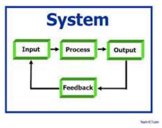


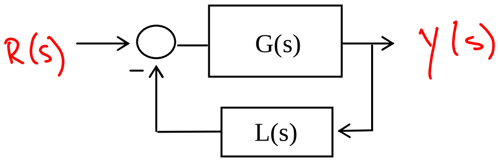
TEST 1

1) Clear your desk top of all **handwritten** papers and personal notes. You may keep **only** your test paper, a calculator and a pencil.
2) Read through the test completely and work the problems you can, leave the difficult ones till last.
3) Keep your eyes on your own paper. Cheating will not be tolerated
4) Work problems on the back of the previous page if necessary.
5) **Show your work!**

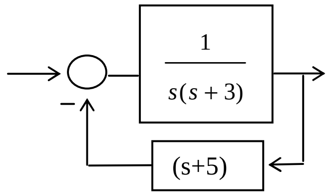
NAME: ____

Question 1

a) For a non-unity gain feedback system such as the one below, find a general expression, in transfer function form, for the steady state error. Show all steps clearly.



b) Then, for the system shown below, find its type and the steady state error to the appropriate input.



a) $Y(s) = G(s) * (R(s) - L(s)Y(s)) = G(s)R(s) - G(s)L(s)Y(s)$

$Y(s) + G(s)L(s)Y(s) = G(s)R(s)$

$(1 + G(s)L(s))\,Y(s) = G(s)R(s) \Rightarrow \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)L(s)}$

$E(s) \triangleq R(s) - Y(s)$

$E(s) = R(s) - \frac{G(s)}{1 + G(s)L(s)} * R(s) = \left(1 - \frac{G(s)}{1 + G(s)L(s)}\right) R(s)$

$E(s) = \frac{1 + G(s)L(s) - G(s)}{1 + G(s)L(s)} * R(s) = \frac{1 + (L(s) - 1)G(s)}{1 + L(s)G(s)} * R(s)$

Definition of $e_{ss} = \lim_{t\to\infty} \varepsilon(t) * t^K = c^{te}$

$\boxed{\text{*IF* Poles in LHP!!}}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} s\,E(s)\,\frac{1}{s^{K+1}} = c^{te}$

$$e_{ss} = \lim_{s\to 0} \frac{1+(L(s)-1)G(s)}{1+L(s)G(s)} * \frac{1}{s^K} = c^{te}$$

b) $$e_{ss} = \lim_{s\to 0} \frac{1+(s+5-1)*\frac{1}{s^2+3s}}{1+(s+5)*\frac{1}{s^2+3s}} * \frac{1}{s^K}$$

$$= \lim_{s\to 0} \frac{s^2+3s+s+4}{s^2+3s+s+5} * \frac{1}{s^K}$$

$$= \lim_{s\to 0} \frac{s^2+4s+4}{s^2+4s+5} * \frac{1}{s^K}$$

poles $\Rightarrow$ $(-2\pm j) \in$ LHP

if $K=0$ $\lim_{s\to 0}(\cdot) = \frac{4}{5} = c^{te}$ $\Rightarrow$ $\underline{\text{type } 0}$

Question 2

(a) Given the TF G(s) = 100/(s^2+15s+100), find below and **write next**:

i. the damping ratio = 0.75

ii. the natural frequency = 10 s^{-1}

iii. peak time = 0.475 s

iv. percent overshoot = 2.84 %

v. settling time = 0.613 s

vi. rise time = 0.18

(b) If the poles of another unity-gain system are at -3 ± j7, find below and again, **write next**:

i. the damping ratio = 0.394

ii. the natural frequency = 7.6 s^{-1}

iii. peak time = 0.449 s

iv. percent overshoot = 26%

v. settling time = 1.533 s

vi. rise time = 0.23 s

a) i) & ii) $G(s) = \dfrac{100}{s^2+15s+100} = \dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\Rightarrow \omega_n = 10$ & $\zeta = 0.75$

iii) $T_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{\pi}{10\sqrt{1-0.75^2}} = 0.475\,s$

iv) $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = e^{-\left(\frac{0.75\pi}{\sqrt{1-0.75^2}}\right)} = 2.84\,\%$

v) $T_s = \dfrac{4.6}{\zeta\omega_n} = 0.613\,s$

vi) $T_r = \dfrac{1.8}{\omega_n} = 0.18\,s$

b) i) & ii) poles $= -3 \pm 7j$

$$\begin{cases} \sigma = 3 = \zeta \omega_n \\ \omega_d = 7 = \omega_n \sqrt{1-\zeta^2} \end{cases} \Rightarrow \begin{array}{l} \omega_n = 7.6 \\ \zeta = 0.394 \end{array}$$

iii) $$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{7.6} = 0.449\,s$$

iv) $$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = e^{-0.394\pi/\sqrt{1-0.394^2}} = 26\%$$

v) $$T_s = \frac{4.6}{\zeta * \omega_n} = \frac{4.6}{0.394 * 7.6} = 1.533\,s$$

vi) $$T_R = \frac{1.8}{\omega_n} = \frac{1.8}{7.6} = 0.23\,s$$