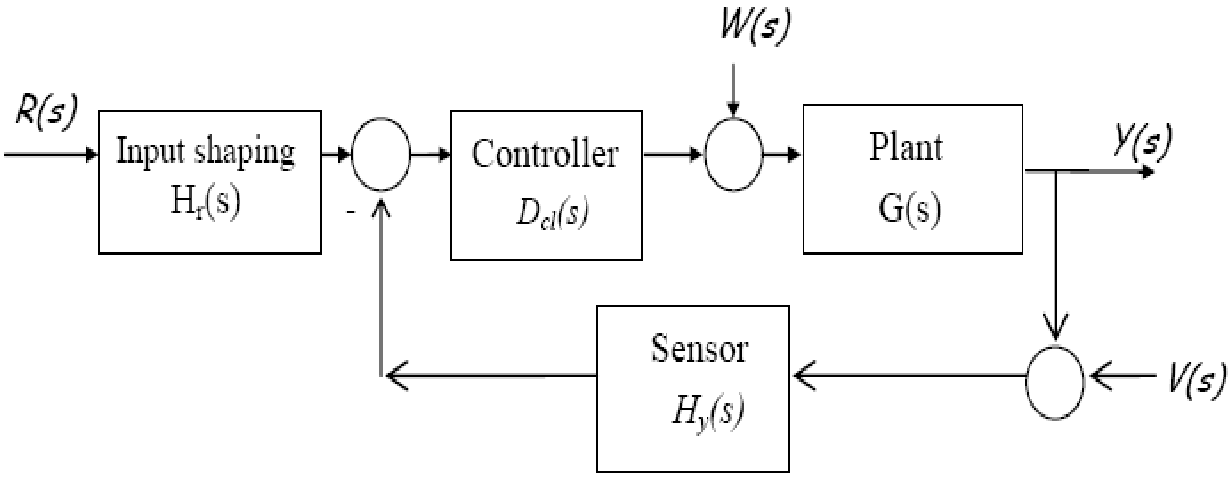
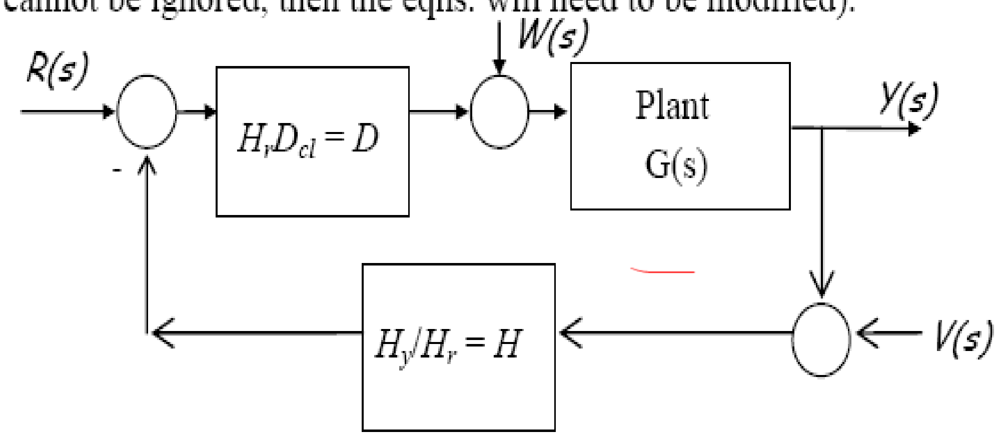
LECTURE 12

4.1 BASIC EQUATIONS OF CONTROL

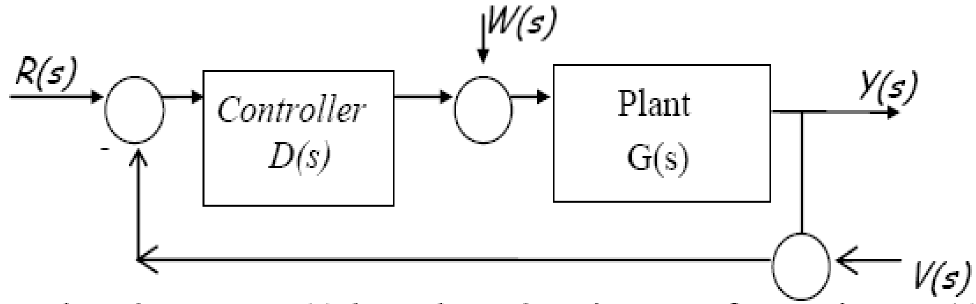


- in a general system, disturbance and noise enter in unspecific ways – for modeling purposes, we assume them to be at the inputs of the process and sensor, respectively
- the sensor if often selected to be fast and accurate, and so usually taken as a constant. An equivalent block diagram is drawn below with the controller transfer function $D(s)=H_r D_{cl}$, and with the feedback function as the ratio $H=H_y/H_r$. (note: if the sensor has dynamics that cannot be ignored, then the eqns. will need to be modified).



(ADAPTED FROM SATISH NAIR)

- It is a standard practice, especially if H_y is constant, to select equal scale factors, so that $H_r = H_y$ and the block diagram can be drawn as a unity feedback structure below:



- For this configuration, the output *Y(s)* depends on three inputs, reference input *R(s)*, disturbance input *W(s)* and noise input *V(s)*, as follows

$$Y_{cl} = \frac{DG}{1+DG}R + \frac{G}{1+DG}W - \frac{DG}{1+DG}V.$$ So, what is the disturbance TF? …….the noise TF?

- Also, defining e=r-y, you can show that the error transfer function is $E_{cl} = \frac{1}{1+DG}R - \frac{G}{1+DG}W + \frac{DG}{1+DG}V$

- Can you derive these in the space below?

- We will explore the four basic objectives of **STABILITY**, **TRACKING**, **REGULATION** and **SENSITIVITY**.
- Real world control problems include all these inputs, and the problem statement for a real world control problem becomes: "**Design a controller D(s) that will track the reference input *r*, i.e., that will make *y* as close to *r* as possible, while simultaneously making the disturbance and noise transfer functions as small as possible (so that they don't contribute to the error – see equation above).**"

4.1.1 Stability: All poles of the TF must be in the LHP

Exercise: Feedback controllers can be defined for unstable systems also. Let $G(s) = 1/(s^2-1)$. Design a controller of the form $D(s)=K(s+\gamma)/(s+\delta)$. A simple solution would be to take $\gamma=1$ so that the common (stable) factor cancels. If we wish to force its characteristic equation to be $s^2+2\zeta\omega_n+\omega_n^2=0$, solve for K and δ in terms of ζ and ω_n.

4.1.2 Tracking: Cause the output to follow the reference input as closely as possible.

If the open loop plant is stable, and has no zeros in the RHP, then in principle the controller can be selected to cancel the TF of the plant, and substitute whatever desired TF the engineer wishes. BUT, this apparent freedom comes with three caveats

1. The controller TF must be proper, i.e., it cannot have more zeros than poles – TO PHYSICALLY BUILD IT
2. An unreastically fast design cannot be expected – WILL RESULTS IN SATURATION
3. Cannot cancel poles close to the imaginary axis – PARAMETER CHANGES MAY CAUSE INSTABILITY

Exercise: For a plant having a TF G(s) = 1/(s2+3s+9), use a unity feedback controller with TF $(c_2s^2+c_1s+c_0)/(s(s+d_1))$. Solve for the parameters of this controller so that the closed-loop will have the characteristic equation $(s+6)(s+3)(s^2+3s+9)=0$.

Now show that if the reference input to the system is a step of amplitude A, the steady-state error will be zero.

4.1.3 Regulation: Keep the error small when the reference is a constant set point and disturbances are present

Look at the long closed loop transfer function you derived earlier (including the reference input, disturbance and noise).

$$E_{cl} = \frac{1}{1+DG}R - \frac{G}{1+DG}W + \frac{DG}{1+DG}V$$

What are your conclusions?

<u>Exercise</u>: Show that if w is a constant bias, and if D_{cl} has a pole at s=0, the error due to this bias will be zero. However, show that if G has a pole at zero, it does not help with a disturbance bias.

4.1.3 Sensivitity: What happens if the plant G itself changes to G + δG?

Suppose the plant is designed with gain G but in operation it changes to G+δG. This represents a fractional change of δG/G. Let the controller gain be fixed at D. In the open loop case, the nominal overall gain is thus $T_{ol} = G*D_{ol}$, and with the perturbed plant gain, the overall gain would be

$$\mathbf{T_{ol} + \delta T_{ol} = D_{ol}*(G+\delta G) = D_{ol}*G + D_{ol}*\delta G = T_{ol} + D_{ol}*\delta G}$$

Therefore, the gain change is $\mathbf{\delta T_{ol} = D_{ol}*\delta G}$.

The sensitivity $S_G^{T_{ol}}$ of a TF T_{ol}, to a plant gain G, is defined as the ratio of the fractional change in T_{ol} to the fractional change in G. In equation form,

$$S_G^{T_{ol}} = \frac{\frac{\delta T_{ol}}{T_{ol}}}{\frac{\delta G}{G}} = \frac{G}{T_{ol}} * \frac{\delta T_{ol}}{\delta G} = ?$$

What happens for the closed loop case? That is, what is S_G^T for that case? YOU CALCULATE $S_G^{T_{cl}}$?

Given $\delta T_{CL} = \frac{dT_{CL}}{dG} \delta G$

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What happens for the closed loop case? That is, what is S_G^T for that case? YOU CALCULATE $S_G^{T_{cl}}$?

$$T_{cl} = \frac{GD_{cl}}{1+GD_{cl}}. \quad \text{This implies that } T_{cl} + \delta T_{cl} = \frac{(G+\delta G)D_{cl}}{1+(G+\delta G)D_{cl}}. \quad \text{Given: } \delta T_{cl} = \frac{dT_{cl}}{dG}\delta G$$

$$\frac{dT_{CL}}{dG} = \frac{(1+GD)D - DGD}{(1+GD)^2}$$

$$S_G^{T_{cl}} = \frac{\frac{\delta T_{cl}}{T_{cl}}}{\frac{\delta G}{G}} = \frac{G}{T_{cl}} * \frac{\delta T_{cl}}{\delta G} = \frac{1}{1+GD_{cl}}.$$

With feedback, the error in the overall transfer function gain is less sensitive to variations in the plant gain by a factor of $S = 1/(1+GD_{cl})$