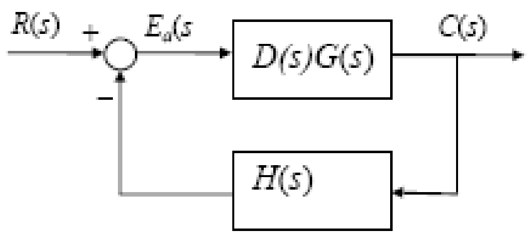
4.4 CONVERTING A NON-UNITY FB SYSTEM TO A UFB SYSTEM

LECTURE 14

$$\text{Error} \triangleq E(s) = R(s) - C(s)\,; \qquad E_a(s) \neq E(s)$$



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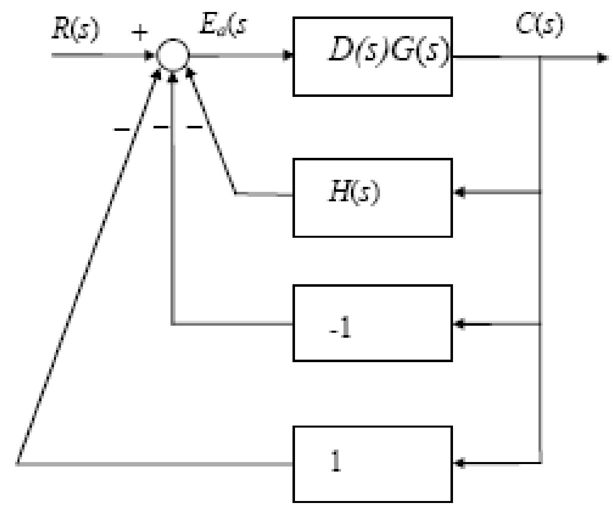

THIS TRICK MAY HELP YOU DETERMINE SYSTEM TYPE FOR NON-UNITY FEEDBACK SYSTEMS…since for the Unity Feedback case, if *L*(*s*) has n free *s* in the denominator, then the system is of type *n*.

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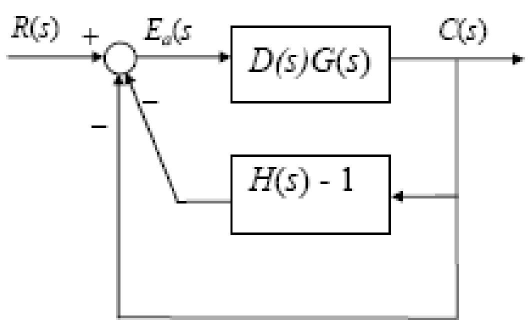

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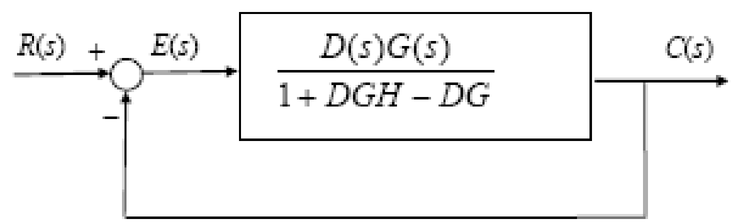


Note: The error constants K_p, K_v and K_a are defined for UFB systems.

(ADAPTED FROM SATISH NAIR)

Consider the system $G(s) = 1/[s(\tau s+1)]$; $D(s) = K_p$; $H(s) = 1+K_t*s$

Determine the system type and relevant error constant with respect to the reference input.

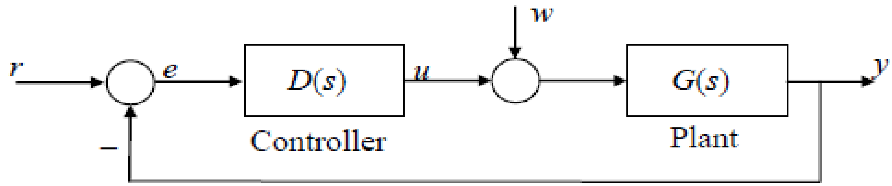
(servo with tach feedback; example 4.3, page 181 of text)

$$\frac{e(s)}{s^k} = \frac{1}{s^k} * \frac{1 + K_p*(1+K_t s)*\frac{1}{s(\tau s+1)} - \frac{K_p}{s(\tau s+1)}}{1 + K_p*(1+K_t s)*\frac{1}{s(\tau s+1)}}$$

$$\lim_{s\to 0} \frac{1}{s^k} * \frac{s(\tau s+1) + K_p(1+K_t s) - K_p}{s(\tau s+1) + K_p(1+K_t s)}$$

$$\frac{1}{s^{k-1}} * \frac{\tau s + (K_p*K_t+1)}{\tau s^2 + (K_p*K_t+1)s + K_p}$$

UNITY FEEDBACK SYSTEMS REVISITED



RECALL -- for the Unity Feedback case, if *D(s)G(s)* has n free *s* in the denominator, then the system is of type *n*

e.g., D(s)G(s) = 1/[s(s+1)] is a type 1 system and will have a zero ss error for a step input and a constant steady state error for a ramp input.

How do we calculate ss error for such systems?

Note that for UFB systems only, the error constants are as follows:

position error constant $K_p = \lim_{s \to 0} D(s)G(s)$;

velocity error constant $K_v = \lim_{s \to 0} sD(s)G(s)$; **and**

acceleration error constant $K_a = \lim_{s \to 0} s^2 D(s)G(s)$ [*these terms are used in industry*]

TYPES? Type 0 – the system has a constant error to a step; Type 1 – system has a constant error to a ramp input,….and so on.

So, for type 0 systems, ss error to a step input $r(t) = 1(t)$ **is** $e_{ss} = \dfrac{1}{1 + K_p}$

for type 1 systems, ss error to a ramp input $r(t) = t \cdot 1(t)$ **is** $e_{ss} = \dfrac{1}{K_v}$

for type 2 systems, ss error to parabolic input $r(t) = t^2 \cdot 1(t)$ **is** $e_{ss} = \dfrac{1}{K_a}$

Find the ss error to the appropriate inputs for the following systems:

H(s) = 1/(s+5); H(s) = 1/[s(s+7)]; H(s) = (s+4)/[s(s+10)]; H(s) = (s+2)/[s (s+3)(s+12)]

System type for regulation and Disturbance Rejection

$$\frac{E(s)}{W(s)} = \frac{-Y(s)}{W(s)} = T_W(s)$$

$$e_{Wss} = Y_{ss} = \lim_{t \to \infty} T_W(t) = \lim_{s \to 0} sT(s)$$

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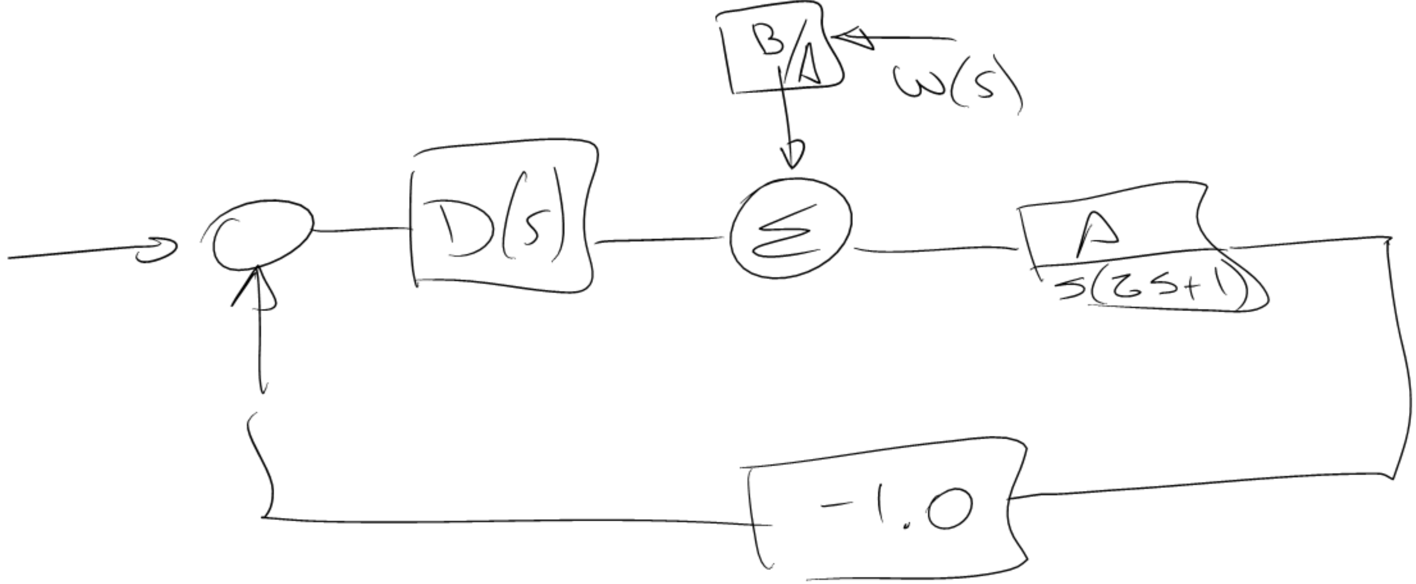
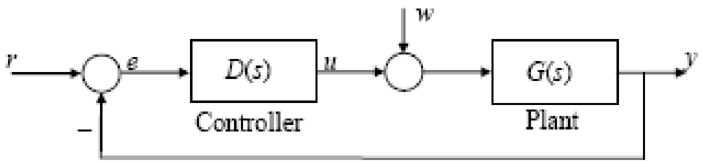


a) $D(s) = K_p$

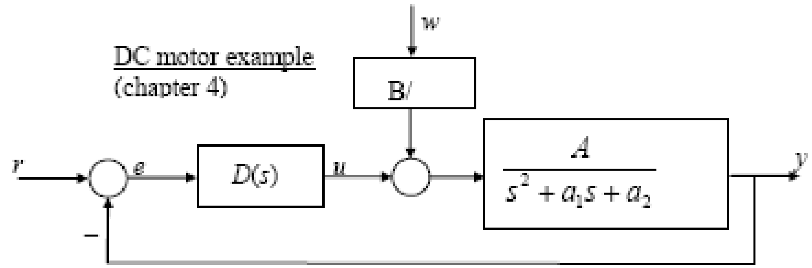
b) $D(s) = K_p + \frac{K_I}{s}$

4.5 PID CONTROL

General



DC motor example
(chapter 4)



RECALL: $\frac{\omega^2}{s^2+2\zeta\omega+\omega^2} \Rightarrow T_p=\frac{\pi}{\omega\sqrt{1-\zeta^2}}$ $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$ $T_s=\frac{4.6}{\zeta\omega}$ (1%) $T_r=\frac{1.8}{\omega}$

Objective: Determine how the C.L. roots are related to the control gains K_p, K_I and K_d (use SIMULINK to check some with G(s) = 1/(s+10), and different D(s))

Control Type	Time Domain Representation	Frequency domain representation of controller	Characteristic equation (denom. of closed loop TF=0), $1+D(s)G(s)=0$	Remarks
P	$u(t)=K_pe(t)$	$D(s)=K_p$	$s^2+a_1s+a_2+AK_p=0$	*1 degree of freedom, and so cannot locate the roots at any location. *Can reduce error magnitude but non-zero steady-state error exists. *Can increase the speed of response but has much larger transient overshoot.
I	$u(t)=K_I\int_{t_o}^{t}e(t)dt$	$D(s)=\frac{K_I}{s}$	$s^3+a_1s^2+a_2s+AK_I=0$	*Can reduce or eliminate constant steady-state errors at the cost of worse transient response.
PI	$u(t)=\left(K_pe(t)+K_I\int_{t_o}^{t}e(t)dt\right)$	$D(s)=\frac{K_ps+K_I}{s}$	$s^3+a_1s^2+(a_2+AK_p)s+AK_I=0$	*2 degrees of freedom and the designer can choose K_p and k_I independently to provide better transient response. *Approximated by lag controller.
D	$u(t)=K_d\dot{e}(t)$	$D(s)=K_ds$	$s^2+(a_1+AK_d)s+a_2=0$	*Pure derivative feedback is not practical to implement.
PD	$u(t)=(K_pe(t)+K_d\dot{e}(t))$	$D(s)=(K_p+K_ds)$	$s^2+(a_1+AK_d)s+(a_2+AK_p)=0$	*Can increase the damping and generally improve the stability of a system. *Approximated by lead controller.
PID	$u(t)=\left(K_pe(t)+K_I\int_{t_o}^{t}e(t)dt+K_d\dot{e}(t)\right)$	$D(s)=\left(K_p+\frac{K_I}{s}+K_ds\right)$	$s^3+(a_1+AK_d)s^2+(a_2+AK_p)s+AK_I=0$	*3 degrees of freedom and can provide an acceptable degree of error reduction simultaneously with acceptable stability and damping

Other controller types: state feedback, adaptive, robust, stochastic, neural, fuzzy,…

Derive an expression for the ss error for a automobile cruise controller assuming a first order plant (assume [10/(s+1)] ??), with throttle angle as input and speed as output and a pure gain for D(s). Will you buy such an automobile? How would you correct it? Use MATLAB if needed.

PROBLEMS FROM TEXT