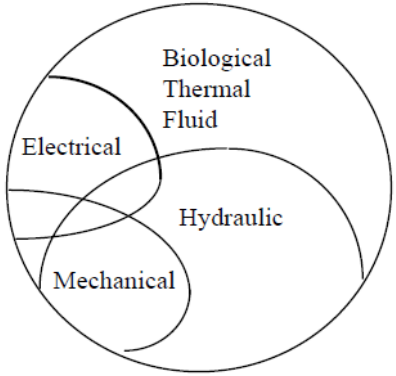
Chapter 2
Dynamic Models

LECTURE 2

Model - a set of DEs describing the dynamic behavior of a system



<u>Fundamentals from the 'basic' courses you have taken come together here – 'connecting the dots':</u>

Differential equations	Physics
Circuits 1/2	Chemistry
Electronic circuits/signals	Biology
Transform analysis	……

lumped parameter vs. distributed parameter representation

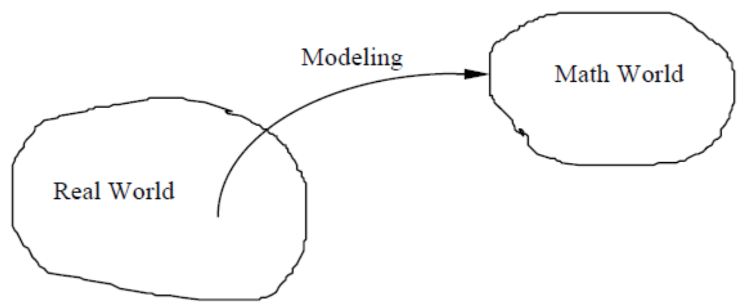
Mechanical
- Newton's laws
- Elements – springs, dampers, Gears, ……

Electrical
- Kirchoff's laws
- Ohms law
- component laws
- ……

Thermal
- Fourier law
- convection, radiation
- ……

Biological
- neuron firing
- bacterial growth
- ……..

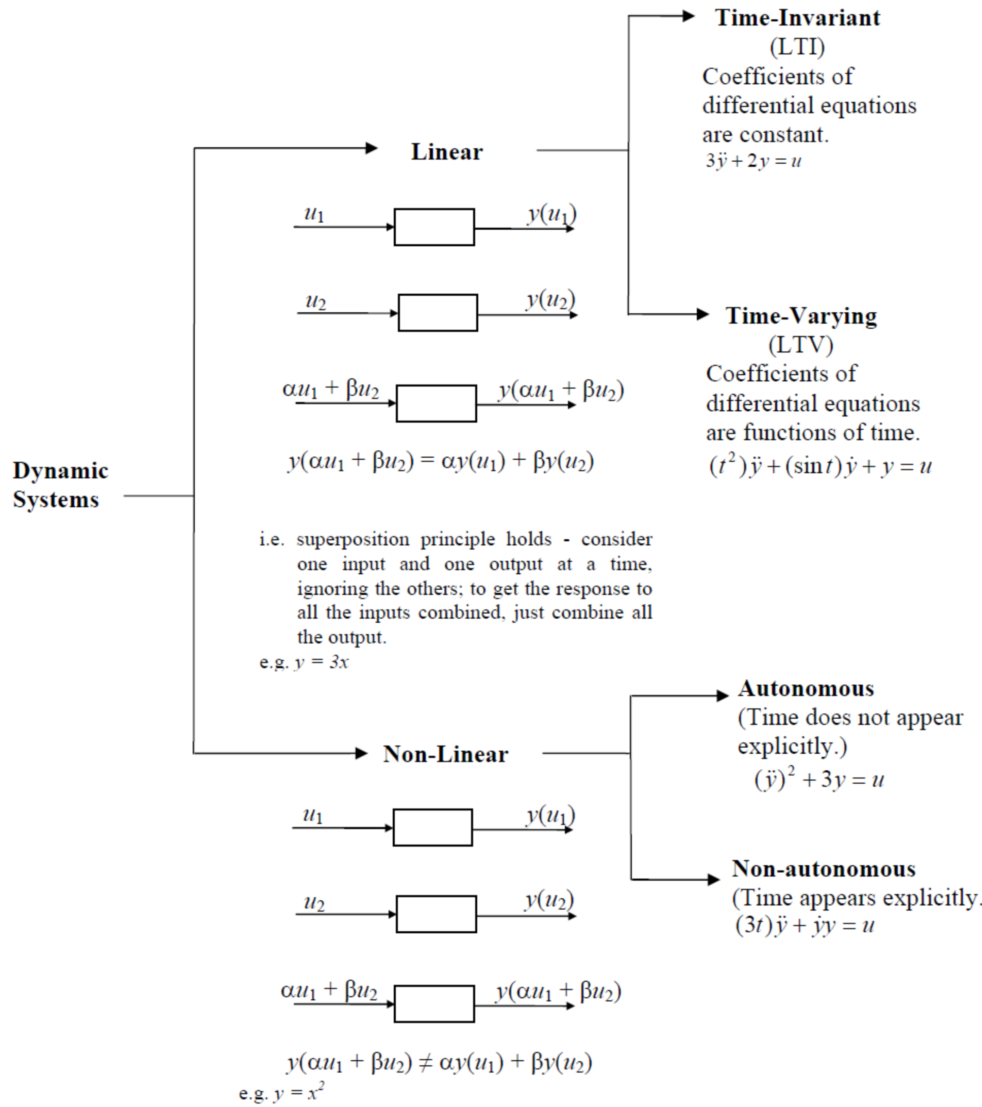
Pneumatic
- compressible flows
- ……



(ADAPTED FROM SATISH NAIR)

Chapter 2
Dynamic Models

2.2 CLASSIFICATION OF DYNAMIC SYSTEMS



Examples: models for cruise control, space shuttle trajectory, robot dynamics, weather prediction,…

Note: In 'static' systems, there are no 'dot' terms. For example: y = kx.

Chapter 2 Dynamic models

(a set of differential equations that describe the dynamic behavior)

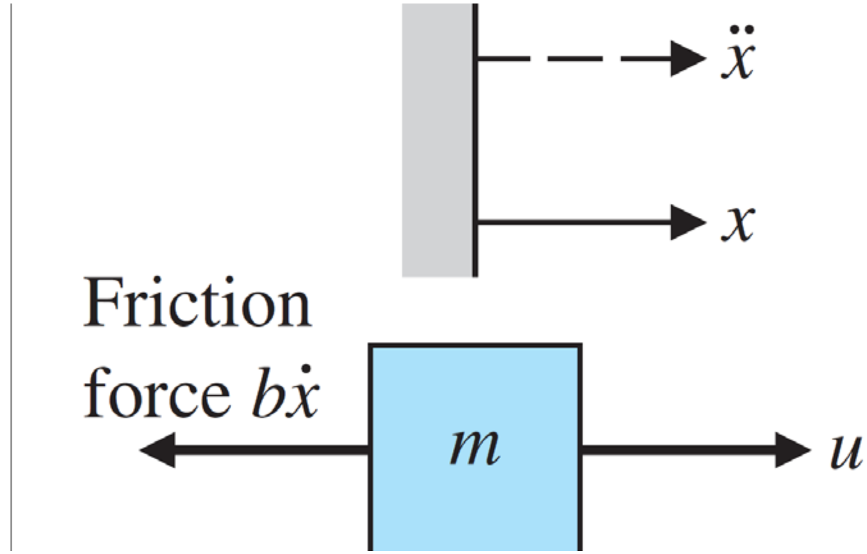
First step in control design is to get the dynamic model of the 'system'

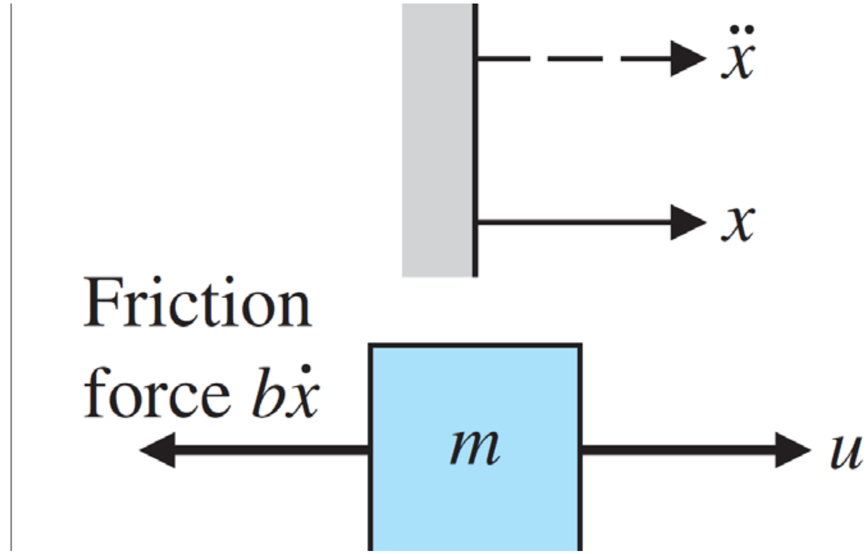
2.1 Dynamics of mechanical systems

F=ma

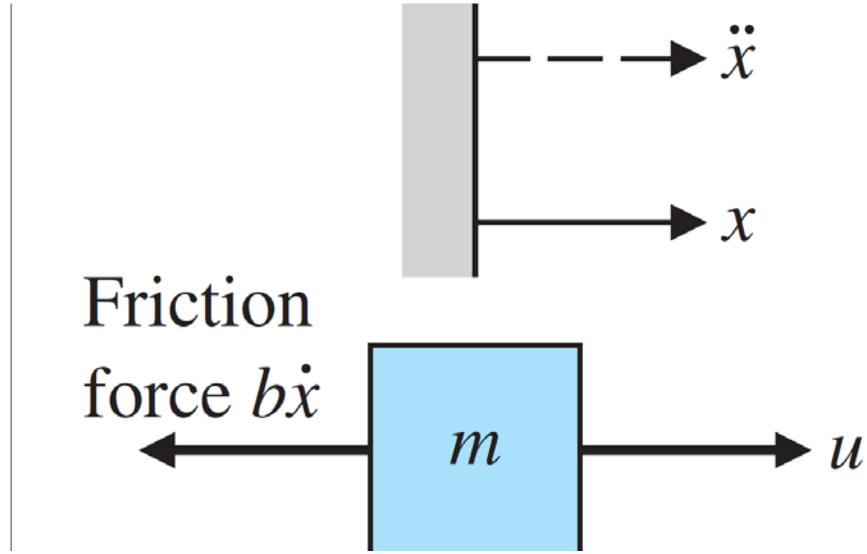
Points to note:

1. **assign variables that are both necessary and sufficient to describe an arbitrary position of the object**
2. **draw a free body diagram of each component**
3. **apply the translational or rotational version of Newton's law**
4. **the no. of independent equations needs to be equal to the no. of unknowns**


$\ddot{x}$
x
Friction
force $b\dot{x}$
m
u

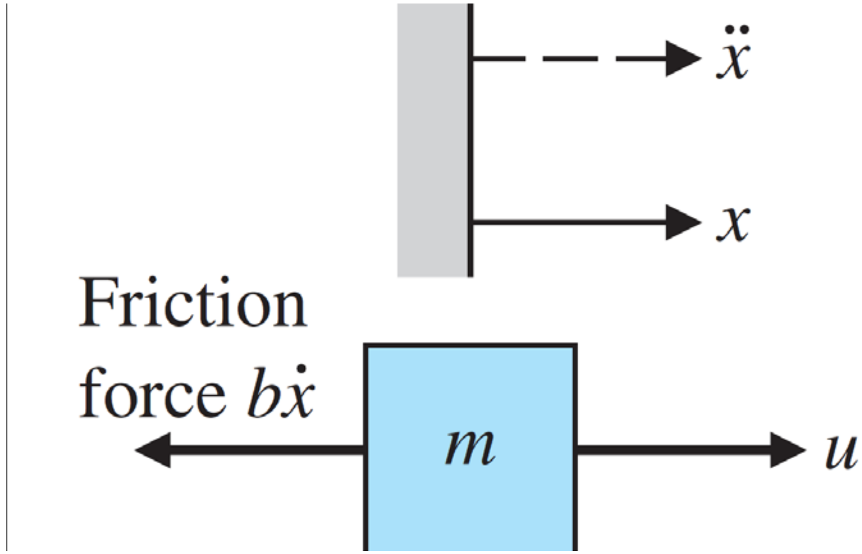


$$m\ddot{x} = u - b\dot{x}$$



$$m\ddot{x} = u - b\dot{x}$$

$$m\ddot{x} + b\dot{x} = u$$



$$m\ddot{x} = u - b\dot{x}$$

$$m\ddot{x} + b\dot{x} = u$$

$$m\dot{v} + bv = u$$

$$v = V_0 e^{st}$$

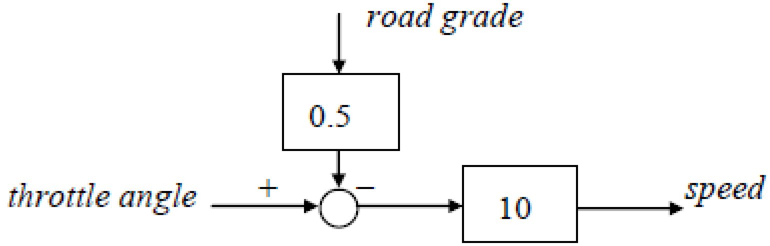
$$u = U_0 e^{st}$$

With $\dot{x} = v,\ \dfrac{V}{U}(s) = \dfrac{1/m}{s + b/m}$

Note : Force u α ***throttle angle***

2.4 BACK TO THE CRUISE CONTROLLER PROBLEM

Recall that the steady state vehicle model was 'static', i.e., modeled as a pure gain element, as follows: (input – throttle angle; output – speed; disturbance – road grade)



The 'auto body' in the model above, is represented by "10" which captures the steady state effect thus: for a given throttle angle u, the car speed 'at steady state' will be 10*u, in the absence of road grade.

In reality, though, the car has 'dynamics', i.e., we are interested in knowing how fast it can accelerate……its transient characteristics, ….."0 to 60 in how many seconds?"

To do this, we need to model the car using a dynamic equation. For that we need to use Newton's laws: Assume the car mass is m, all effects of friction have been modeled using a viscous term $b\dot{x}$ and that the force acting on it is 'u'. The dynamic equation relating the input force u to the position of the car, x, is

$$m\ddot{x} + b\dot{x} = u$$

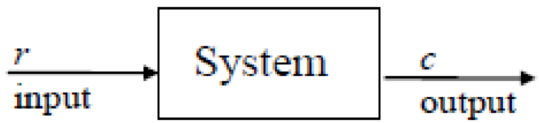
For cruise control design, the variable of interest is velocity v, rather than position x. So, we can replace $v = \dot{x}$, to get a first order equation

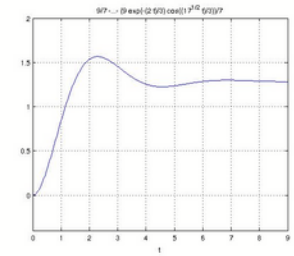
$$m\dot{v} + bv = u \quad \text{or, in transfer function form} \quad \frac{V}{U}(s) = \frac{1/m}{s + b/m}$$

After you get an equation, how do you solve it?

Use MATLAB!

2.3 MATHEMATICAL REPRESENTATION OF LTI SYSTEMS





1. Differential Equation Form (time domain)

e.g. $3\ddot{c} + 4\dot{c} + 7c = 9r$

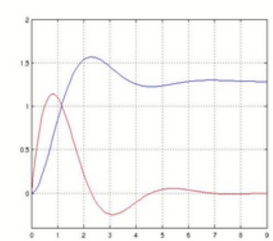
{Matlab code assuming r=1; syms c; c=dsolve('3*D2c+4*Dc+7*c=9,c(0)=0,Dc(0)=0','t'); figure(1);ezplot(c,[0,10]); axis([0 9 -0.4 2]);grid on;}

2. Transfer Function Form (frequency domain)

For the same e.g. as in (1), taking Laplace transforms of both sides (with zero ICs), the transfer function form is obtained as

$$\frac{C(s)}{R(s)} = \frac{C}{R}(s) = \frac{9}{3s^2 + 4s + 7}$$

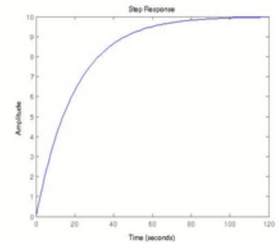
{Matlab code: num=[9]; [den]=[3,4,7]; sys = tf(num,den);[y1,t1,x1] = step(sys);[y2,t2,x2] = impulse(sys); figure(2); plot(t1,y1,'b',t2,y2,'r');grid on; axis([0 9 -0.4 2]); }



Let us simulate this system using MATLAB, to find the velocity v of the car as a function of a 'step' input in force u.

```
num=1/1000;
den=[1 50/1000];
sys=tf(500*num,den);
step(sys)
```

$\frac{1/m}{s + b/m}$



This gives the velocity v as a function of time, for an input $u = 500$. This model can now replace the 'static' model above, if we assume that the throttle angle results in a constant force (good approximation). So, you can replace the block '10' above with the transfer function V(s)/U(s) if you are interested in controlling the dynamic characteristics.

Let's now switch to active suspension control which some of the cars have presently. How do we model the car for designing an active suspension?

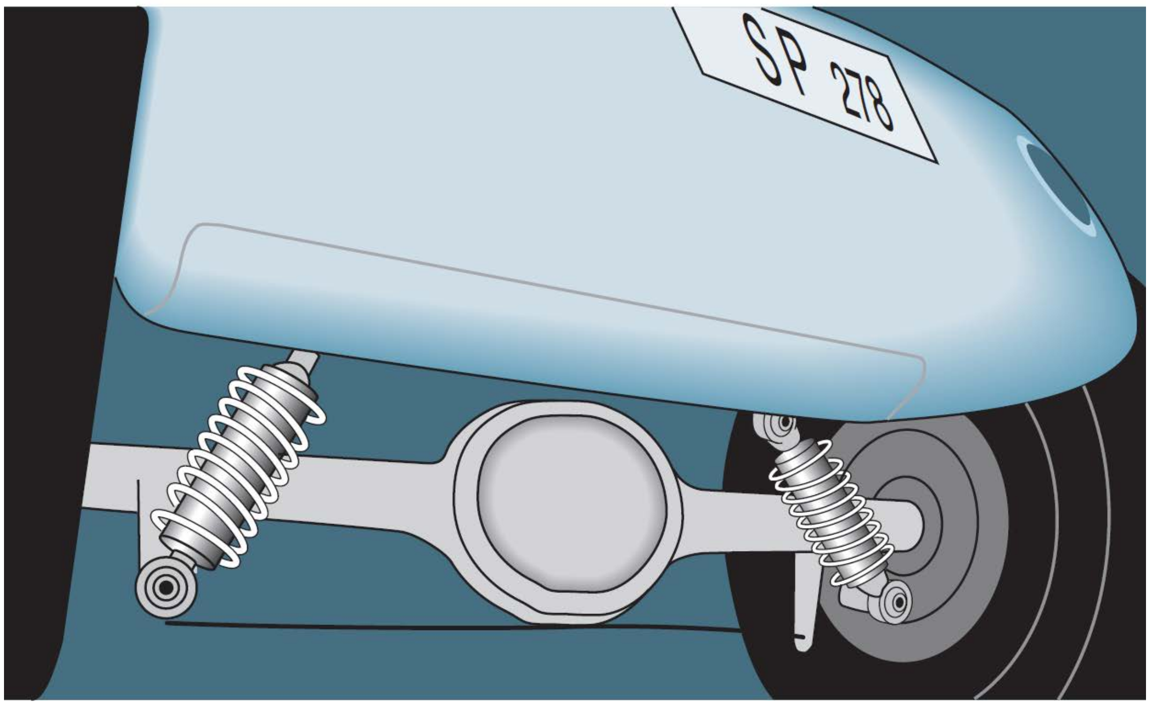
We will need the model of the wheel and the car body, with the shocks and springs in between. The control objective here will be to provide a 'smooth' ride for the passenger, even if the road is 'bumpy'.

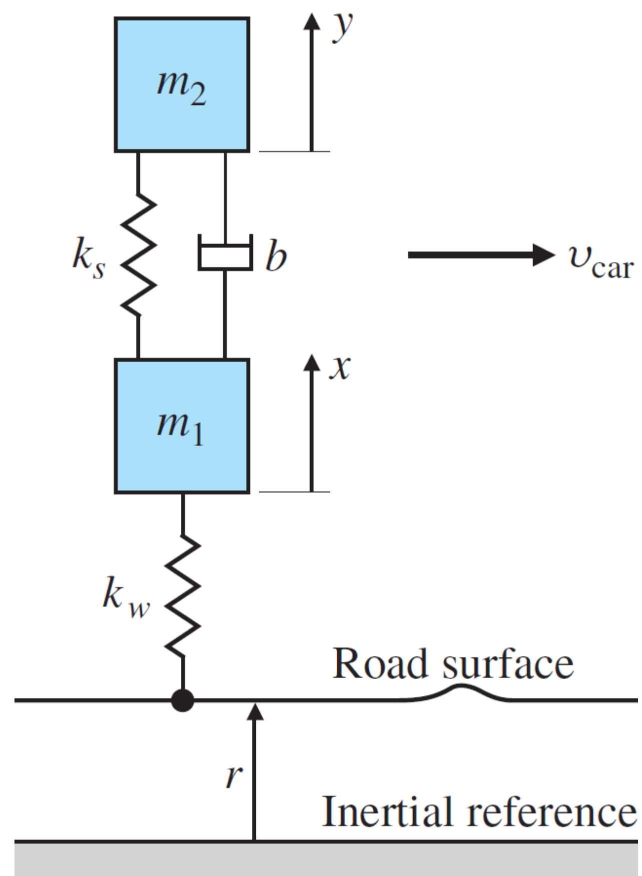
How will you proceed with this modeling?

Automobile suspension system – Detroit uses such models

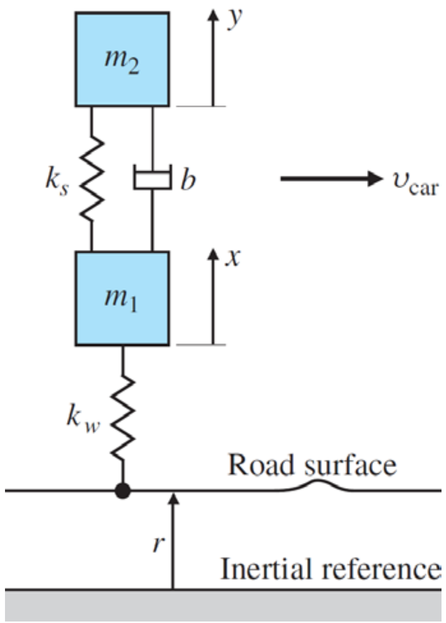
Quarter car model

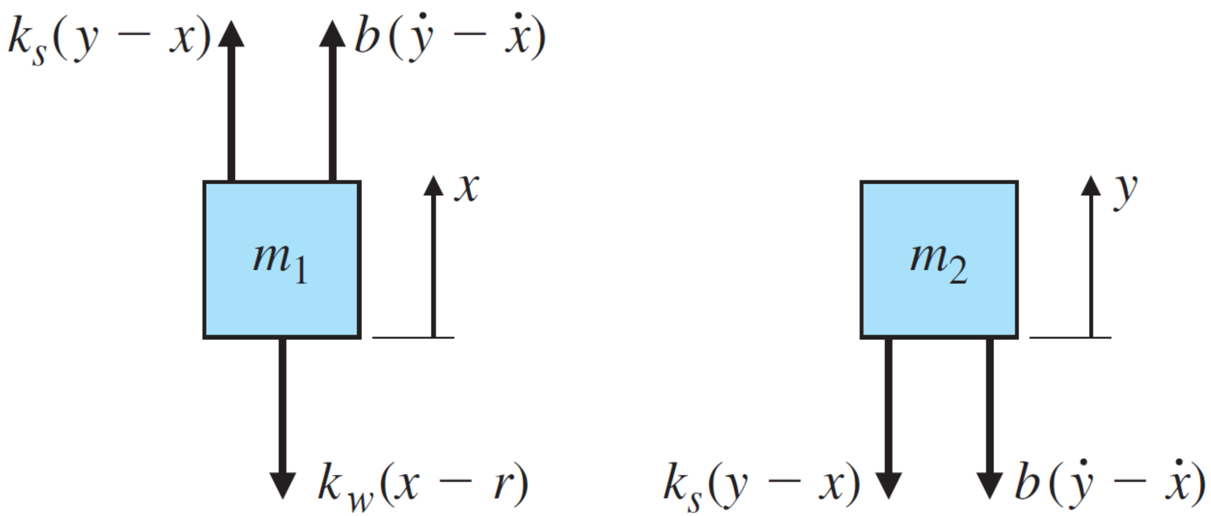
(one of the 4 wheels)





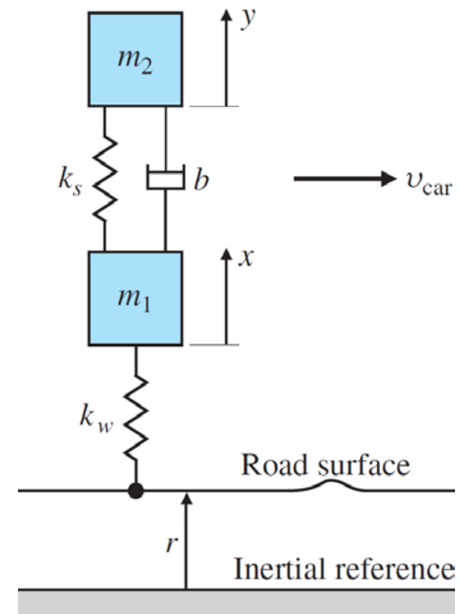
Write the equations of motion for the quarter car model

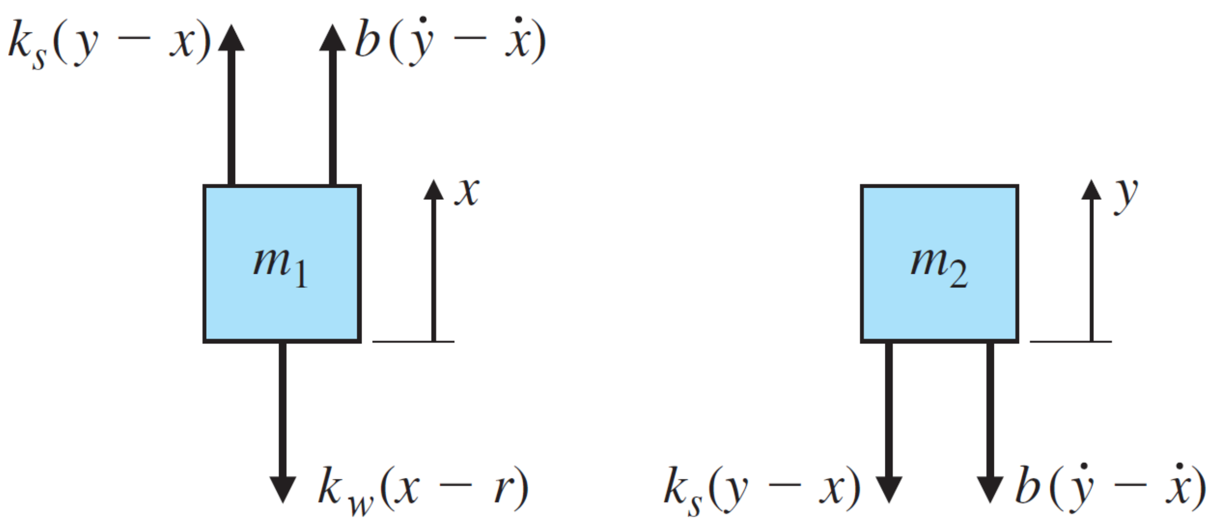




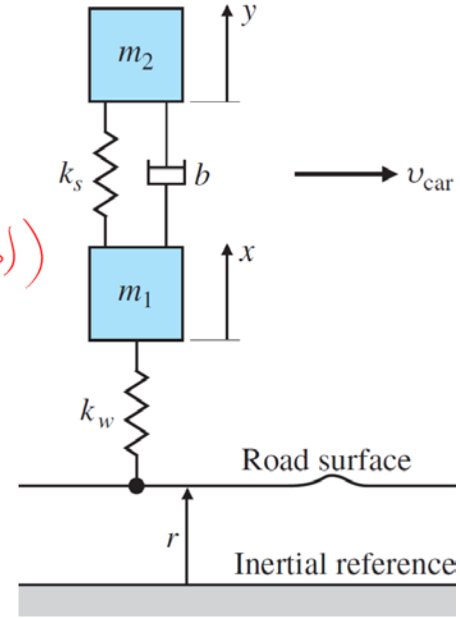
Write the equations of motion for the quarter car model

$$m_1\ddot{x}_1 = k_s(y - x) + b(\dot{y} - \dot{x}) - k_w(x - y) \quad \underline{Eq.1}$$





Write the equations of motion for the quarter car model



$$m_1\ddot{x} = k_s(y-x) + b(\dot{y}-\dot{x}) - k_w(x-y) \quad \underline{Eq.1}$$

$$m_1 s^2 X(s) = K_s(Y(s) - X(s)) + b(sY(s) - sX(s)) - K_w(X(s) - Y(s))$$

$$m_2\ddot{y} = -k_s(y-x) + b(\dot{y}-\dot{x}) \quad \underline{Eq.2}$$

$$m_2 s^2 Y(s) = -K_s(Y(s) - X(s)) + b(sY(s) - sX(s))$$

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{\frac{K_w b}{m_1 m_2}\left(s + \frac{K_s}{b}\right)}{s^4 + \left(\frac{b}{m_1} + \frac{b}{m_2}\right)s^3 + \left(\frac{K_s}{m_1} + \frac{K_s}{m_2} + \frac{K_w}{m_1}\right)s^2 + \left(\frac{K_w b}{m_1 m_2}\right)s + \frac{K_w K_s}{m_1 m_2}}$$