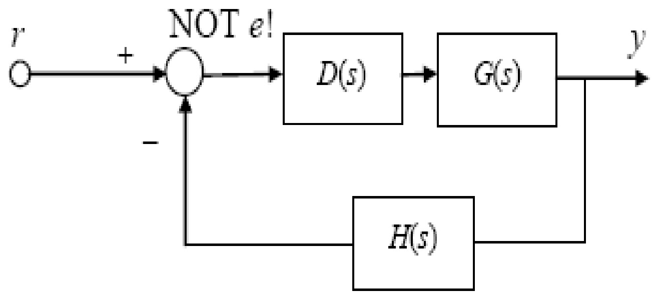
4.2 CONTROL OF STEADY-STATE ERROR TO POLYNOMIAL INPUTS: SYSTEM TYPE

LECTURE 13



<u>General case</u>

Let us assume that the reference input is a polynomial of degree k and compute the steady-state error for a general system with plant G(s) and controller D(s). If the reference input is

$r(t) = \frac{t^k}{k!} 1(t)$, then the transform of the input is $R(s) = \frac{1}{s^{k+1}}$ (no k! terms in the Laplace transform!)

If $k = 0$, the input is a step function of unit amplitude; if $k = 1$, the input is a ramp function with a unit slope; if $k = 2$, the input is a parabola with a unit second derivative, and so on. ***Note that system type is defined by the degree of input polynomial for which the steady-state tracking error is constant***. Define $e = r - y$. The reference input to error transfer function is

$\frac{E(s)}{R(s)} = 1 - T(s)$ where $T(s) = \frac{Y(s)}{R(s)}$ is the CLTF. Let us also define loop transfer function $L(s)=D(s)G(s)H(s)$.

The steady-state error is given by applying the Final Value Theorem (assuming all poles in the left half plane).

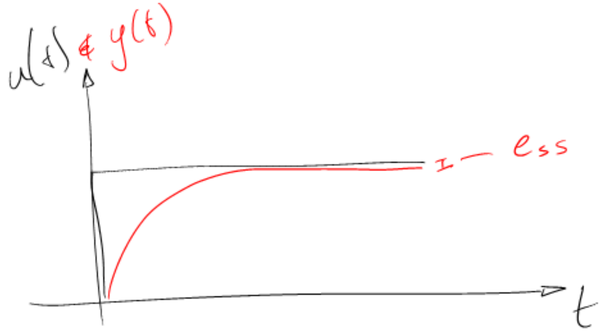
$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s[1-T(s)]R(s) = \lim_{s\to 0} \frac{1-T(s)}{s^k} \quad [= \lim_{s\to 0} \frac{1+D(s)G(s)H(s)-D(s)G(s)}{s^k(1+L(s))})]$$

This system is referred to as type k if e_{ss} is a non-zero constant for that k. For example, if $k = 0$ (step input), and e_{ss} is a non-zero constant then the system is type 0, i.e., a type 0 system will give a constant error with a step input. Similarly, if $k = 1$ and e_{ss} is a non-zero constant then the system is type 1.

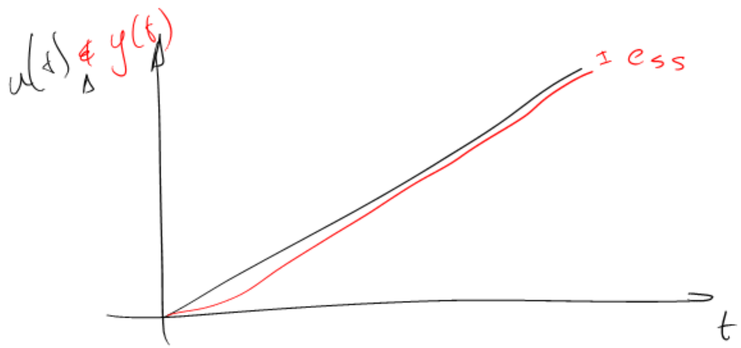
(ADAPTED FROM SATISH NAIR)

System Types

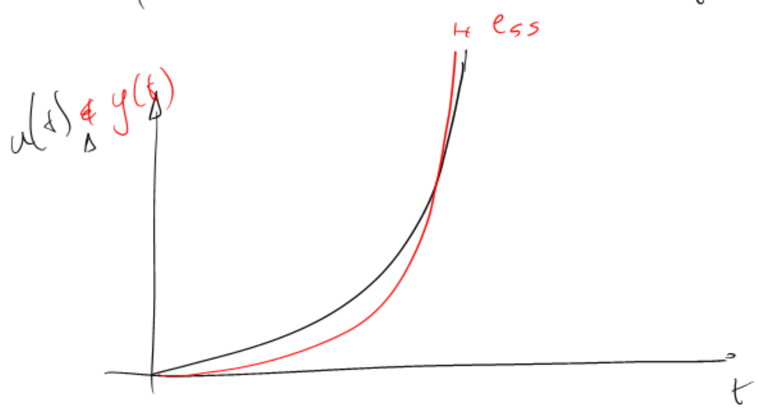
Type ∅



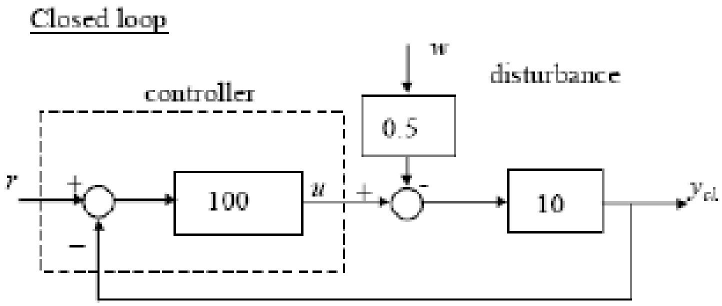
Type 1



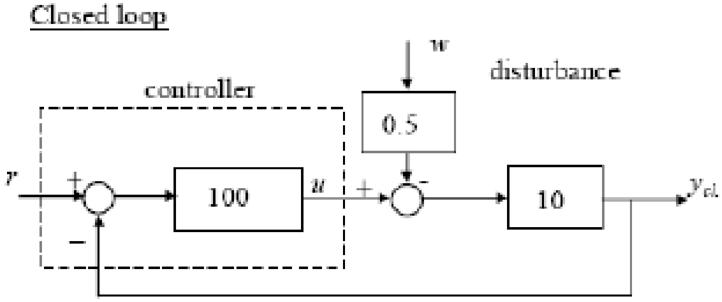
Type 2



Find an expression for the ss error for the cruise controller you studies in chapter 1:



Find an expression for the ss error for the cruise controller you studies in chapter 1:



Now derive an expression for the ss error for a unit step input for the system 1/(s+3) in feedback configuration with gain K – check with MATLAB. What is the effect of gain K?

$$e(s) = \frac{1 + K * 1 * 1/s+3 - 1 * 1/s+3}{1 + K * 1/s+3 * 1}$$

$H = K$

$D = 1$

$G = 1/s+3$

$$= \frac{s+3+K-1}{s+3+K}$$

$$\lim_{s \to 0} \frac{s+3+K-1}{s+3+K} * \frac{1}{s^l} \Rightarrow l = 0$$

$$e(s) = \frac{1 + K * 1/s+3 - K * 1/s+3}{1 + K * 1/s+3 * 1}$$

$H = 1$

$D = K$

$G = \frac{1}{s+3}$

$$= \frac{s+3}{s+3+K} * \frac{1}{s^l}$$

A special case: Unity Feedback

This is a case when $H(s) = 1$, and $L(s) = D(s)*G(s)$. Note that the system error expression is simplified for this case to

$$E(s) = \frac{1}{1+L(s)} R(s)$$. Substituting R(s) and using the Final Value theorem $e_{ss} = \lim_{s \to 0} \frac{1}{[1+L(s)]s^k}$

CASES: step input $r(t) = 1(t)$ (assuming type 0 system), $e_{ss} = \lim_{s \to 0} \frac{1}{[1+L(s)]} = \frac{1}{\lim_{s \to 0}[1+L(s)]} = \frac{1}{1+K_p}$

ramp input $r(t) = t \cdot 1(t)$ (assuming type I system), $e_{ss} = \lim_{s \to 0} \frac{1}{[1+L(s)]s} = \frac{1}{\lim_{s \to 0} sL(s)} = \frac{1}{K_v}$

parabola input $r(t) = t^2 \cdot 1(t)$ (assuming type II system), $e_{ss} = \lim_{s \to 0} \frac{1}{[1+L(s)]s^2} = \frac{1}{\lim_{s \to 0} s^2 L(s)} = \frac{1}{K_a}$

where (for UFB only) position error constant $K_p = \lim_{s \to 0} L(s)$; velocity error constant $K_v = \lim_{s \to 0} sL(s)$; and acceleration error constant $K_a = \lim_{s \to 0} s^2 L(s)$ [*terms are used in industry*]. For non-UFB systems, first find e_{ss} and get them from that.

The errors for different system types for unity feedback case are shown at table below

	Step	Ramp	Parabola
Type 0	$1/(1+K_p)$	∞	∞
Type I	0	$1/K_v$	∞
Type II	0	0	$1/K_a$

Note that for the Unity Feedback case, if *L*(*s*) has n free *s* in the denominator, then the system is of type *n*.