# LECTURE 29

What if: $CE = s^2 + cs + 1 = 0$

ROOT LOCUS AS $c$ CHANGES?

EVANS EQ (RECALL)

$$1 + DGH = 0 \Rightarrow 1 + K L(s) = 0$$

$$1 + K\frac{b(s)}{a(s)} = 0$$

IN THIS CASE:

$1 + K L(s) = 0$ & $K = c$

$$\Rightarrow s^2 + 1 + cs = 0$$

$$1 + \frac{cs}{s^2+1} = 0$$

$L(s) = \frac{s}{s^2+1}$ & $b(s) = s$ $m = 1$ $z_1 = 0$

$a(s) = s^2 + 1$ $n = 2$ $p_{1,2} = \frac{-c}{2} \pm \frac{\sqrt{c^2-4}}{2}$

(ADAPTED FROM SATISH NAIR)

What is the root Locus for the system w/ CE as above?

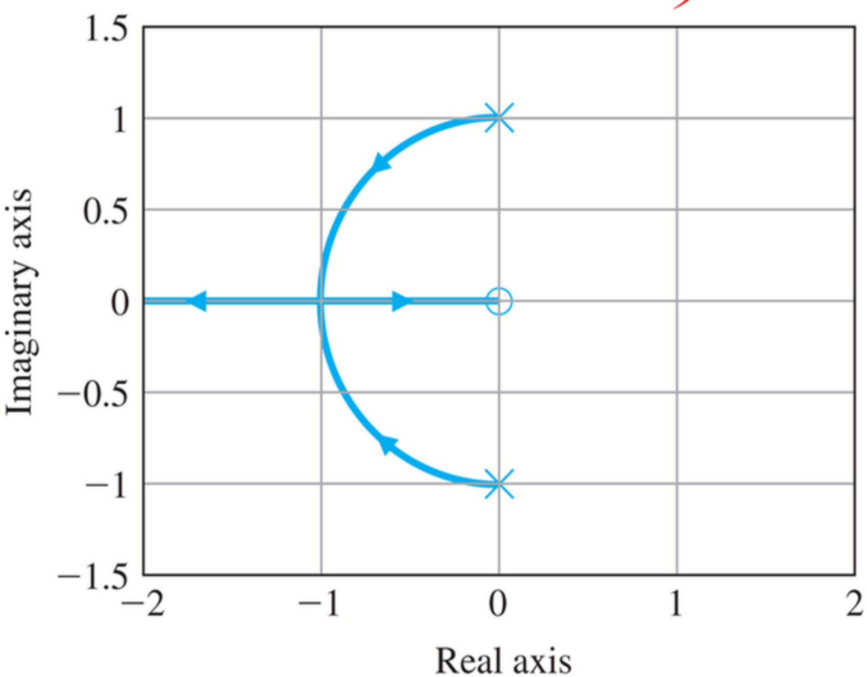
$c = 0 \Rightarrow s^2 + \cancel{cs} + 1 = 0 \qquad s = \pm j$

$c = 2 \Rightarrow s^2 + 2s + 1 = 0 \qquad s = -1$

$c = \infty \Rightarrow$ i) $s = \infty$

ii) $L(s) = -1/\cancel{k}\ \infty = 0 = \dfrac{b(s)}{a(s)}$

$\Rightarrow b(s) = 0 \qquad s = 0$



Ex FIND THE EQNS EQ FOR THE CASES BELOW.

(iii) What is the root locus for the system with the CE $1+KG(s)=1+\dfrac{K}{s[(s+4)^2+16]}$ ?

Use the same approach as for the two examples above. But, it is not as simple to sketch in this case! For such complex system, Evans came up with guidelines to sketch the root locus, using a 7-step procedure, as shown in the text. [Note: Matlab can sketch the root locus for any G(s) easily using the rlocus command.]

RECALL

A **complex number** may be defined as

$$A = \sigma + j\omega,$$

where $\sigma$ is the real part and $\omega$ is the imaginary part, denoted, respectively, as

$$\sigma = \mathrm{Re}(A), \quad \omega = \mathrm{Im}(A).$$

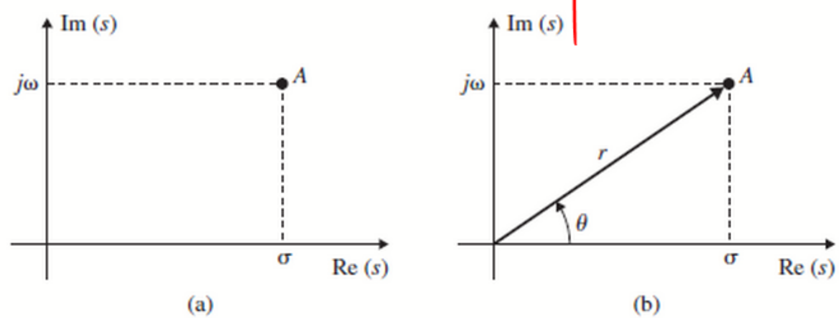
$$A = |A| \cdot \angle \arg A = r \cdot \angle\theta = re^{j\theta}, \quad 0 \le \theta \le 2\pi,$$

$$r = |A| = \sqrt{\sigma^2 + \omega^2},$$

$$\tan\theta = \frac{\omega}{\sigma}$$

$$\theta = \arg(A) = \tan^{-1}\left(\frac{\omega}{\sigma}\right)$$

**Figure WA.1**
The complex number $A$ represented in
(a) Cartesian and
(b) polar coordinates



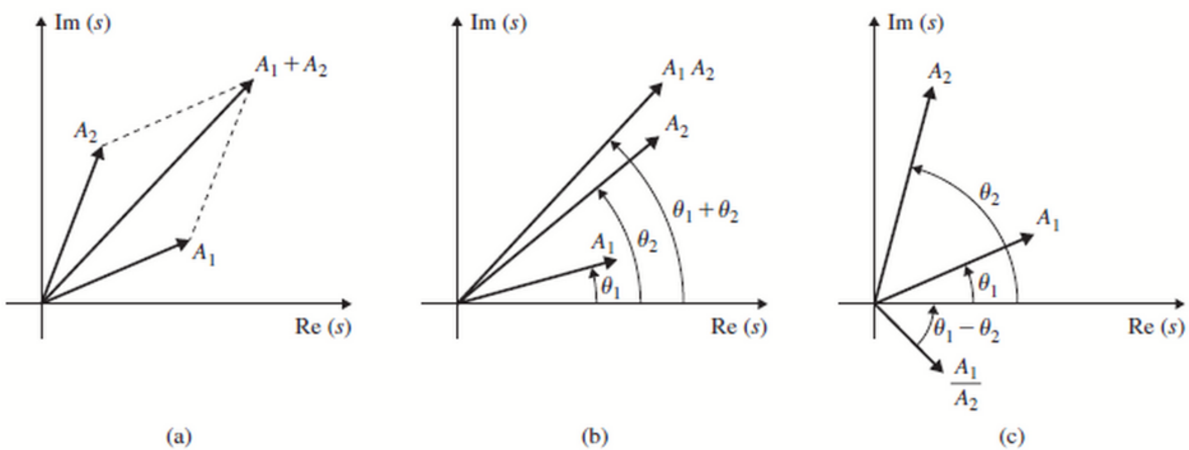




**Figure WA.2**
Arithmetic of complex numbers: (a) addition; (b) multiplication; (c) division

Consider the transfer function

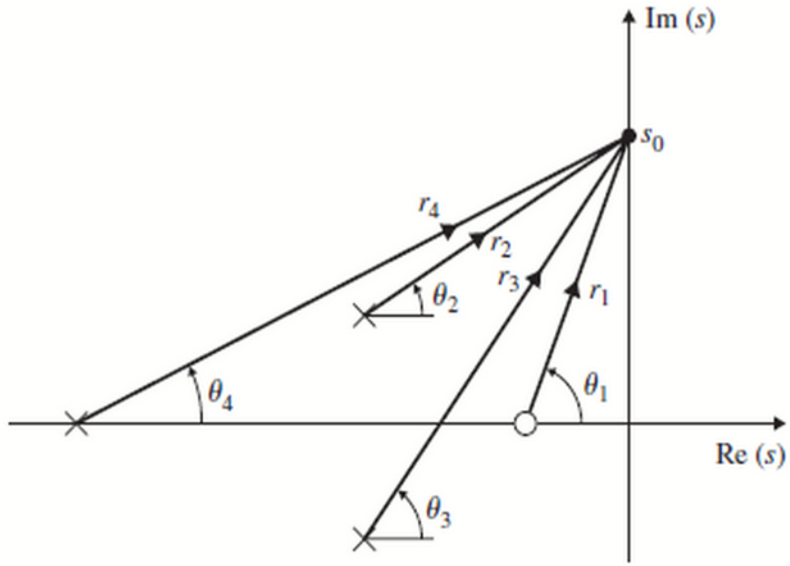
$$G(s) = \frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{i=1}^{n}(s+p_i)}.$$

if $m=1$

$n=3$

$s_0 = j\omega_0$

$$|G(j\omega_0)| = \frac{r_1}{r_2 r_3 r_4}.$$




$\angle G(s_0) =$

$\sum_{i=1}^{m} \angle(s_0+z_i) -$

$\sum_{i=1}^{n} \angle(s_0+p_i)$