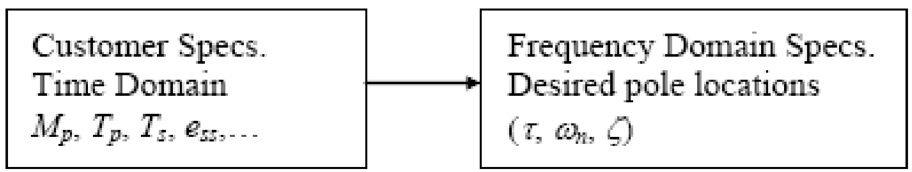
LECTURE 28

5.1 THE CONTROL PROBLEM

Problem Statement

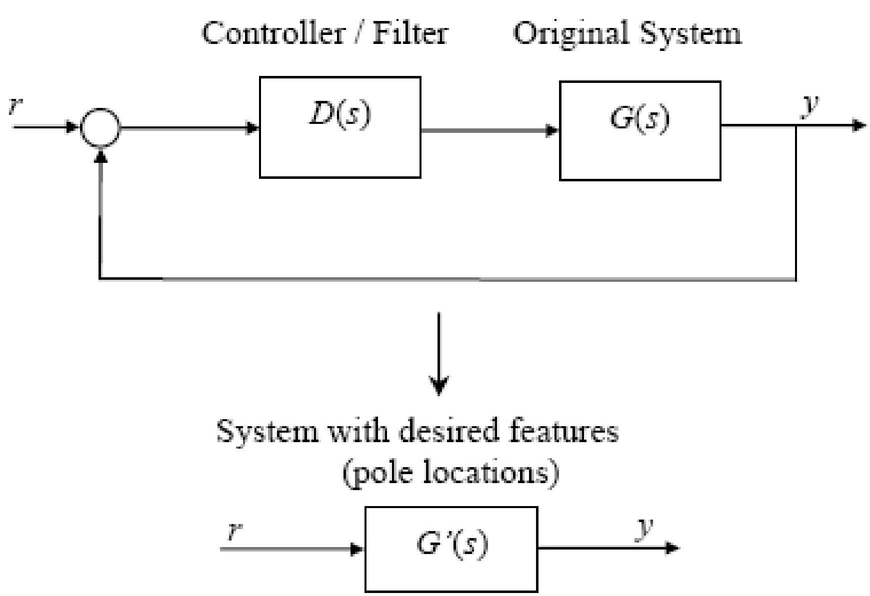
- Given dynamic system does not have desired characteristics -
  transient, steady state, stability (i.e., pole locations not good)
- Define or find the desired characteristics in terms of M_p, T_p, T_s, steady state error, etc., from custome specifications (e.g. overshoot, steady state error)

This permits specification of desired pole locations (closed loop)



Solution

- The problem is well posed now – what $D(s)$ will allow the closed loop poles to be at the desired locations?



(ADAPTED FROM SATISH NAIR)

HOW? - compensator design using root locus (Chapter 5), and using frequency response (Chapter 6)

Thus, control design mathematically changes $G(s)$ to $G'(s) = \dfrac{D(s)G(s)}{1 + D(s)G(s)}$ by adding a controller $D(s)$.

This transfer function G'(s) has its poles at the 'desired' locations.
[<u>Note: 1 + D(s)G(s) = 0 is called the *characteristic equation*, and its roots are the CL poles</u>]

<u>**Types of Controllers, *D*(*s*)**</u>

ECE 4310

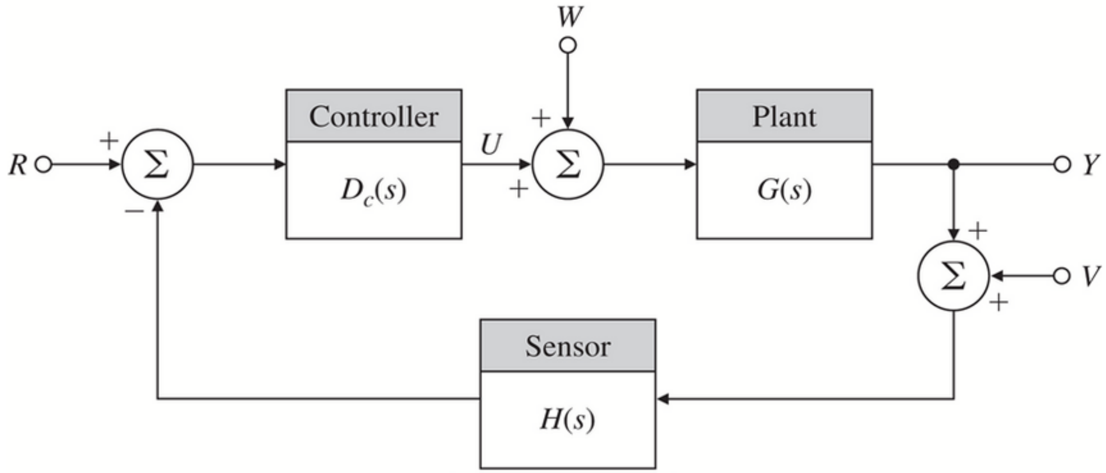
$D(s)$ is either
---- Pure gain
---- PID
---- Lead and Lag

→

Advanced Control Courses

$D(s)$ can be, in addition,
---- State space
---- Adaptive
---- Robust
---- Neural / Fuzzy
---- …

IN GENERAL



CHARACTERISTIC EQUATION

$$\frac{DG}{1+DGH} \Rightarrow \quad CE = 1 + DGH = 0$$

WE WANT TO DETERMINE THE ROOT LOCI AS A CERTAIN PARAMETER K VARIES [0 ... ∞)

ROOT LOCUS FORMS = EVANS EQUATIONS

$$1 + K L(s) = 0 \qquad 1 + K \frac{b(s)}{a(s)} = 0$$

$$a(s) + K b(s) = 0 \qquad L(s) = -1/K$$

5.2 ROOT LOCUS OF A SYSTEM

Find the closed loop TF for the system G(s) = 1/(s(s+1)), with a pure gain (K) controller. What is its characteristic equation? $H(s) = 1$

CE is: ____

[Note: 1 + D(s)G(s) = 0 is called the *characteristic equation*, and its roots are the CL poles]

Now find how the roots of the CE (i.e., poles of the CLTF) move as K varies.

K	**Root 1**	**Root 2**	**Plot**
0			
1/4			
1			
∞			

Now plot them in the Re-Imag plane.

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$\text{roots} = \frac{-1}{2} \pm \frac{\sqrt{1-4K}}{2}$

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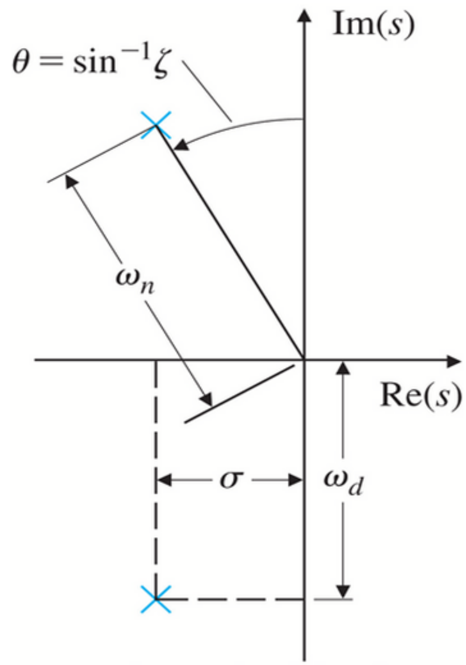
1	$\frac{-1}{2} + \frac{\sqrt{3}}{2}j$	$-\frac{1}{2} - \frac{\sqrt{3}}{2}j$	
∞	$-\frac{1}{2} + \infty j$	$-\frac{1}{2} - \infty j$	

Now plot them in the Re-Imag plane.

RECALL (CHAP 3)

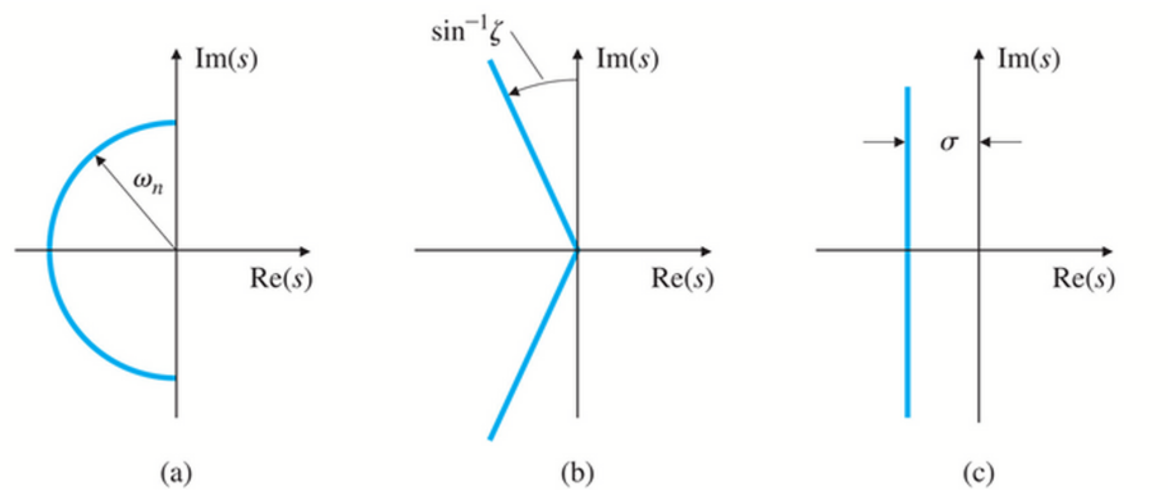


$$a(s) = (s + r_1)(s + r_2)$$

$$r_{1,2} = -\sigma \pm j\omega_d$$

$$a(s) = (s+\sigma)^2 + \omega_d^2$$

$$\sigma = \zeta\omega_n \quad \& \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$



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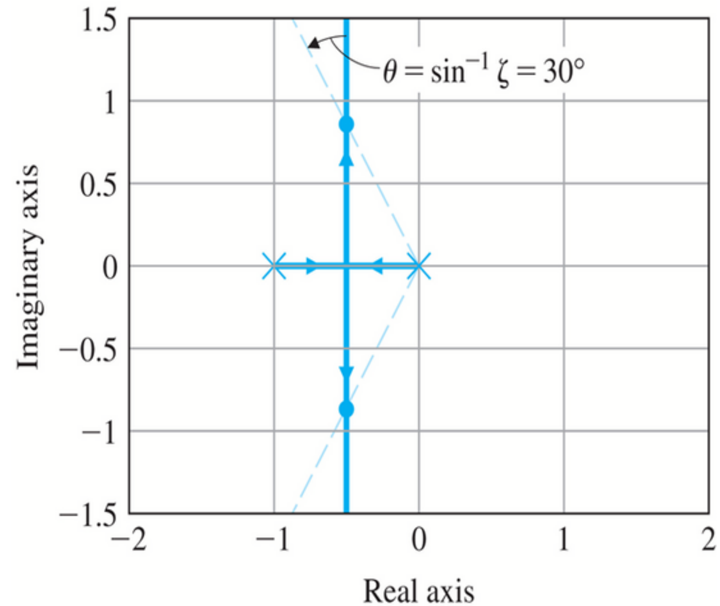
Plot



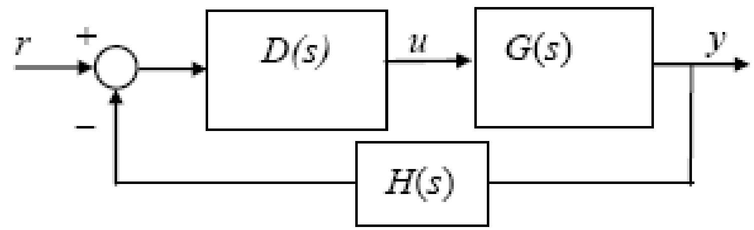
Now plot them in the Re-Imag plane.

Repeat for the CE $1+ K/[(s(s+1)(s+5)]=0$

K	**Root 1**	**Root 2**	**Root 3**	**Plot**
0				
0.1				
…				
…				
1				
…				
…				
10				
….				
…				
100				

Now plot them in the Re-Imag plane.

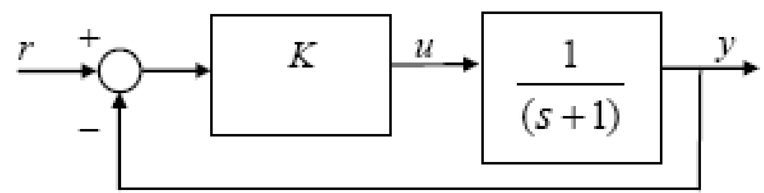
Problem: What is meant by the root locus of a system?



Consider the system shown alongside. For this system, the CLTF is T(s) = D(s)G(s)/[1+D(s)G(s)H(s)], and the CE is 1 + D(s)G(s)H(s) = 0 (gives locations of CL poles). How do the locations of the roots of this system change as one of its parameter changes? To study this, we put the eqn. in polynomial form and select the parameter of interest, which we will call K. That is, rewrite the CE as 1 + K* P(s) = 0, where P(s) is a polynomial b(s)/a(s). Typically, the parameter K is the gain of the controller, and then P(s) is proportional to D(s)G(s)H(s), i.e., remove K out of D(s) and the rest is P(s). Note that the loop transfer function of the system is L(s)=D(s)G(s)H(s).

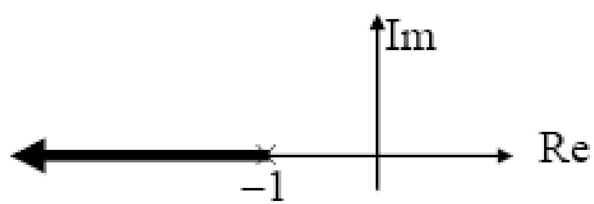
Examples:

(i) What is the root locus for the system shown alongside? Find the CE for the system to determine the poles of the CLTF, i.e., roots of the CE.



CE is $1 + \dfrac{k}{(s+1)} = 0$, i.e., s+1+K = 0.

So, the root locus for $G(s) = \dfrac{1}{(s+1)}$ is

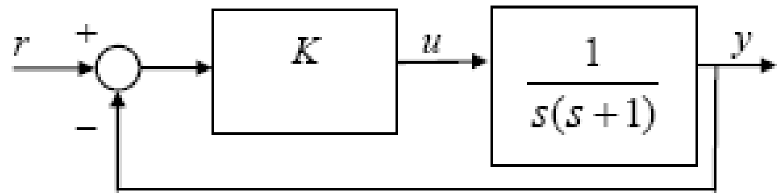


Evans Eq.

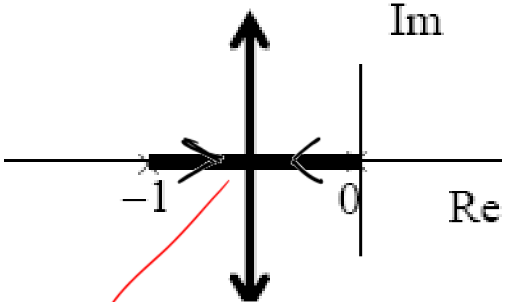
$b(s) = 1$ (No zeros) $m = 0$ (order of b(s)) $L(s) = \frac{1}{s+1}$

$a(s) = s+1$ (pole = −1) $n = 1$ (order of a(s))

(ii) What is the root locus for 1+K/[s(s+1)] = 0?



The CE is $1 + \dfrac{K}{s(s+1)} = 0$, i.e., $s^2+s+K = 0$ (see Example 5.1 in text, page 234). You can get this easily using MATLAB – just one command! But, you have to know how to do this by hand first.



breakaway point

EVANS EQ

$b(s) = 1$ (NO ZEROS) $m = 0$

$a(s) = s^2 + s$ (POLES $= 0, -1$) $n = 2$

$L(s) = \dfrac{1}{s^2+s}$

KEY POINT: The math above suggests that if you need to determine how the CL roots vary with ANY parameter in the system, say 'c', then just rewrite the CE in the from 1 + c*G(s) = 0, which allows us to use the rlocus command in MATLAB (or the technique we discuss next) to get the root locus for the closed loop system.
SO, THE TECHNIQUE CAN BE EXTENDED TO STUDYING HOW THE ROOTS OF THE CE CHANGE WITH ANY PARAMETER!!! e.g., How do we determine of the roots of G(s) in the structure for the examples discussed above as c changes if G(s) = 1/[s(s+c)] and K=1? ………………
CE is 1+G(s) = 0, i.e., 1+ 1/[s(s+c)] = 0, which results in $s^2 + cs + 1 = 0$. How do we get this to the standard form?