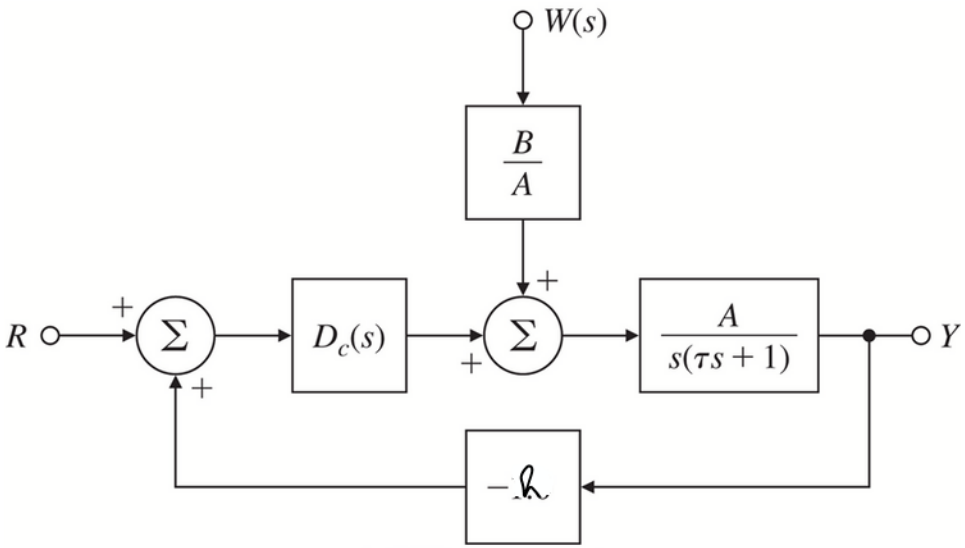
LECTURE 22



i) $y_{ss} = ?$ if $D_c(s) = $ PROP

ii) $y_{ss} = ?$ if $D_c(s) = $ PI

(ADAPTED FROM SATISH NAIR)

i) $D = K_p$

$$Y_{ss} = \lim_{s \to 0} -s\, T_w(s) \frac{1}{s^{k+1}}$$

$$T_w = \frac{G}{1 + DGH} \frac{B}{A} = \frac{\frac{B}{s(\tau s+1)}}{1 + K_p \frac{A h}{s(\tau s+1)}}$$

$$T_w = \frac{B}{\tau s^2 + s + K_p A h} \Rightarrow Y_{ss} = \lim_{s \to 0} \frac{-B}{\tau s^2 + s + K_p A h} \frac{1}{s^k} \Rightarrow$$

$\underline{\text{type } 0}$

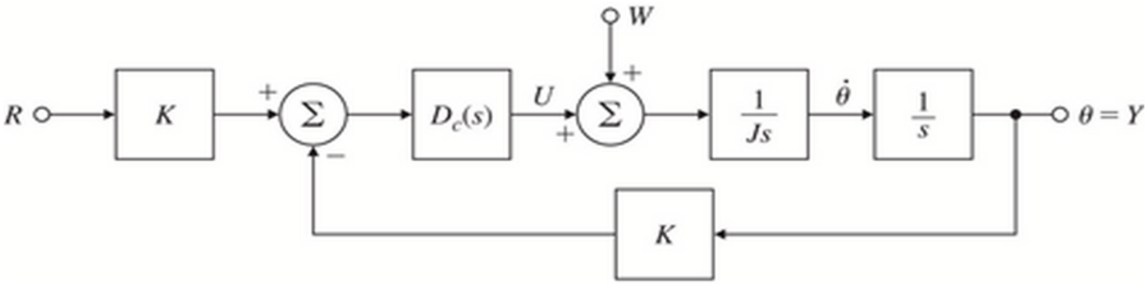
$$Y_{ss} = \frac{-B}{K_p A h}$$

iii) Similarly, for $D = \frac{K_I}{s} + K_D$

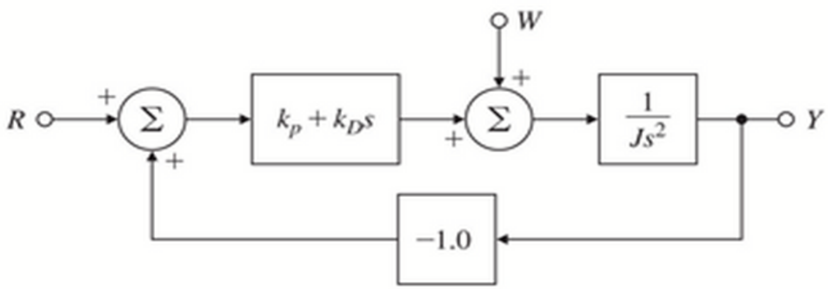
$$T_w = \frac{Bs}{\tau s^3 + s^2 + K_D A h s + K_I A h}$$

$\underline{\text{type } 1}$

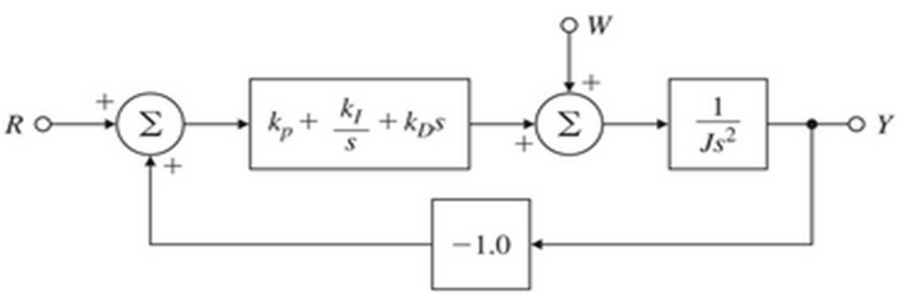
$$Y_{ss} = \lim_{s \to 0} \frac{-Bs}{\tau s^3 + s^2 + K_D A h s + K_I A h} * \frac{1}{s^k} \Rightarrow Y_{ss} = \frac{-B}{K_I A h}$$



(a)



(b)



(c)

i) System type for tracking (b & c)

ii) Sytem type for regulation (disturbance rejection)

4.6 ZIEGLER-NICHOLAS TUNING OF PID REGULATORS

Problem statement: For many real world systems, models are difficult to obtain. How do we perform control design in such cases?

J. G. Ziegler and N. B. Nichols (1948) recognized that the step responses of a large number of process control systems exhibit a process reaction curve. This curve can be generated from either experimental data or dynamic simulation of the plant. The S- shape of the curve is characteristic of many high-order systems, and such that plant transfer functions may be approximated by

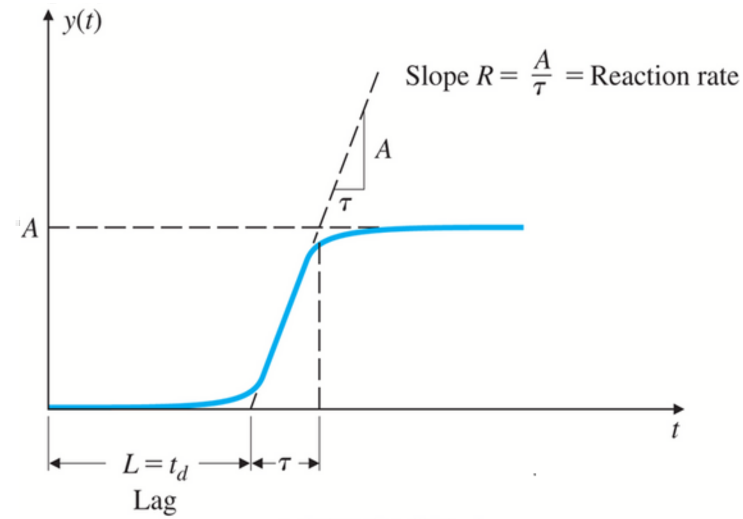
$$\frac{Y(s)}{U(s)} = \frac{Ae^{-t_d s}}{\tau s+1}$$

which is simply a first order system plus a time delay of t_d seconds. The constants can be determined from unit step response of the process, experimentally. These are

$L = t_d \qquad R = \frac{A}{\tau}$

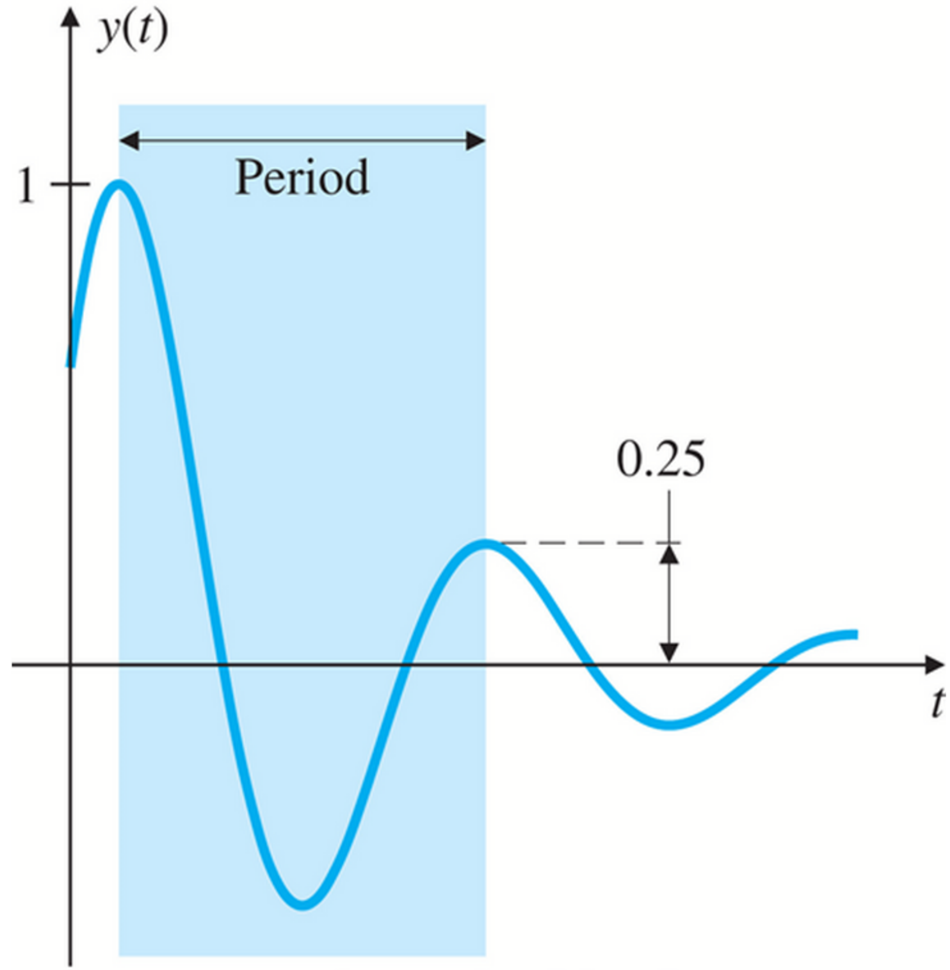
Ziegler and Nichols gave two methods for determining the constants for a controller

$D(s) = K_p(1 + \frac{1}{T_I s} + T_D s)$ for such a process



Quarter decay ratio method

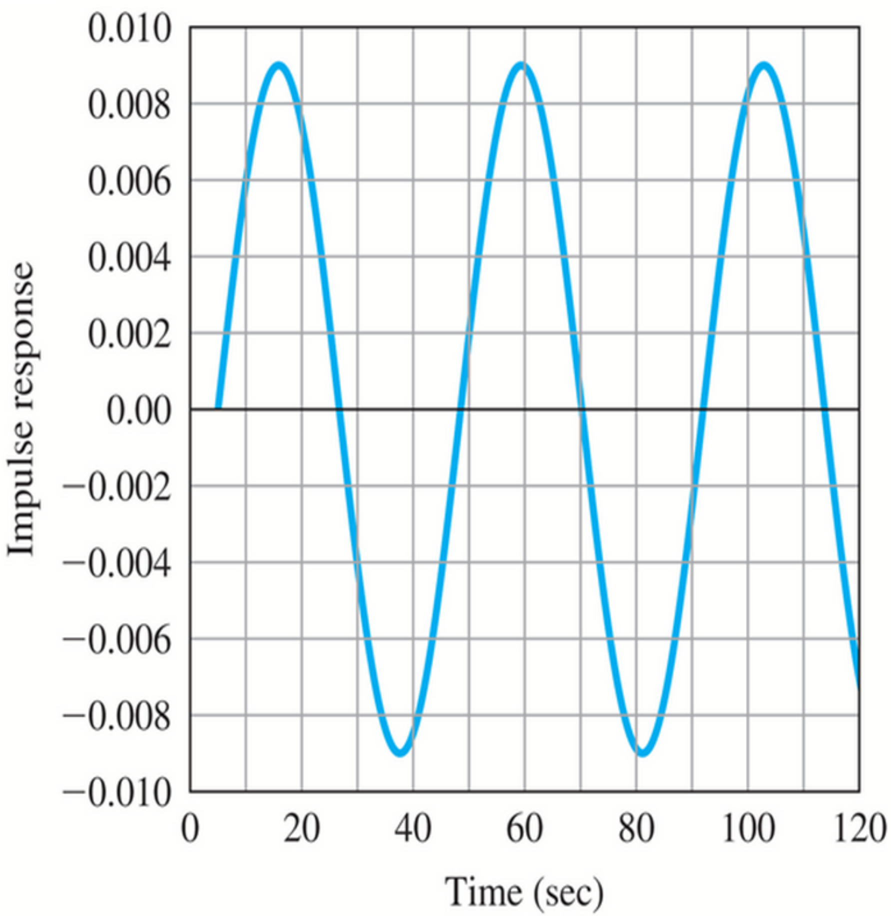
The choice of controller parameters is based on a decay ratio of approximately 0.25. This means that a dominant transient decays to quarter of its value after one period of oscillation. A quarter decay corresponds to $\zeta = 0.21$ and is a good compromise between quick response and adequate stability margins.



Type of Controller	Optimum Gain
P	$k_P = 1/RL$
PI	$k_P = 0.9/RL$ $T_I = L/0.3$
PID	$k_P = 1.2/RL$ $T_I = 2L$ $T_D = 0.5L$

$$D_c(s) = k_P(1 + 1/T_I s + T_D s)$$

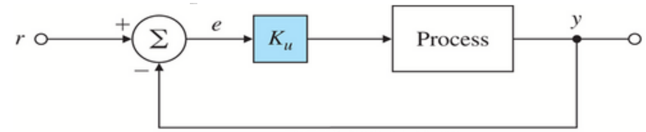
Note: The process operator starts with these gain values and 'tunes' them till a satisfactory response is obtained.



$$D_c(s) = k_P(1 + 1/T_I s + T_D s)$$

Ultimate sensitivity method

The criteria for adjusting the parameters are based on evaluating the system at the limit of stability rather than on taking a step response.

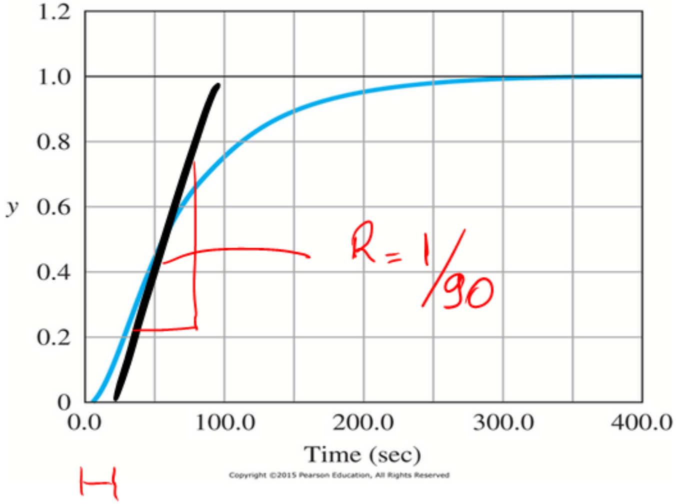


We increase the proportional gain K_u until we observe continuous oscillation, that is, until the system becomes marginally stable. The period of oscillation P_u should be measured when the amplitude of oscillation is quite small. Then we back off from this gain as shown in the table at below.

Type of Controller	Optimum Gain
P	$k_P = 0.5K_u$
PI	$\begin{cases} k_P = 0.45K_u \\ T_I = \frac{P_u}{1.2} \end{cases}$
PID	$\begin{cases} k_P = 1.6K_u \\ T_I = 0.5P_u \\ T_D = 0.125P_u \end{cases}$

Note: The process operator starts with these gain values and 'tunes' them till a satisfactory response is obtained.

Ex Design the P & PI controllers for a system w/ the following process reaction using the QDR method.



$L = 13s$

$$D_c(s) = k_P(1 + 1/T_I s + T_D s)$$

PROPORTIONAL - P

$$\Rightarrow \quad K_p = \frac{1}{RL} = \frac{90}{13} = 6.92$$

$$\boxed{D_c(s) = 6.92}$$

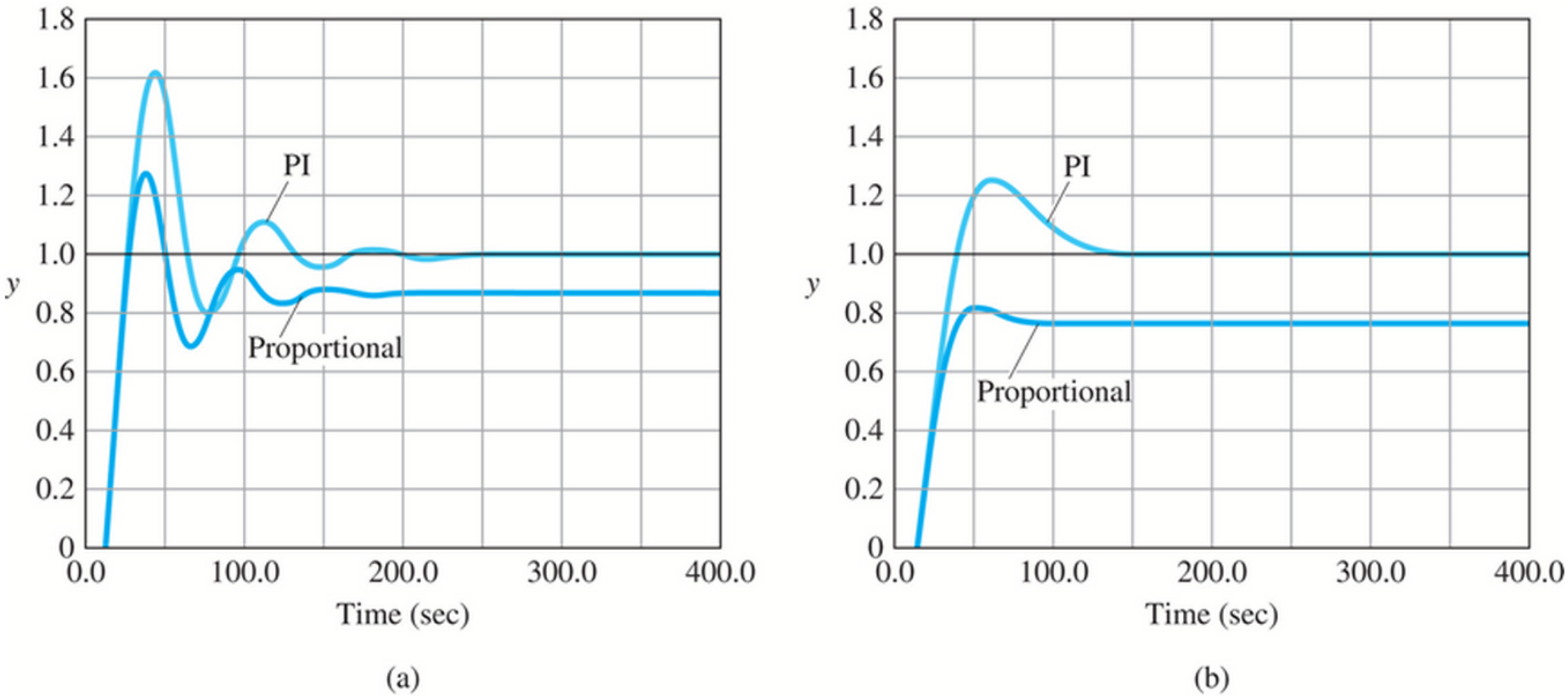
PROPORTIONAL INTEGRAL - PI

$$\Rightarrow \quad K_p = \frac{0.9}{RL} = 6.22 \qquad T_I = \frac{L}{0.3} = \frac{13}{0.3} = 43.3$$

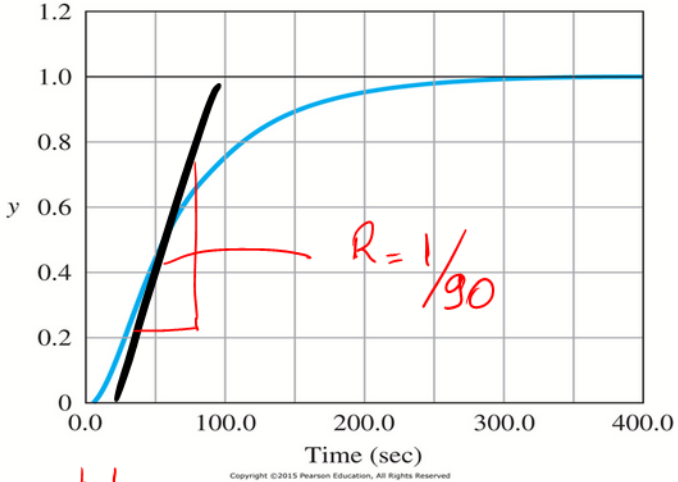
$$D_c(s) = 6.22 + \frac{6.22}{43.3 s} = \boxed{6.22 + \frac{0.14}{s}}$$

P: $D_c(s) = 6.92$ $\xrightarrow{(\div 2)}$ P: $D_c(s) = 3.46$

PI: $D_c(s) = 6.22 + \frac{0.14}{s}$ PI: $D_c(s) = 6.11 + \frac{0.07}{s}$

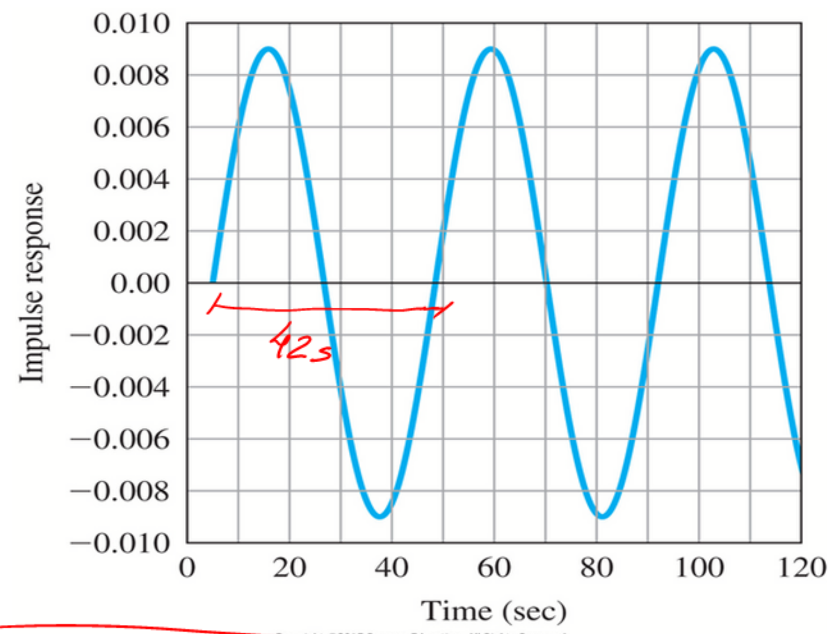


Ex Design the P & PI controllers for a system w/ the following process reaction using the US method given that the ultimate gain (K_u) was measured to be 15.3 and the ultimate period, 42 secs



$L = 13\,s$

$$D_c(s) = k_P(1 + 1/T_I s + T_D s)$$



Proportional - P

$K_p = 0.5 K_u = 7.65$

$$\boxed{D_c(s) = 7.65}$$

Proportional Integral - PI

$K_p = 0.45 K_u = 6.885$

$T_I = \frac{1}{1.2} P_u = 35$

$$\Rightarrow \boxed{D_c(s) = 6.885 + \frac{0.197}{s}}$$

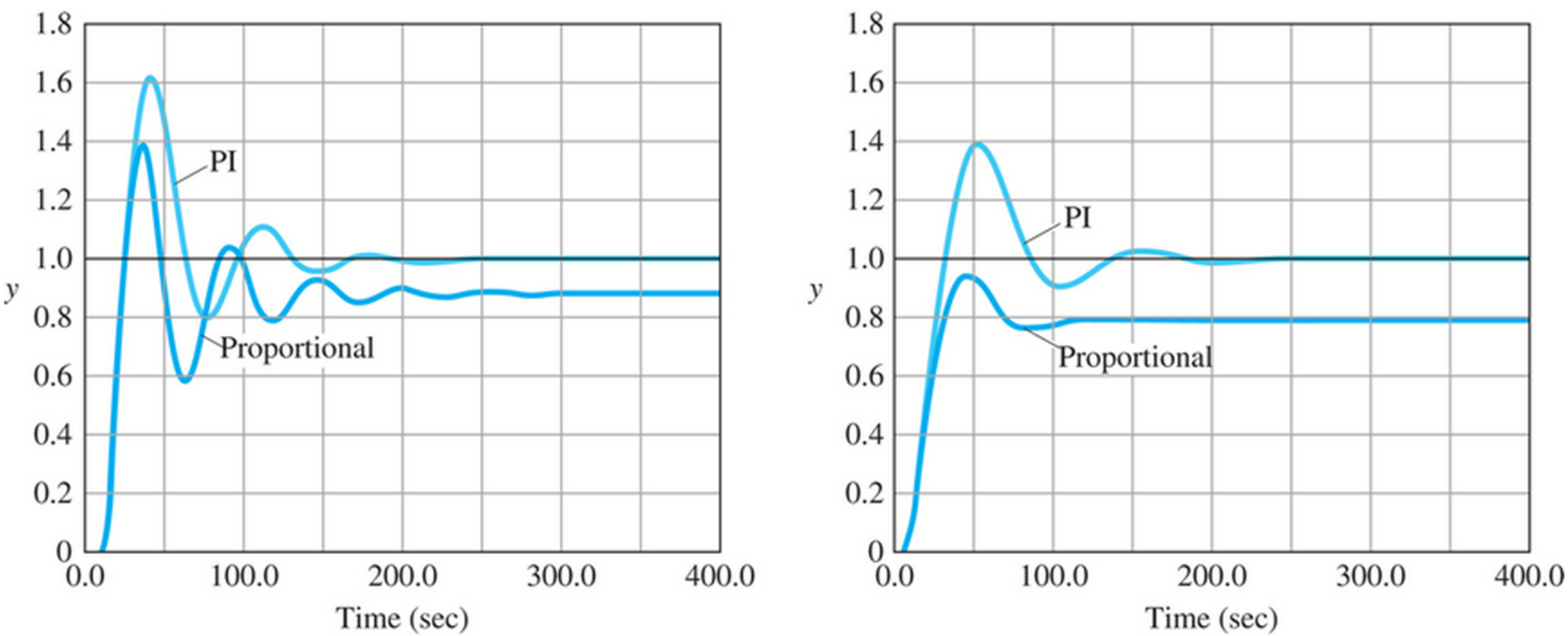
P: $D_c(s) = 7.65$

$(\div 2)$

P: $D_c(s) = 3.87$

PI: $D_c(s) = 6.885 + \frac{0.197}{s}$

PI: $D_c(s) = 3.44 + \frac{0.098}{s}$



(a) (b)