

## LAB 5: Design of Controller for a Hybrid Vehicle using Root Locus

### Objective:

- Learn Root Locus techniques
- Design via Root Locus

**Description of system:** The use of hybrid cars is becoming increasingly popular. A hybrid electric vehicle (HEV) combines electric machine(s) with an internal combustion engine (ICE), making it possible (along with other fuel consumption–reducing measures, such as stopping the ICE at traffic lights) to use smaller and more efficient gasoline engines. Thus, the efficiency advantages of the electric drive train are obtained, while the energy needed to power the electric motor is stored in the onboard fuel tank and not in a large and heavy battery pack.

There are various ways to arrange the flow of power in hybrid car. In a serial HEV (Figure: 1), the ICE is not connected to the drive shaft. It drives only the generator, which charges the batteries and/ or supplies power to the electric motor(s) through an inverter or a converter.

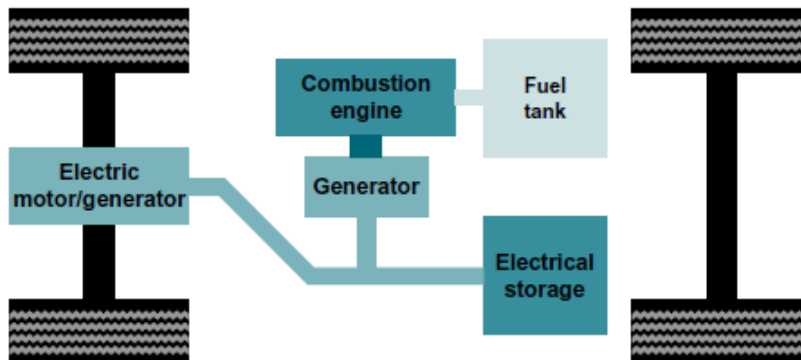


Figure: 1 Serial hybrid-electric vehicle

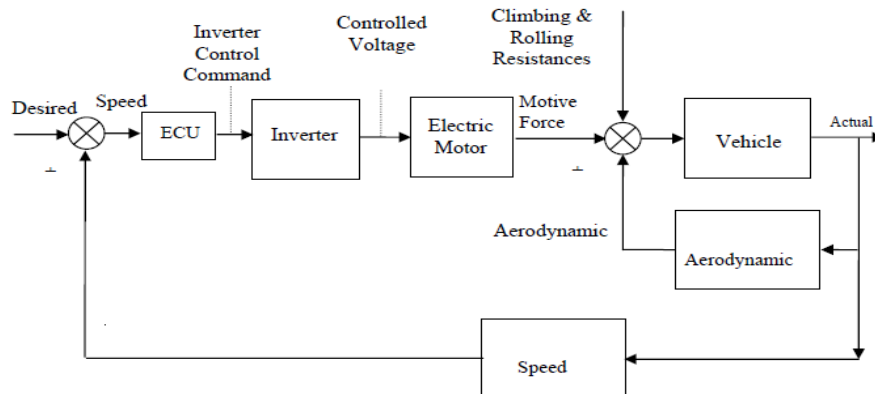
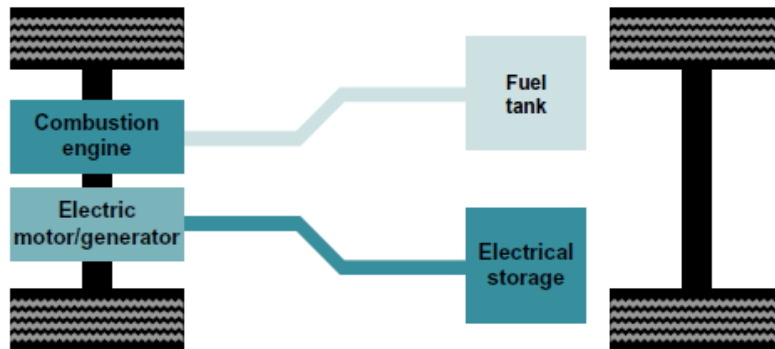
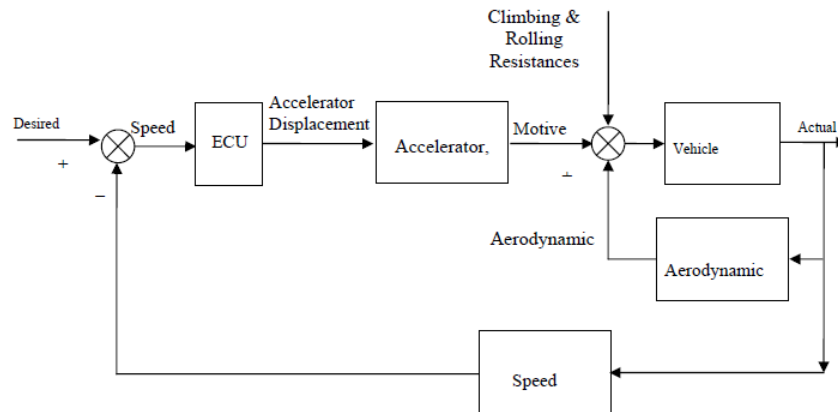


Figure: 2 Functional block diagram of serial hybrid-electric vehicle

The HEVs sold today are primarily of the parallel or split-power variety. If the combustion engine can turn the drive wheels as well as the generator, then the vehicle is referred to as a parallel hybrid, because both an electric motor and the ICE can drive the vehicle. A parallel hybrid car (Figure: 3) includes a relatively small battery pack (electrical storage) to put out extra power to the electric motor when fast acceleration is needed. See (Bosch 5th ed., 2000), (Bosch 7<sup>th</sup> ed., 2007), (Edelson, 2008), (Anderson, 2009) for more detailed information about HEV.



**Figure: 3 Parallel hybrid drive**



**Figure: 4 Functional block diagram of parallel hybrid drive**

As shown in (Figure: 5), split-power hybrid cars utilize a combination of series and parallel drives (Bosch, 5th ed., 2007). These cars use a planetary gear (3) as a split-power transmission to allow some of the ICE power to be applied mechanically to the drive. The other part is converted into electrical energy through the alternator (7) and the inverter (5) to feed the electric motor (downstream of the transmission) and/or to charge the high-voltage battery (6). Depending upon driving conditions, the ICE, the electric motor, or both propel the vehicle.

1. internal-combustion engine; 2. tank
3. planetary gear; 4. electric motor; 5. inverter;
6. battery; 7. alternator.

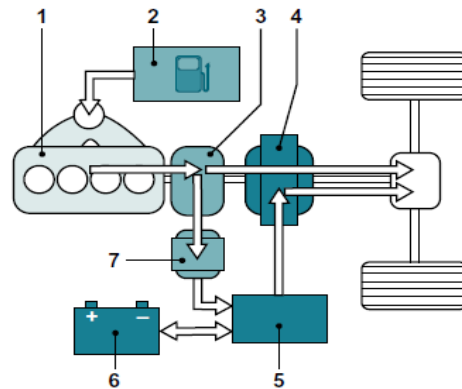


Figure: 5 Split-power hybrid electric vehicle

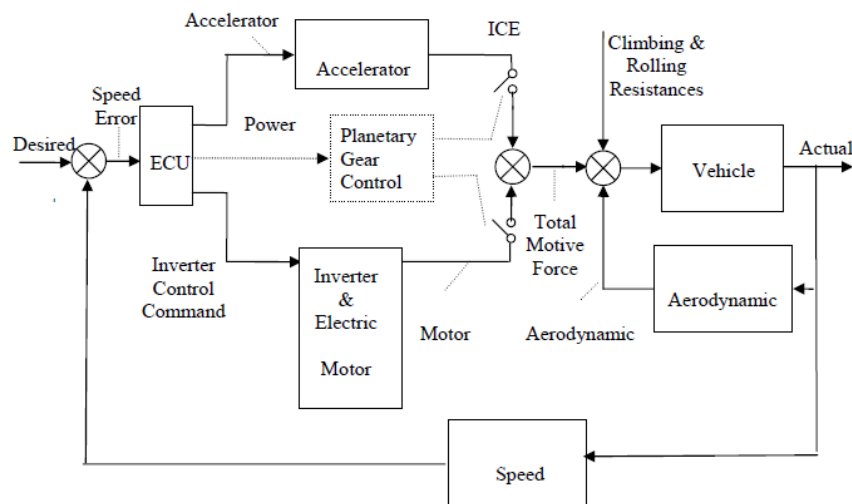


Figure: 6 Functional block diagram of split-power hybrid electric vehicle

**Pre lab:** No pre-lab this time. Participation in lab discussion is counted towards pre-lab.

**Background:** The functional block diagrams (Figure: 2, 4, 6) developed for these HEVs indicated that the speed of a vehicle depends upon the balance between the motive forces (developed by the gasoline engine and/or the electric motor) and running resistive forces. The resistive forces include the aerodynamic drag, rolling resistance, and climbing resistance. (Figure: 7) illustrates the running resistances for a car moving uphill (Bosch, 2007).

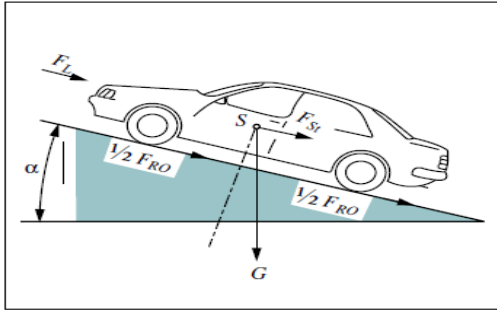


Figure: 7 Running resistances

The total running resistance,  $F_w$ , is calculated as  $F_w = F_{Ro} + F_L + F_{Si}$ , where  $F_{Ro}$  is the rolling resistance,  $F_L$  is the aerodynamic drag, and  $F_{Si}$  is the climbing resistance. The aerodynamic drag is proportional to the square of the sum of car velocity,  $v$ , and the head-wind velocity,  $v_{hw}$ , or  $v + v_{hw}$ . The other two resistances are functions of car weight,  $G$ , and the gradient of the road (given by the gradient angle,  $\alpha$ ), as seen from the following equations:

$$F_{Ro} = fG \cos \alpha = fmg \cos \alpha$$

where

- $f$  = coefficient of rolling resistance,
- $m$  = car mass, in kg,
- $g$  = gravitational acceleration, in  $m/s^2$ .

$$F_L = 0.5\rho C_w A(v + v_{hw})^2$$

where

- $\rho$  = air density, in  $kg/m^3$ ,
- $C_w$  = coefficient of aerodynamic drag,
- $A$  = largest cross-section of the car, in  $kg/m^2$ .
- $F_{Si} = G \sin \alpha = mg \sin \alpha$

The motive force,  $F$ , available at the drive wheels is:

$$F = \frac{T i_{tot}}{r} \eta_{tot} = \frac{P \eta_{tot}}{v}$$

where

- $T$  = motive torque,
- $P$  = motive power,
- $i_{tot}$  = total transmission ratio,
- $r$  = tire radius,
- $\eta_{tot}$  = total drive-train efficiency.

The surplus force,  $F - F_w$ , accelerates the vehicle (or retards it when  $F_w > F$ ). Letting  $a = \frac{F - F_w}{k_m \cdot m}$ , where  $a$  is the acceleration and  $k_m$  is a coefficient that compensates for the apparent increase in vehicle mass due to rotating masses (wheels, flywheel, crankshaft, etc.):

Acceleration 'a' can be determined from the equation below

$$F = fmg \cos \alpha + mg \sin \alpha + 0.5\rho C_w A(v + v_{hw})^2 + k_m ma \tag{1}$$

This is a nonlinear equation. The liberalized equation will be calculated as shown below for the following parameters

- ters:  $m = 1590 \text{ kg}$ ,  $A = 2 \text{ m}^2$ ,  $f = 0.011$ ,  $\rho = 1.2 \text{ kg/m}^3$ ,  $C_w = 0.3$ ,  $\eta_{tot} = 0.9$ ,  $k_m = 1.2$ . Further-

Substituting car parameters to Eq—1 yields..

$$F = 0.011 \times 1590 \times 9.8 + 0.5 \times 1.2 \times 0.3 \times 2 v^2 + 1.2 \times 1590 dv/dt \quad (2)$$

$$\text{or } F(t) = 171.4 + 0.36 v^2 + 1908 dv/dt \quad (3)$$

Let's say we want to linearize this equation about  $v_0 = 50$  kmph, for this we use truncated Taylor series as below

$$v^2 - v_0^2 \approx \left. \frac{d(v^2)}{dv} \right|_{v=v_0} (v - v_0) = 2v_0 \cdot (v - v_0) \quad (4), \text{ from which we obtain:}$$

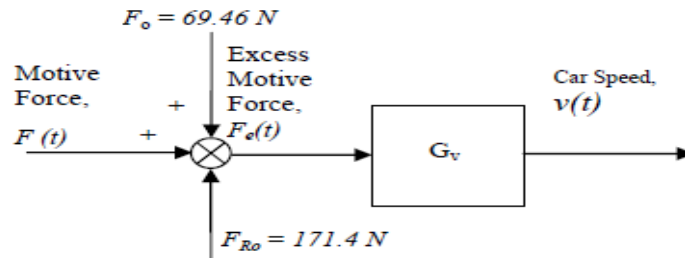
$$v^2 = 2v_0 \cdot v - v_0^2 = 27.78 \cdot v - 13.89^2 \quad (5)$$

Substituting from equation (5) into (3) yields:

$$F(t) = 171.4 + 10 v - 69.46 + 1908 dv/dt \text{ or}$$

$$F_e(t) = F(t) - F_{Ro} + F_o = F(t) - 171.4 + 69.46 = 10 v + 1908 dv/dt \quad (6)$$

Equation (6) may be represented by the following block-diagram:



Taking the Laplace transform of the left and right-hand sides of equation (6) gives,

$$F_e(s) = 10 V(s) + 1908 sV(s) \quad (7)$$

Thus the transfer function,  $G_v(s)$ , relating car speed,  $V(s)$  to the excess motive force,  $F_e(s)$ , when the ad at speeds around  $v_0 = 50$  km/h = 13.89 m/s under windless conditions is:

$$G_v(s) = \frac{V(s)}{F_e(s)} = \frac{1}{10 + 1908 s} \quad (8)$$

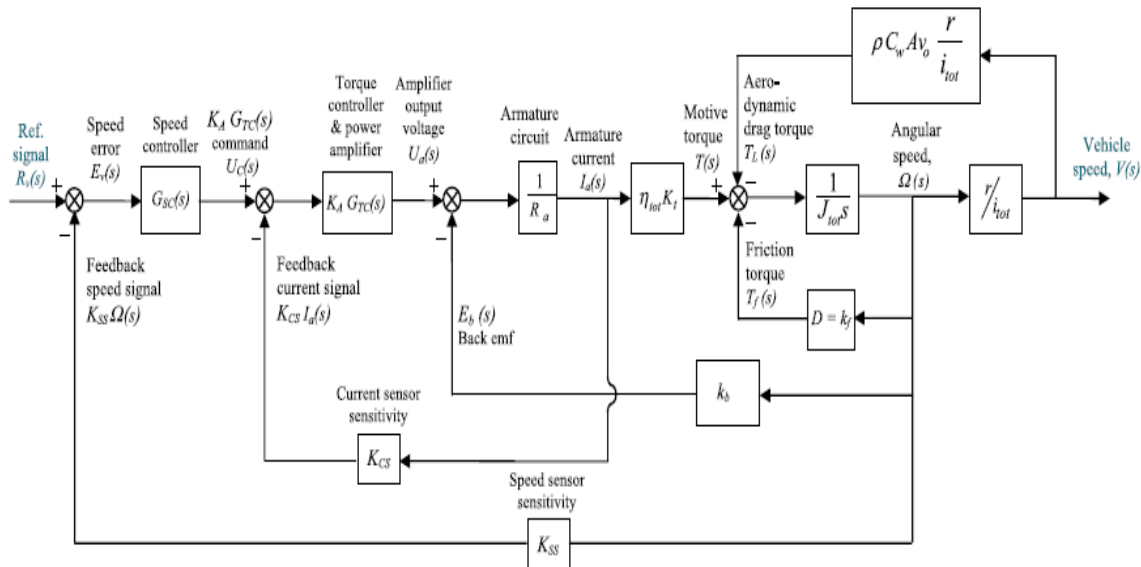


Figure: 8 Block diagram of a possible cascade control scheme for an HEV driven dc motor (Preitl, 2007)

**Lab:**

**Task 1:** Using (Figure: 8) substitute the parameters below and find the transfer function,  $T(s) = V(s)/R_v(s)$ , using block-diagram reduction rules.

Let the speed controller  $G_{SC}(s) = 100 + \frac{40}{s}$ , the torque controller and power amp  $K_A G_{TC}(s) = 10 + \frac{6}{s}$ , the current sensor sensitivity  $K_{CS} = 0.5$ , the speed sensor sensitivity  $K_{SS} = 0.0433$ . Also following the development in previous chapters  $\frac{1}{R_a} = 1$ ;  $\eta_{tot} K_t = 1.8$ ;  $k_b = 2$ ;  $D = k_f = 0.1$ ;  $\frac{1}{J_{tot}} = \frac{1}{7.226}$ ;  $\frac{r}{i_{tot}} = 0.0615$ ; and  $\rho C_w A v_0 \frac{r}{i_{tot}} = 0.6154$ .

**Task 2:** Develop a Simulink model for the original system in Figure 8. Set the reference signal input,  $r_v(t) = 4 u(t)$ , as a step input with a zero initial value, a step time = 0 seconds, and a final value of 4 volts. Use X-Y graphs to display (over the period from 0 to 8 seconds) the response of the following variables to the step input: (1) change in car speed (m/s), (2) car acceleration (m/s<sup>2</sup>), and (3) motor armature current (A).

To record the time and the above three variables (in array format), connect them to four Workspace sinks, each of which carry the respective variable name. After the simulation ends, utilize MATLAB plot commands to obtain and edit the three graphs of interest.

**Task 3:** Figure 9 shows the block diagram of the speed control of an HEV taken from Figure 8, and rearranged as a unity feedback system (Preitl, 2007). Here the system output is,  $C(s) = K_{SS} \cdot V(s)$ , the output voltage of the speed sensor/transducer.

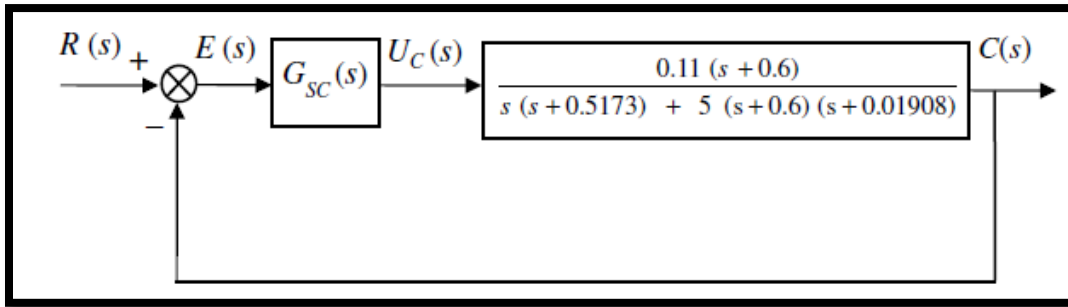


Figure: 9 HEV rearranged as a unity feedback system

- Assume the speed controller is given as  $G_{SC}(s) = K_{PSC}$ . Find the gain,  $K_{PSC}$ , that yields a **steady-state error**,  $e_{step}(\infty) = 1\%$ .
- Now assume that in order to reduce the steady state error for step inputs, integration is added to the controller yielding  $G_{SC}(s) = K_{PSC} + (K_{ISC}/s)$ , where  $K_{PSC} = 100$ . Find the value of integral gain,  $K_{ISC}$ , that results in a steady-state  $e_{ramp}(\infty) = 2.5\%$

**Task 4:** For Figure: 8 with values from task 1 substituted Use the **Routh-Hurwitz stability method** to find the range of positive  $K_p$  for which the system is closed-loop stable.

**Task 5:** For Figure: 9 let speed controller be  $G_{SC}(s) = K_{PSC} + (K_{ISC}/s)$

- Assume first that the speed controller is configured as a proportional controller  $K_{ISC} = 0$  and  $G_{SC}(s) = K_{PSC}$ . Calculate the **forward-path open-loop poles**. Now use MATLAB to plot the system's **root locus** and find the gain,  $K_{PSC}$  that yields a critically damped closed-loop response. Finally, plot the time-domain response,  $c(t)$ , for a unit-step input using MATLAB. Note on the curve the rise time,  $T_r$ , and settling time,  $T_s$ .
- Now add an integral gain,  $K_{ISC}$ , to the controller, such that,  $K_{ISC} / K_{PSC} = 0.4$ . Use MATLAB to plot the root locus and find the proportional gain,  $K_{PSC}$ , that could lead to a closed-loop unit-step response with 10% overshoot. Plot  $c(t)$  using MATLAB and note on the curve the peak time,  $T_p$ , and settling time,  $T_s$ . Does the response obtained resemble a second-order underdamped response?

Task 6:

- Use the open loop transfer function found in task 5 (a) and itemize the performance specifications in table 1 below.
- Now assume that the system specifications require zero steady-state error for step inputs, a steady-state error for ramp inputs  $\leq 2\%$ , a  $\%OS \leq 4.32\%$ , and a settling time  $\leq 4$  sec. It should be evident that this is not accomplished with a proportional

controller. Thus, start by designing a PI controller to meet the requirements. If necessary add a PD mode to get a PID controller. Simulate your final design using MATLAB. Fill in the results of this design in respective columns.

c) Did you observe any limitations to your design?

	Uncompensated	PD/PI compensated	PID compensated
Plant and compensator			
Dominant poles			
K			
$\zeta$			
$\omega_n$			
%OS			
$T_s$			
$T_p$			
$K_p$			
$e(\infty)$			
Other poles			
Zeros			
Comments			

**Table 1: Itemized performance characteristics**

**Lab Procedure:**

1. Read the description of the system and background carefully and ask TA questions if you are not sure about anything.
2. Substitute the values given in task 1 in the block diagram (Figure: 8)
3. Reduce the block diagram using reduction techniques studied in Chapter 3, section 3.2 or a useful link is provided in appendix section. [Hint: Start by moving the last (r/I<sub>tot</sub>) block to the right past the pickoff point]
4. For Task 2, all you have to do is put together the simulink diagram (Figure: 8) and get the graphs asked.

$$e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s \rightarrow 0} G(s)}$$

5. Calculate the steady state error using this.

$$e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s \rightarrow 0} s G(s)}$$

6. Calculate the steady state error using this.
7. Get the characteristic polynomial and you can use routh.m in the useful Matlab commands to do this or by hand.



8. For task 5, calculate the open loop transfer function to get the poles.
9. Write a Matlab script to get desired graphs .The command to get root locus in Matlab is rlocus.
10. From the step response get the rise time and settling time.
11. Task 6 (a): you can use results from task 5 to fill the table.

Note: We will update procedure as we go along.

**Post Lab:** Write a report in abstract, objective, theory, procedure, results, conclusion and appendices format. All steps should be clearly mentioned. Include all plots and results.

## **References:**

Anderson, S. Field Guide: Hybrid Electric Powertrains, part 4 of 5. Automotive Design & Production. Gardner Publication, Inc. Available at <http://www.autofieldguide.com/articles/020904.html>. Accessed October 13, 2009.

Bosch, R. GmbH. Automotive Electrics and Automotive Electronics, 5th ed. John Wiley & Sons Ltd., UK, 2007.

Bosch, R. GmbH. Bosch Automotive Handbook, 7th ed. John Wiley & Sons Ltd., UK, 2007.  
Cannon, R. H., Jr. Dynamics of Physical Systems. McGraw-Hill, New York, 1967.

Edelson, J., et al. Facing the Challenges of the Current Hybrid Electric Drivetrain. SMMA Technical Conference of the Motor and Motion Association. Fall 2008. Available at [www.ChorusCars.com](http://www.ChorusCars.com).

Preitl, Z., Bauer, P., and Bokor, J. A Simple Control Solution for Traction Motor Used in Hybrid Vehicles. Fourth International Symposium on Applied Computational Intelligence and Informatics. IEEE, 2007.

Franklin, G.F., Powell, J.D., and Emami-Naeini, A. *Feedback Control of Dynamic Systems*. Pearson, 2015, 7/E

Nise, N.S., *Control Systems Engineering*. John Wiley & sons, Inc, 1995. 6/E

### *Useful Links*

<http://vigir.ee.missouri.edu/~gdesouza/ece4310/index.htm> (intro slides)

[http://vigir.ee.missouri.edu/~gdesouza/ece4310/Lab\\_Assignments/ECE\\_Feedback\\_Lab5.pdf](http://vigir.ee.missouri.edu/~gdesouza/ece4310/Lab_Assignments/ECE_Feedback_Lab5.pdf) (Lab 5 document)

[http://ctms.engin.umich.edu/CTMS/index.php?aux=Basics\\_Matlab](http://ctms.engin.umich.edu/CTMS/index.php?aux=Basics_Matlab) (Matlab basics)

[http://ctms.engin.umich.edu/CTMS/index.php?aux=Basics\\_Simulink](http://ctms.engin.umich.edu/CTMS/index.php?aux=Basics_Simulink) (Simulink basics)

<http://www.msubbu.in/sp/ctrl/BD-Rules.htm> (Block- Diagram Reduction)

[https://en.wikibooks.org/wiki/Control\\_Systems/Routh-Hurwitz\\_Criterion](https://en.wikibooks.org/wiki/Control_Systems/Routh-Hurwitz_Criterion) (Stability Criteria)

<http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=ControlPID> (Introduction to PID control design)