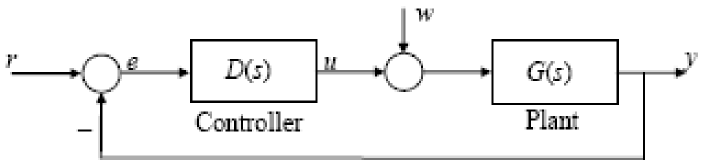
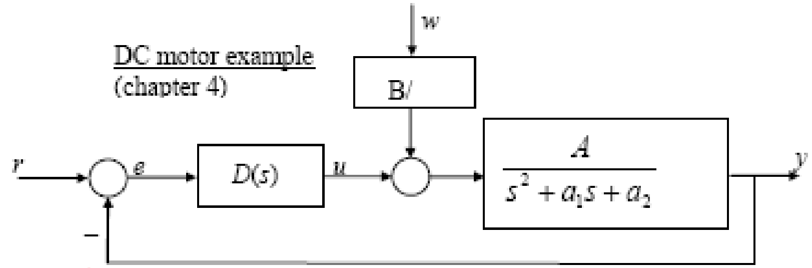
LECTURE 21

4.5 PID CONTROL

General



DC motor example (chapter 4)



RECALL: $\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2} \Rightarrow T_p = \frac{\pi}{\omega\sqrt{1-\zeta^2}}$ $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$ $T_s = \frac{4.6}{\zeta\omega}$ (1%) $T_r = \frac{1.8}{\omega}$

Objective: Determine how the C.L. roots are related to the control gains K_p, K_I and K_d (use SIMULINK to check some with G(s) = 1/(s+10), and different D(s))

Control Type	Time Domain Representation	Frequency domain representation of controller	Characteristic equation (denom. of closed loop TF=0), $1+D(s)G(s)=0$	Remarks
P	$u(t)=K_p e(t)$	$D(s)=K_p$	$s^2+a_1 s+a_2+AK_p=0$	*1 degree of freedom, and so cannot locate the roots at any location. *Can reduce error magnitude but non-zero steady-state error exists. *Can increase the speed of response but has much larger transient overshoot.
I	$u(t)=K_I\int_{t_0}^{t} e(t)dt$	$D(s)=\frac{K_I}{s}$	$s^3+a_1 s^2+a_2 s+AK_I=0$	*Can reduce or eliminate constant steady-state errors at the cost of worse transient response.
PI	$u(t)=\left(K_p e(t)+K_I\int_{t_0}^{t} e(t)dt\right)$	$D(s)=\frac{K_p s+K_I}{s}$	$s^3+a_1 s^2+(a_2+AK_p)s+AK_I=0$	*2 degrees of freedom and the designer can choose K_p and k_I independently to provide better transient response. *Approximated by lag controller.
D	$u(t)=K_d\dot{e}(t)$	$D(s)=K_d s$	$s^2+(a_1+AK_d)s+a_2=0$	*Pure derivative feedback is not practical to implement.
PD	$u(t)=(K_p e(t)+K_d\dot{e}(t))$	$D(s)=(K_p+K_d s)$	$s^2+(a_1+AK_d)s+(a_2+AK_p)=0$	*Can increase the damping and generally improve the stability of a system. *Approximated by lead controller.
PID	$u(t)=\left(K_p e(t)+K_I\int_{t_0}^{t} e(t)dt+K_d\dot{e}(t)\right)$	$D(s)=\left(K_p+\frac{K_I}{s}+K_d s\right)$	$s^3+(a_1+AK_d)s^2+(a_2+AK_p)s+AK_I=0$	*3 degrees of freedom and can provide an acceptable degree of error reduction simultaneously with acceptable stability and damping

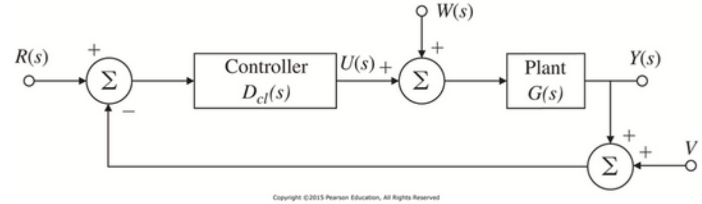
Other controller types: state feedback, adaptive, robust, stochastic, neural, fuzzy,…

(ADAPTED FROM SATISH NAIR)

RECALL :

$$Y = TR + GSW - TV$$

$$E = SR - GSW + TV$$



$(E = R - Y)$ $\qquad T = \frac{GD}{1+GD} \qquad S = \frac{1}{1+GD}$

$$T = 1 - S$$

OPEN LOOP

$$G = \frac{b}{a} \qquad D = \frac{c}{d} \qquad T = GD = \frac{bc}{ad}$$

IF G IS UNSTABLE – i.e. ROOTS (POLES) OF $\underline{a}$ IN RHP – ONE MIGHT THINK THAT THE SAME ROOTS IN $\underline{c}$ (ZEROS) COULD CANCEL THOSE POLES ⇒ UNSTABLE (SENSITIVITY !) (*)

SIMILARLY, IF G HAS POOR RESPONSE DUE TO SOME ZEROS OF $\underline{b}$ IN RHP, POLES OF $\underline{d}$ TO CANCEL THOSE ZEROS WOULD, AGAIN, LEAD TO AN UNSTABLE SYSTEM.

(*) SENSITIVITY

$$S_G^{T_{OL}} \triangleq \frac{\frac{\delta T_{OL}}{T_{OL}}}{\frac{\delta G}{G}} = \frac{\frac{\cancel{D}\,\delta G}{\cancel{D}\,G}}{\frac{\delta G}{G}} = 1 \qquad 10\% \text{ in } G \Rightarrow 10\% \text{ in } T_{OL}$$

FEEDBACK

$$G = \frac{b}{a} \qquad D = \frac{c}{d} \qquad T = \frac{\frac{bc}{ad}}{1+\frac{bc}{ad}} = \frac{bc}{ad+bc}$$

NOW, IF G IS UNSTABLE - POLES OF $\underline{a}$ IN RHP -

THE EXACT SAME ZEROS IN $\underline{b}$ WILL NOT CANCEL

THE POLES $\Rightarrow$ HOW TO CANCEL UNSTABLE POLES ?

BY ADDING STABLE ZEROS ?!

Ex: $$G = \frac{b}{a} = \frac{1}{s^2-1} = \frac{1}{(s+1)(s-1)}$$

$$D = \frac{c}{d} = \frac{K(s+1)}{(s+\delta)}$$

$$\Rightarrow T = \frac{bc}{ad+bc} = \frac{K\cancel{(s+1)}}{\cancel{(s+1)}(s-1)(s+\delta) + K\cancel{(s+1)}}$$

$$T = \frac{K}{(s-1)(s+\delta)+K} = \frac{K}{s^2+(\delta-1)s+(K-\delta)}$$

ROUTH CRITERIA $\Rightarrow$ CHOICE OF K, δ

$(K > \delta > 1)$

ALSO,

$$T_{CL} + \delta T_{CL} = \frac{(G+\delta G)D}{1+(G+\delta G)D} \qquad \delta T_{CL} \approx \frac{dT_{CL}}{dG}\,\delta G$$

$$S_G^{T_{CL}} \triangleq \frac{\frac{\delta T_{CL}}{T_{CL}}}{\frac{\delta G}{G}} = \frac{G}{T_{CL}}\frac{\delta T_{CL}}{\delta G} \approx \frac{G}{T_c}\frac{dT_{CL}}{dG} =$$

$$S_G^{T_{CL}} = \frac{G}{T_c}\frac{d}{dG}\left(\frac{GD}{1+GD}\right) = \dots = \frac{1}{1+GD}$$

IF GAIN OF $1+GD = 100 \Rightarrow$ 10% in $G \Rightarrow$ 0.1% in T_{CL}

NOTE: $S_G^{T_{CL}} = S = \dfrac{1}{1+GD}$

SYSTEM TYPE (FOR TRACKING THE INPUT)

(FOR $W = V = 0$)

K | FOR $R(t) = t^k$ $\therefore$ $e_{ss} = \lim_{t \to \infty} e(t) = c^{te}$

$e_{ss} = \lim_{s \to 0} s\, e(s) = c^{te}$ if roots $e(t) \in$ LHP

$e_{ss} = \lim_{s \to 0} s\, S R = \lim_{S \to 0} \frac{s}{1+GD} * \frac{1}{s^{k+1}} = \lim_{s \to 0} \frac{1}{1+GD} * \frac{1}{s^k}$

if $D = \frac{c}{d} = \frac{c}{s^n}$ n integrators

$e_{ss} = \lim_{s \to 0} \frac{1}{1+\frac{Gc}{s^n}} * \frac{1}{s^k} = \frac{s^n}{s^n + Gc} * \frac{1}{s^k}$

IF $Gc(s=0) = K_n \Rightarrow$ SYSTEM TYPE $= n$

RECALL:

$n = 2$

INPUT:	STEP	RAMP	PARABOLE
e_{ss}:	0	0	c^{te}

System Type (for regulation - disturbance rejection)

(for $R = V = 0$)

K | For $W(t) = t^k$ ∴ $e_{ss} = \lim_{t \to \infty} e(t) = c^{te}$

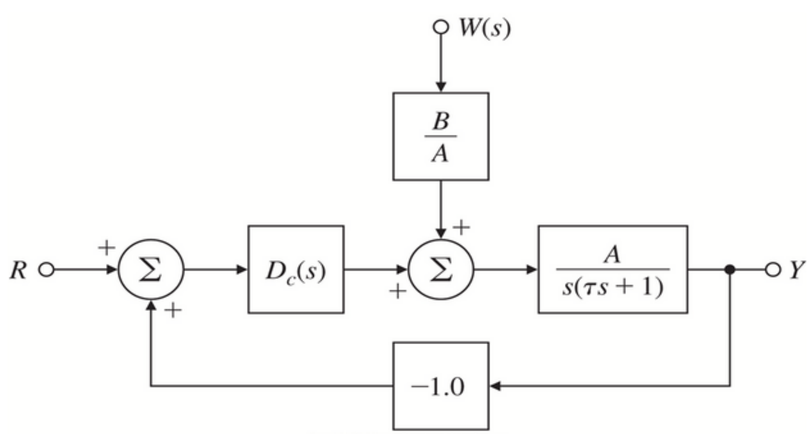
$$e(s) = \cancel{R}^{0}(s) - Y(s) = -Y(s) \Rightarrow \frac{e(s)}{W(s)} = -\frac{Y(s)}{W(s)} = T_W(s)$$

$= GS$ for unit feedback

taking $\lim_{t \to \infty}(\cdot)$ or $\lim_{s \to 0} s(\cdot)$

$$y_{ss} = \lim_{s \to 0} s\, T_W(s) * \frac{1}{s^{k+1}} = \lim_{s \to 0} T_W(s) \frac{1}{s^k}$$

Ex



$D = K_p$ & $D = K_p + \frac{K_I}{s}$

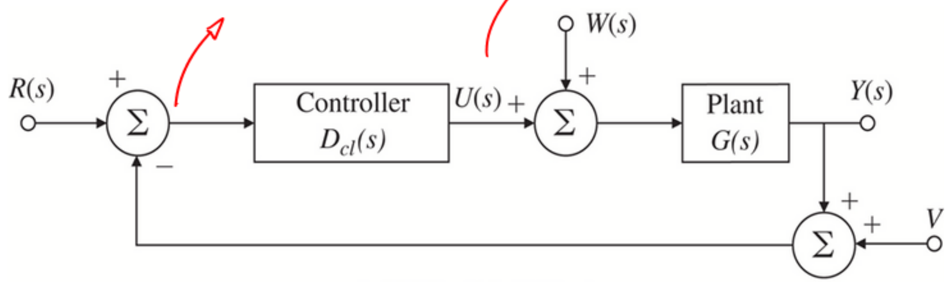
⇓ type 0 ⇓ type 1

$Y_{ss} = \frac{-B}{AK_p}$ $Y_{ss} = \frac{-B}{AK_I}$

PROPORTIONAL CONTROL

E(s)

CONTROL VARIABLE



$u(t) = K_p\, e(t)$ = CONTROL is Proportional to error

$$\frac{U(s)}{E(s)} = D_{cl}(s) = K_p \quad \& \quad T(s) = \frac{K_p G(s)}{1 + K_p G(s)} \qquad e_{ss} = \lim_{s \to 0} \frac{s}{1+K_pG(s)} * \frac{1}{s^{k+1}}$$

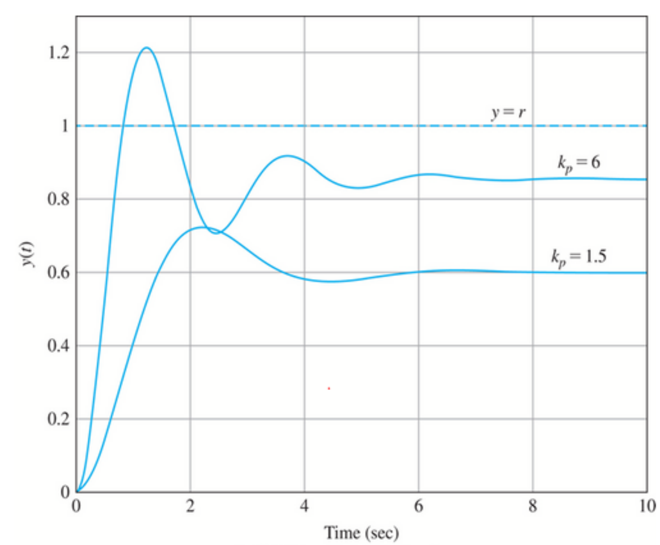
$$\text{if} \quad G(s) = \frac{b}{a} = \frac{A}{s^2 + a_1 s + a_2} \qquad \left(D = \frac{c}{d} = K_p\right)$$

$$T(s) = \frac{bc}{ad + bc} = \frac{A}{s^2 + a_1 s + a_2 + K_p A}$$

ω_n Adjust

SYSTEM TYPE FOR TRACKING = 0

$$\Rightarrow \quad e_{ss} = \frac{1}{a_2 + K_p A}$$



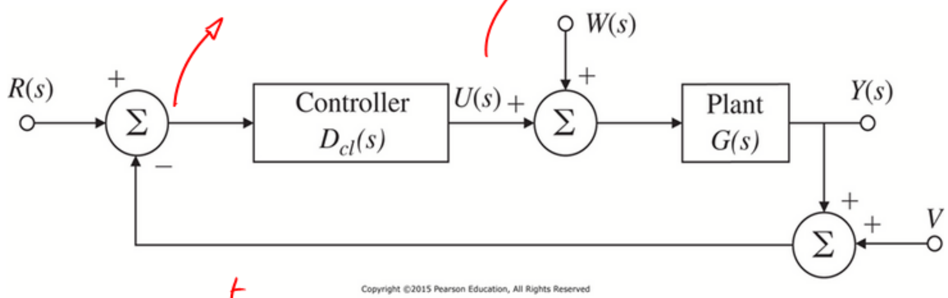
e_{ss}

e_{ss}

INTEGRAL CONTROL

$E(s)$

CONTROL VARIABLE



$$u(t) = K_I \int_{t_0}^{t} e(t) \quad = \quad \text{CONTROL IS THE ACCUMULATED ERROR}$$

$$\frac{U(s)}{E(s)} = D_{cL}(s) = \frac{K_I}{s} \quad \& \quad T(s) = \frac{\frac{K_I}{s} G(s)}{1 + \frac{K_I}{s} G(s)} = \frac{K_I G(s)}{s + K_I G(s)}$$

IF $G(s=0) = 1$

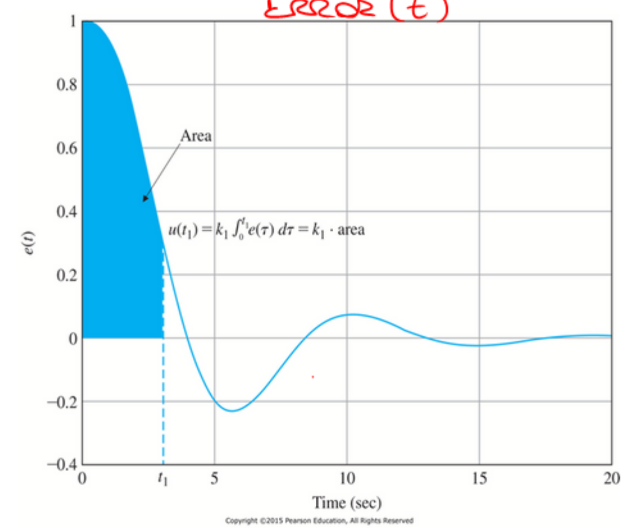
$$e_{ss} = \lim_{s \to 0} \frac{s}{s + GK_I} * \frac{1}{s^k}$$

SYSTEM TYPE FOR TRACKING = 1

$e_{ss} = 0$ for $R(s) = \frac{1}{s}$ (unit step)

$e_{ss} = \frac{1}{K_I}$ for $R(s) = \frac{1}{s^2}$ (ramp)

ERROR (t)



IF $G(s=0) = 1$ (AS ABOVE)

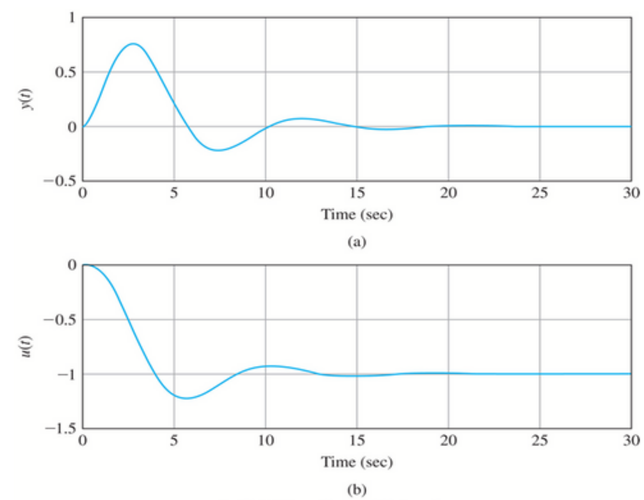
$$y_{ss} = \lim_{s \to 0} s\, T_w(s) * \frac{1}{s^{k+1}} = \lim_{s \to 0} s \frac{G}{1+DG} * \frac{1}{s^{k+1}} = \lim_{s \to 0} \frac{G s}{s + K_I G} * \frac{1}{s^k}$$

$\hookrightarrow \frac{K_I}{s}$

<u>SYSTEM TYPE FOR REGULATION = 1</u>

$y_{ss} = 0$ for $w(s) = \frac{1}{s}$ (step)

$y_{ss} = \frac{1}{K_I}$ for $w(s) = \frac{1}{s^2}$ (ramp)



$$u_{ss} = \lim_{s \to 0} s \frac{-DG}{1+DG} * \frac{1}{s^{k+1}} = \lim_{s \to 0} - \frac{K_I G}{s + K_I G} * \frac{1}{s^k}$$

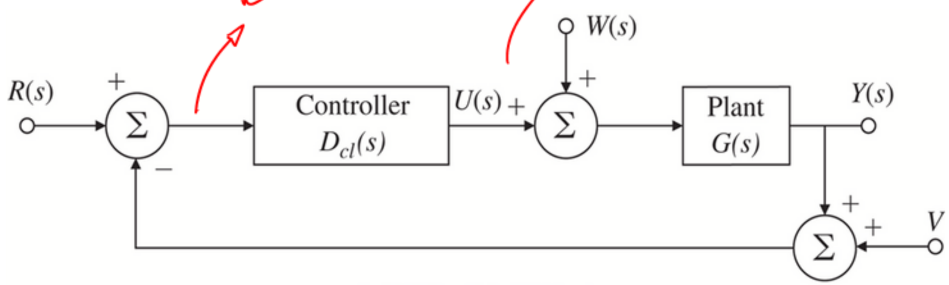
$u_{ss} = -1$ for $w(s) = \frac{1}{s}$ (step)

$u_{ss} = \infty$ for $w(s) = \frac{1}{s^2}$ (ramp)

DERIVATIVE CONTROL

$E(s)$

CONTROL VARIABLE



$u(t) = K_D \, \dot{e}(t)$ = CONTROL IS THE RATE OF THE ERROR

$$\frac{U(s)}{E(s)} = D_{cl}(s) = K_D s \qquad T(s) = \frac{K_D s G(s)}{1 + K_D s G(s)} = \frac{K_D s G(s)}{1 + K_D s G(s)}$$

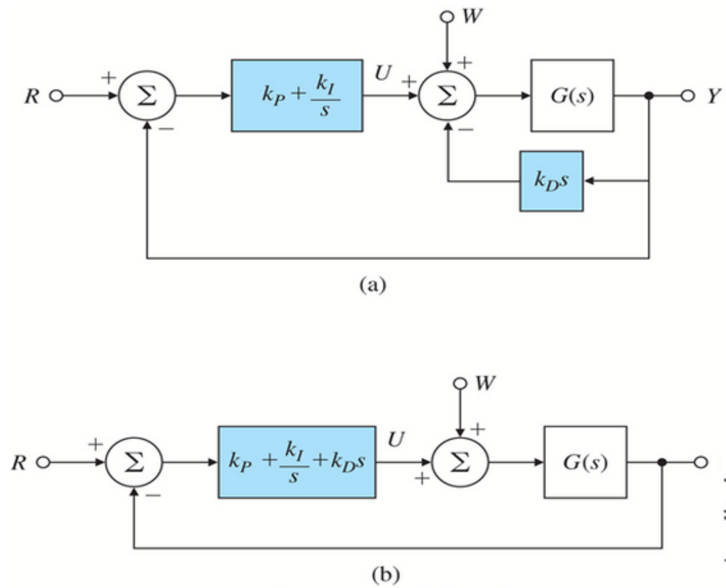
If $G(s) = \frac{b}{a} = \frac{A}{s^2 + a_1 s + a_2}$

$$T(s) = \frac{bc}{ad + bc} = \frac{A K_D s}{s^2 + (a_1 + K_D A) s + a_2}$$

ζ

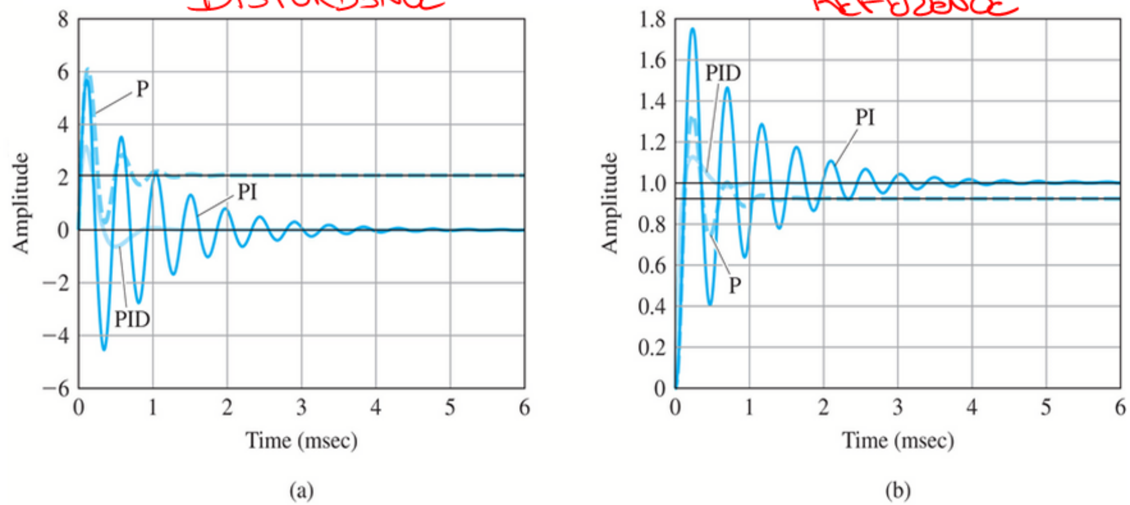
PID

$$G(s) = s^2 + a_1 s + a_2$$



STEP DISTURBANCE

STEP REFERENCE



$$u(t) = K_p e(t) + K_I \int_{t_0}^{t} e(t) + K_D \dot{e}(t)$$

$$u(s) = \left(K_p + \frac{K_I}{s} + K_D s\right) e(s)$$