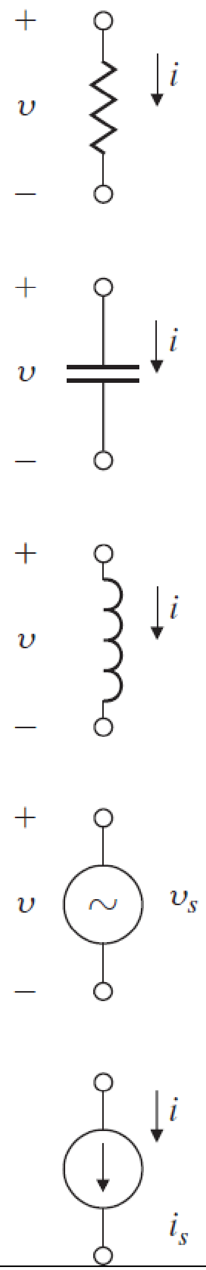
LECTURE 3

Basic Equations

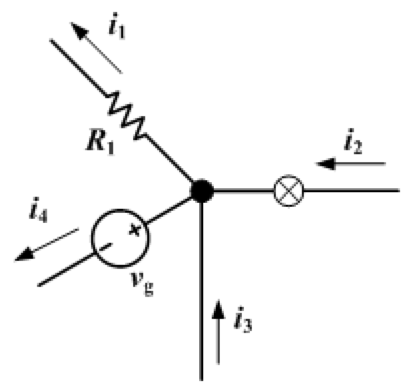


	Symbol	Equation
Resistor		$v = Ri$
Capacitor		$i = C\frac{dv}{dt}$
Inductor		$v = L\frac{di}{dt}$
Voltage source		$v = v_s$
Current source		$i = i_s$

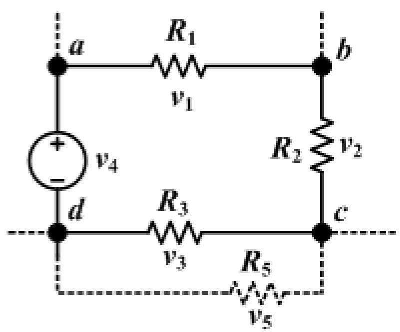
Elemental equations

Kirchoff's Laws

1. The current entering any junction is equal to the current leaving that junction. $i_1 + i_4 = i_2 + i_3$



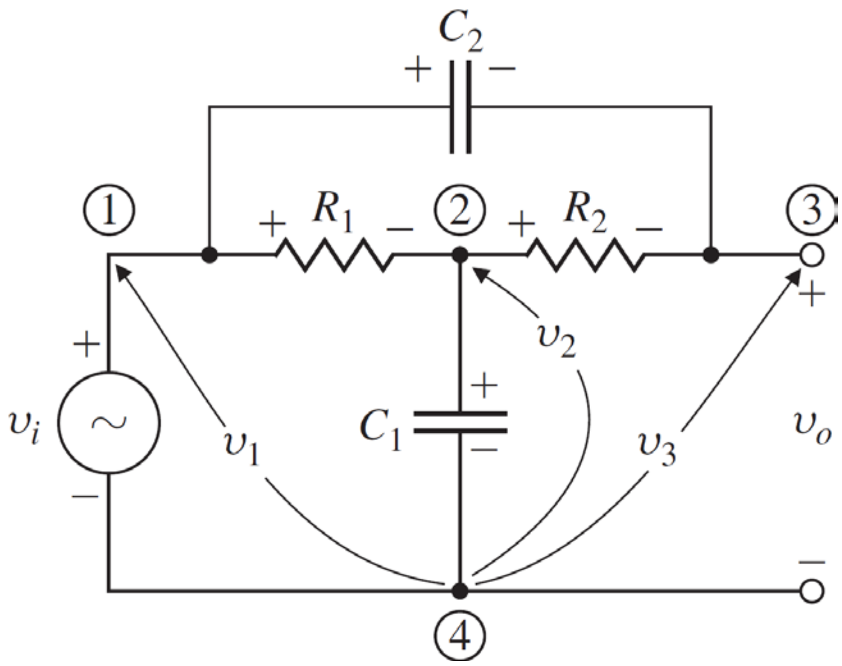
2. The sum of all the voltages around the loop is equal to zero. $v_1 + v_2 + v_3 + v_4 = 0$



(ADAPTED FROM SATISH NAIR)

Models of electrical circuits

Bridged Tee Circuit



Write the equations here

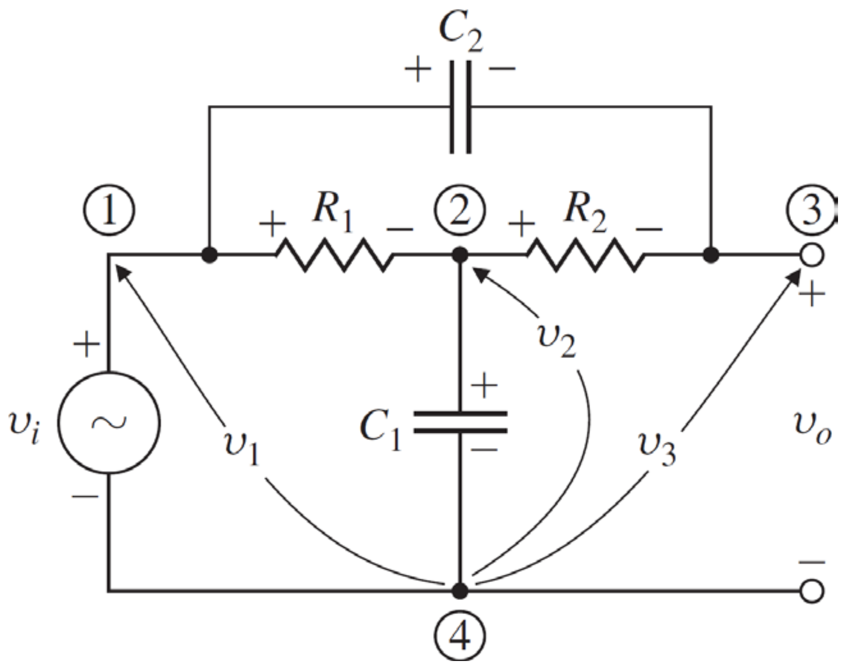
for v_c & i_L as the unknown.

In this case,

v_2 & $(v_1 - v_3)$

Models of electrical circuits

Bridged Tee Circuit



Write the equations here

KEY first step – decide on the reference node

Take v_4 = reference node = 0

Models of electrical circuits

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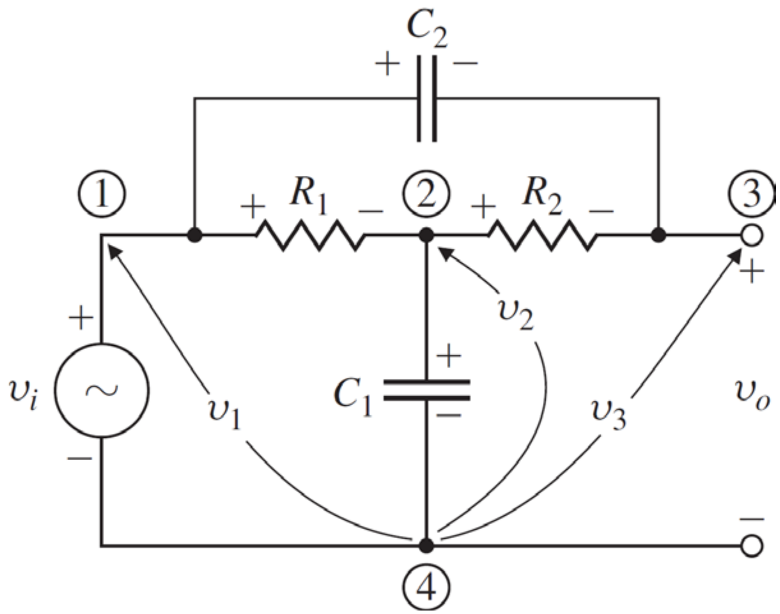
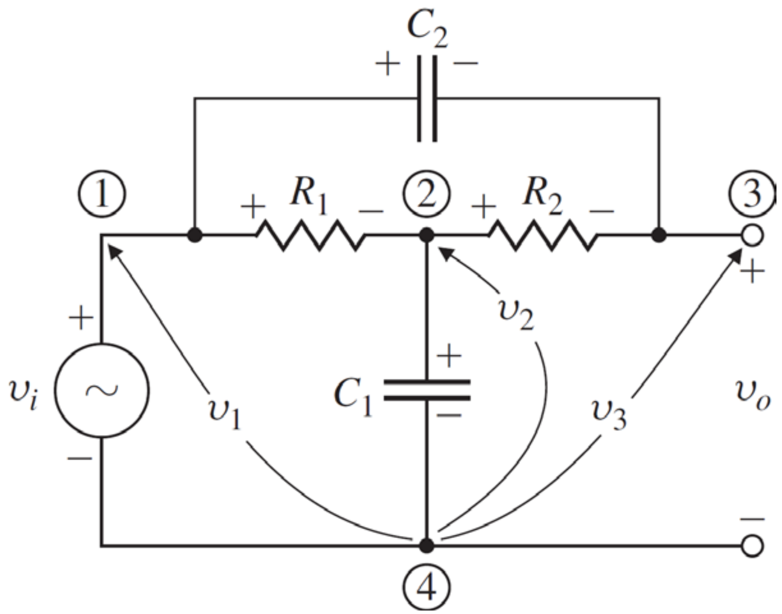


$KCL\ at\ node\ 2\ gives$

$$\frac{v_1 - v_2}{R_1} = \frac{v_2 - v_3}{R_2} + C\frac{dv_2}{dt} \qquad \underline{Eq.1}$$

Models of electrical circuits

Bridged Tee Circuit



KEY first step – decide on the reference node

Take v_4 = reference node = 0

KCL at node 2 gives

$$\frac{v_1 - v_2}{R_1} = \frac{v_2 - v_3}{R_2} + C\frac{dv_2}{dt} \qquad \underline{Eq.1}$$

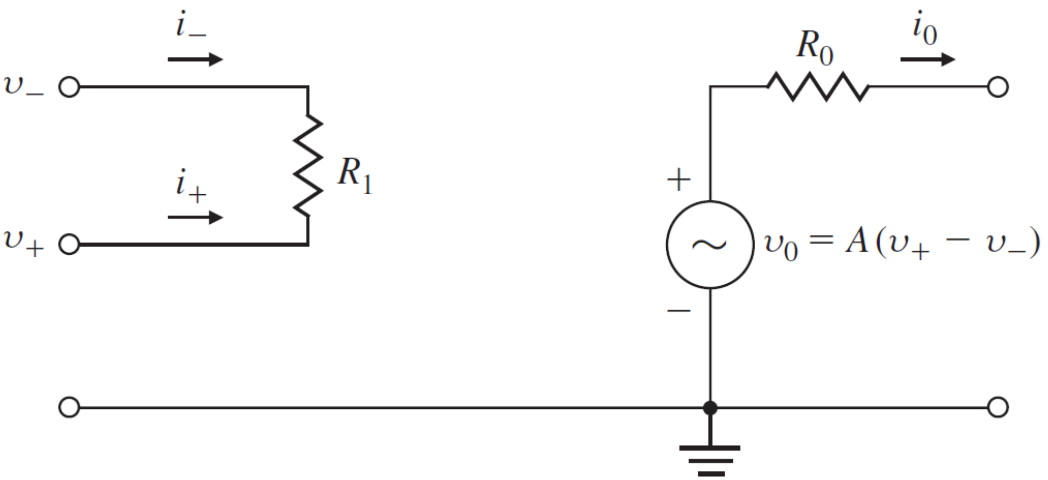
KCL at node 3 gives

$$\frac{v_2 - v_3}{R_2} + C_2\frac{d(v_1 - v_3)}{dt} = 0 \qquad \underline{Eq.2}$$

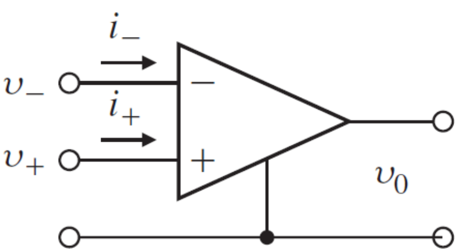
MATLAB or Pspice or some such software can now solve this set of two differential equations (model) easily

Models of electrical circuits – contd

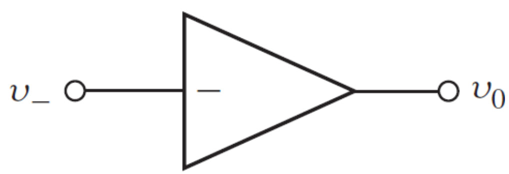
Idea Op-Amp: Input resistance is infinite; Output resistance is zero; and Gain is infinite



(a)



(b)



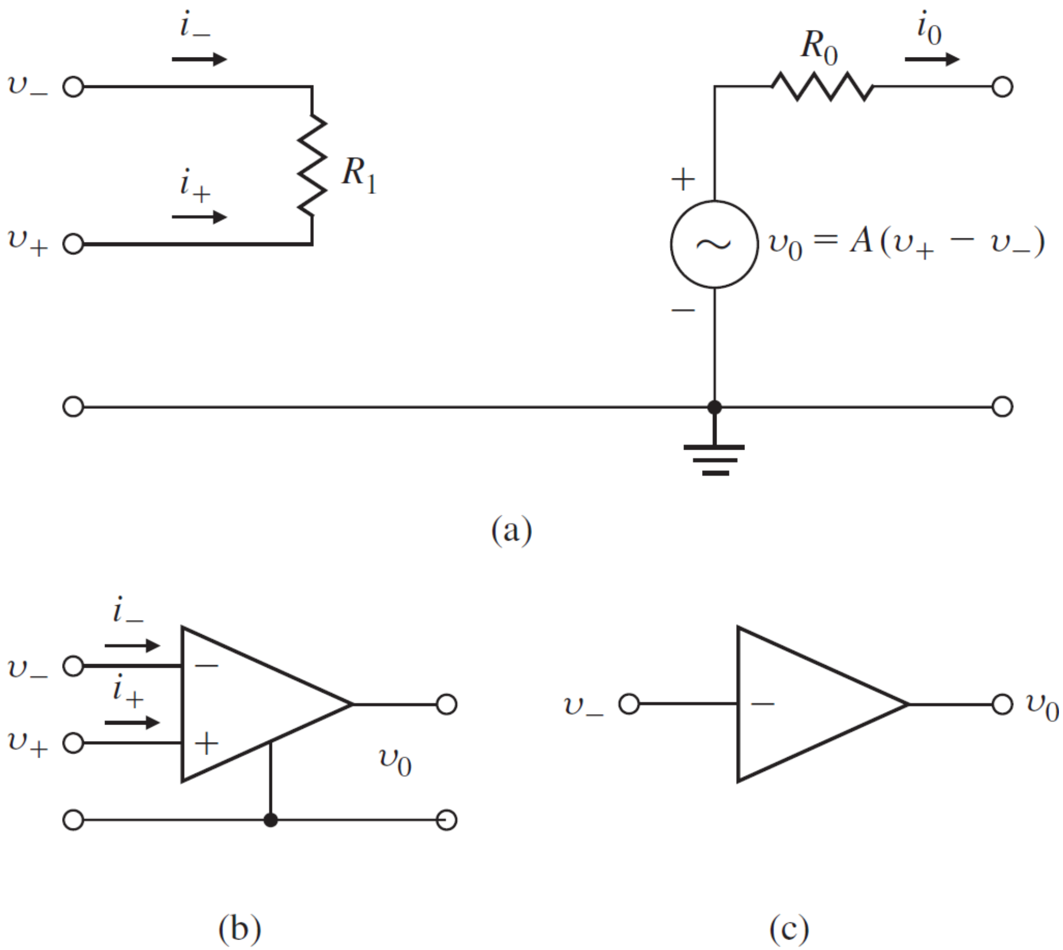
(c)

a. Op-amp simplified circuit
b. Op-amp schematic symbol
c. Reduced symbol for $v_+=0$

Write the basic op-amp equations

Models of electrical circuits – contd

Idea Op-Amp: Input resistance is infinite; Output resistance is zero; and Gain is infinite



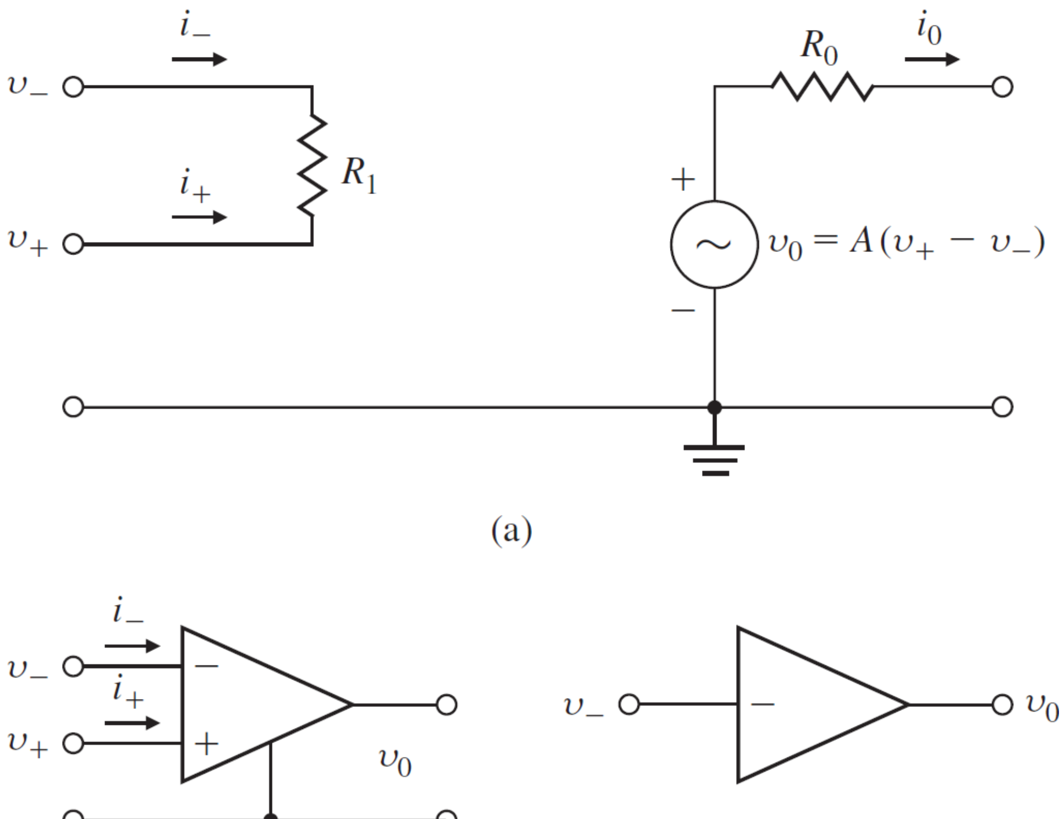
a. Op-amp simplified circuit
b. Op-amp schematic symbol
c. Reduced symbol for v_+=0

Infinite input resistance implies

$i_+ = i_- = 0$ (Eqn. 1)

Models of electrical circuits – contd

Idea Op-Amp: Input resistance is infinite; Output resistance is zero; and Gain is infinite



(a)

(b) (c)

a. Op-amp simplified circuit
b. Op-amp schematic symbol
c. Reduced symbol for v_+=0

Infinite input resistance implies

$i_+ = i_- = 0$ (Eqn. 1)

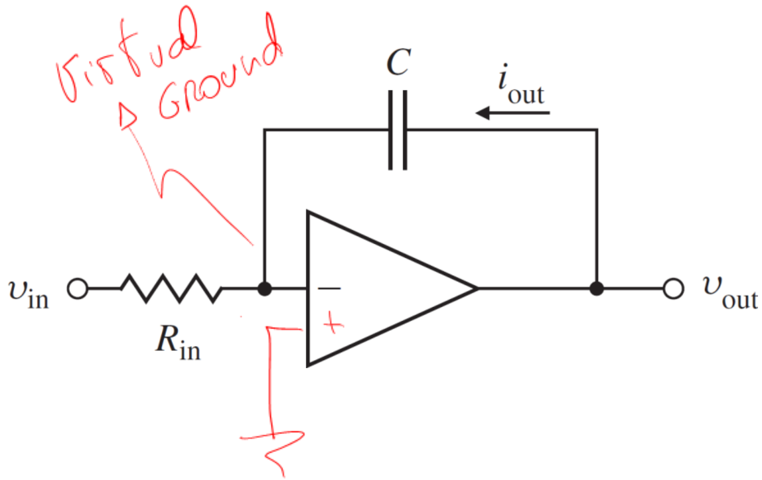
Gain of such op-amps are in the range of $10^6 – 10^9$. Note that $A*(v_+ - v_-) = v_o$, and since v_o is in the range of 5 V, this implies that **$v_+ = v_-$ (Eqn. 2)**

virtual short

The two equations in red are used to solve op-amp circuits, together with KCL and KVL eqns.

Models of electrical circuits – contd

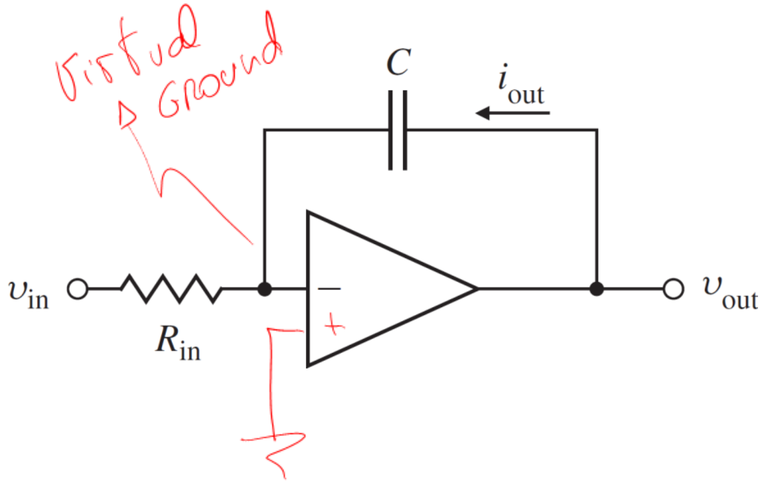
Idea Op-Amp: Input resistance is infinite; Output resistance is zero; and Gain is infinite



Write the equations and find the transfer function for this "integrator" circuit

Models of electrical circuits – contd

Idea Op-Amp: Input resistance is infinite; Output resistance is zero; and Gain is infinite



Write the equations and find the transfer function for this "integrator" circuit

KCL gives

$$i_{in} + i_{out} = 0 \quad \underline{Eq.1}$$

Substituting values

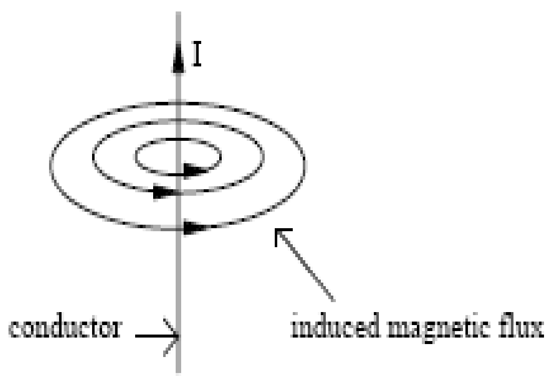
$$\frac{v_{in}}{R_{in}} + C\frac{dv_{out}}{dt} = 0, \textit{which then gives}$$

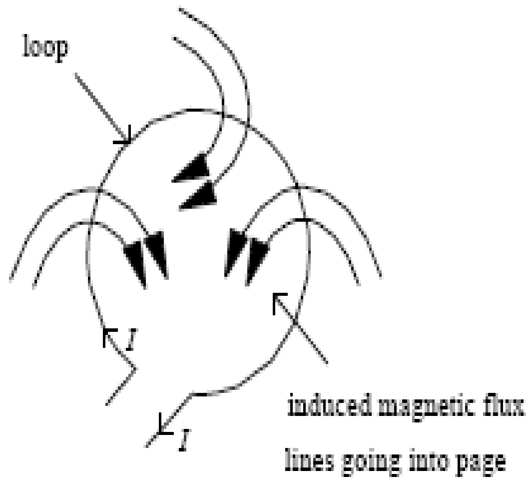
$$v_{out} = -\frac{1}{R_{in}C}\int_0^t v_{in}(\tau)\mathrm{d}\tau + v_{out}(0)$$

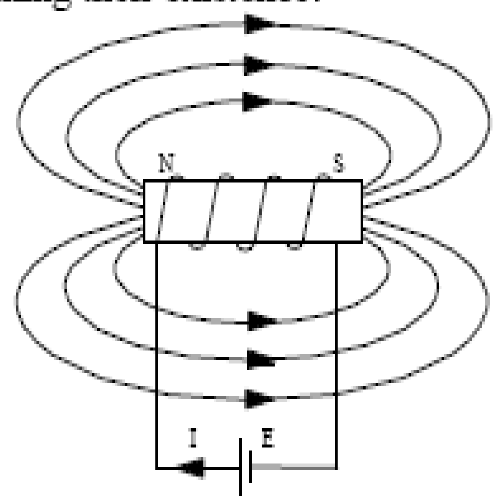
INTEGRATOR!

2.7 ELECTRICITY AND MAGNETISM

- Electromagnetic field: Electricity and magnetism can be considered as one physical phenomenon, the *electromagnetic field*, since they are very closely related to one another. A conductor carrying an electric current I exhibits a magnetic field around it…represented by invisible lines of force called magnetic flux (Φ) – a compass needle placed in their vicinity will show a deflection confirming their existence!

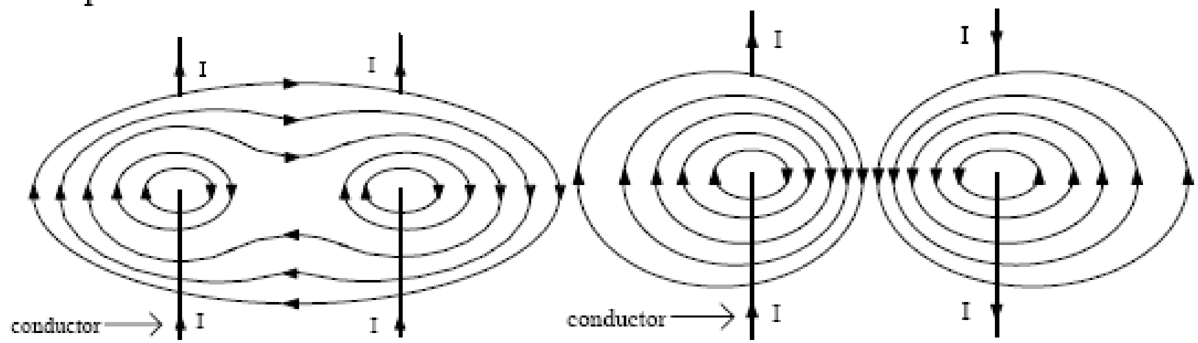




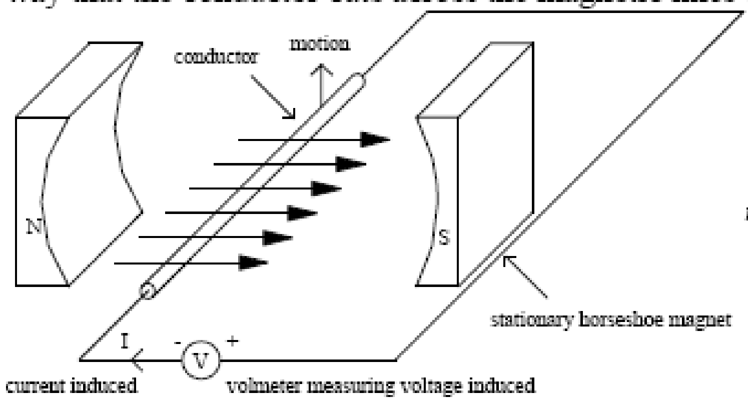


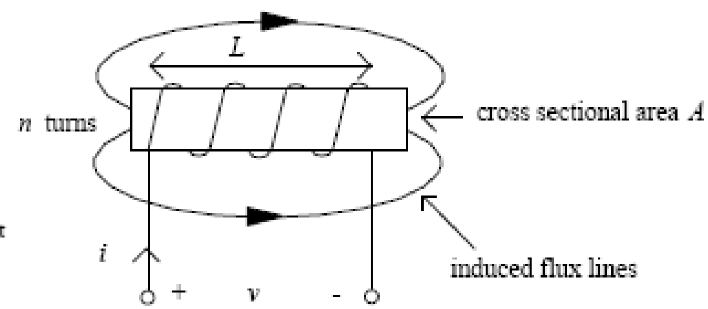
- The current induced magnetic flux lines are in the form of concentric circles as shown above, and if the conductor is bent in the shape of a loop, the induced magnetic flux lines will pass through the inside of the loop in the same direction, into the plane of the loop, concentrating and thus strengthening it. Further concentration of the magnetic field is achieved if the conductor is formed into a coil (solenoid) of n turns or loops. It can be strengthened further by adding a magnet in the center!

- When to parallel conductors carry current in the same direction, the magnetic lines of force between them cancel out in the middle, and the lines of force on the outside will encircle both the conductors – this results in a force of attraction between the conductors. Using the same logic, if the currents are in the opposite direction, the two conductors repel each other since the fields crowd each other in the middle.



Faraday's law: A conductor moving through a magnetic field generates an emf that is directly proportional to the rate at which the flux lines are cut, $v(t) = d\Phi/dt$, where $B \equiv$ magnetic field. This means that when the magnetic lines of force (flux) linking a conductor are changed by moving either the conductor or the magnetic field itself in such a way that the conductor cuts across the magnetic lines of force, voltage will be induced across the conductor.





Lenz's law: The direction of current induced by moving a conductor through a magnetic field is such that it produces opposition to the motion that produced it. This means that in order to move the conductor upwards in the circuit shown above, force must be used to overcome the opposing downward force produced by the induced current.

Inductance (Self Inductance): The inductance L is the property of the electric circuit element that exhibits opposition to the change in current flowing in it. (due to Faraday's law !)

Development of Electromotive Force

- Faraday's Law
- Application of Faraday's Law
- A Single Coil DC Motor
- Motor Constants

Faraday's Law

In the early 1830's, Michael Faraday and Joseph Henry independently discovered the relationship between changing magnetic fields and induced EMF in circuits. If B is the flux density of a constant magnetic field and a conductor is moved through this field at a velocity V, an EMF E is generated in the conductor such that:

$$E = B \times V$$

If the conductor is part of a complete electrical circuit with a resistance R, then the EMF will produce a current in the conductor such that:

generator (motion ⇒ voltage)

$$I = E / R = B \times V / R$$

The development of an EMF in a conductor moving in a magnetic field is the principle on which many types of tachometers are based. By using the commutation techniques described in the next section, a rotary device can be constructed which has, as its input, a rotary mechanical motion and, as its output, a voltage proportional to that input rotational velocity.

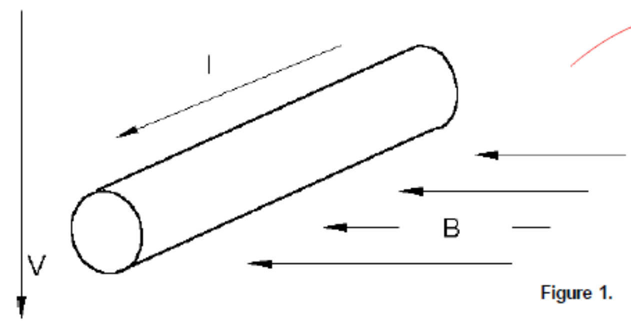


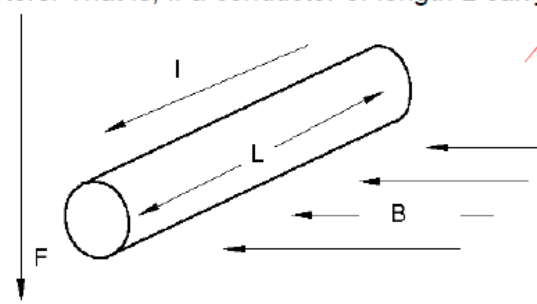
Figure 1.

Another specific application of Faraday's Law is used in electric motors. That is, if a conductor of length L carrying a current I is placed in a magnetic field B, a magnetic force F is created such that:

motor (current ⇒ motion)

$$F = BLI \sin A$$

where A is the included angle between B and I.
The force vector F is a vector perpendicular to both vector quantities B and I.



2.7 THE ELECTROMECHANICAL WORLD

- Relationship between current and magnetism: Magnetic field is proportional to current strength i and the number of turns of wire around the toroid N.

  $B = \frac{\mu N i}{2\pi R}$ where $B \equiv$ magnetic field (T), $\mu \equiv$ material property,

  $N \equiv$ number of turns of wire around toroid, $R \equiv$ radius of toroid

- Law of motors: A current carrying conductor in a magnetic field B experiences a force F given by $\vec{F} = i\vec{l} \times \vec{B}$. If the moving charge is composed of a current of i amperes in a conductor of length l meters arranged at right angles to the field strength of B tesla (as in a DC motor), then the force can be found by the following equation.

  $F = Bli$ Newtons,
  where $B \equiv$ magnetic field (T), $i \equiv$ current (A), $l \equiv$ length of the conductor (m)

- Law of generators: Faraday's law describes how a changing magnetic flux through a loop, Φ_B, causes an emf e, to be induced around the loop. An electromotive force (voltage) is generated in a conductor of length l meters. If the conductor is moved at a velocity v meters per second through a constant field of B teslas at right angles to the direction of the field, then the voltage between the ends of the conductor is given by $\vec{e} = B\vec{l} \times \vec{v}$.

  $$e(t) = Blv \quad (also = -\frac{d\Phi_B}{dt}) \text{ Volts,}$$

  where $B \equiv$ magnetic field (T), $l \equiv$ length of the conductor (m), $v \equiv$ velocity of moving conductor (m/sec)