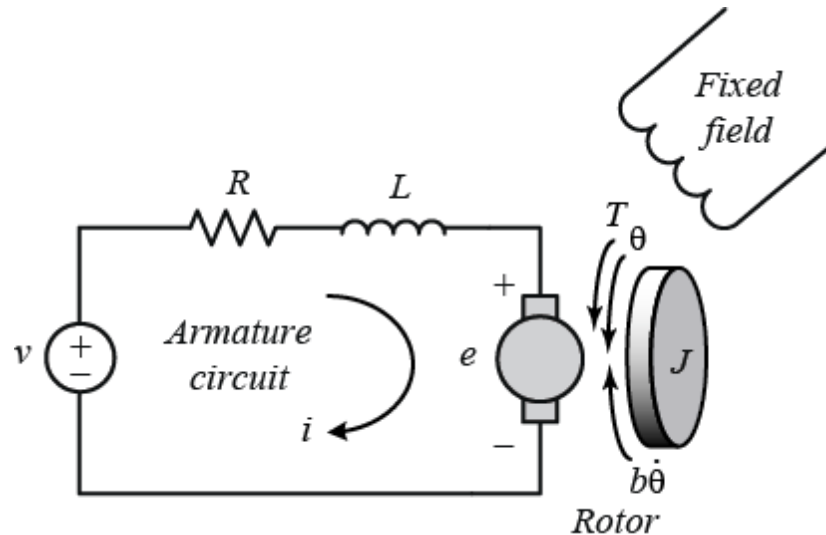


Figure 1: Schematic representation of an example DC motor

The rotor and the shaft are assumed to be rigid. The input is the armature voltage  $v$  in Volts (driven by a voltage source). Measured variables are the angular velocity of the shaft  $\dot{\theta}$  in radians per second, and the shaft angle  $\theta$  in radians.

System equations:

$$J \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} = Ki$$

$$L \frac{di}{dt} + Ri = V - K \frac{d\theta}{dt}$$

Transfer function:

$$G(s) = \frac{\dot{\theta}(s)}{V(s)} = \frac{s\theta(s)}{V(s)} = \frac{K}{(R + Ls)(Js + b) + K^2} \left[ \frac{\text{rad/sec}}{\text{V}} \right]$$

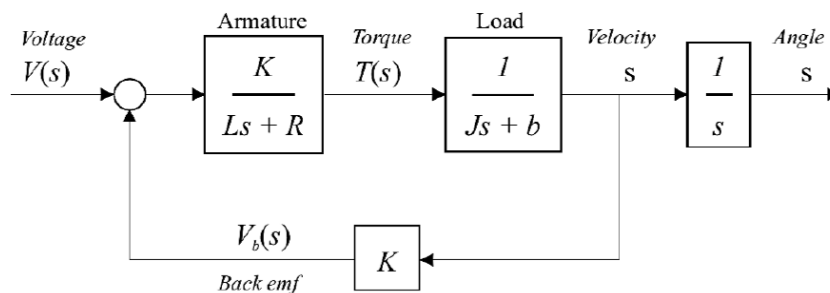


Figure 2: A block diagram of the DC motor

Please refer to section 3.3 for theory about effect of poles

Reference for pole zero plots:

- 1) [https://en.wikipedia.org/wiki/Pole%E2%80%93zero\\_plot](https://en.wikipedia.org/wiki/Pole%E2%80%93zero_plot)
- 2) pzmap(sys) creates a pole-zero plot of the continuous- or discrete-time dynamic system model sys in MATLAB

To get Simulink model from system equations:

- 1) <http://ctms.engin.umich.edu/CTMS/index.php?example=MotorPosition&section=SimulinkModeling>
- 2) <http://vigir.ee.missouri.edu/~gdesouza/ece4310/index.htm>

### **Lab:**

1. Reproduce the first order time response of Figure 3.16 ('natural response for various pole locations'). Also verify in Matlab.
2. Reproduce Fig. 3.19 (a and b; Responses of 2<sup>nd</sup> order system for various damping ratios). Also verify in Matlab.
3. Reproduce Figs. 3.27 and 3.36 (Effects of zeros and additional poles). Also verify in Matlab.

### **Lab Procedure:**

- 1) Open the simulink 'Library Browser' by clicking on Start → Simulink. Open a new model file (.mdl). Build a model of the above closed loop system by using the respective blocks.
- 2) Double-click on the blocks to open its property editor. This will give you options of changing the parameters of the blocks.
- 3) Arrange the blocks in the proper order, and connect them.
- 4) Also save your workspace to mat lab.
- 5) Go to Simulation → Configuration Parameters. Set the Stop Time as needed.
- 6) Simulate the model and look at the output by double-clicking on the Scope.
- 7) Save the result for first order pole locations.
- 8) Save the result for second order pole locations.
- 9) Save the result for effect of zeros and additional poles.
- ❖ Also show the MATLAB PLOTS along with SIMULINK plots.

**Post Lab:** Write a report in Abstract, Objective, theory, procedure, results, conclusion and appendices. All steps should be clearly mentioned. Include all plots and results.