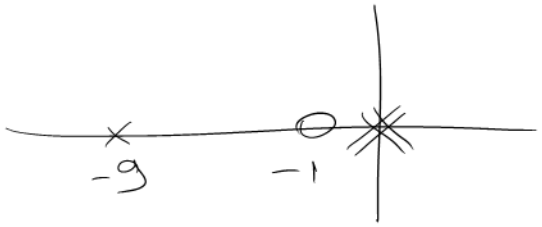
LECTURE 32

Ex 5 $e_c = 1 + K\dfrac{s+1}{(s+9)s^2}$

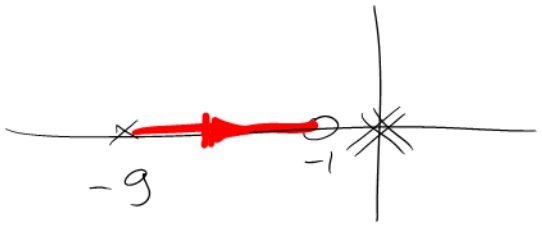
$L(s) = \dfrac{s+1}{s^2(s+9)}$ — $a(s) \Rightarrow m = 1$ zero @ -1

— $b(s) \Rightarrow n = 3$ poles @ $0, -9$



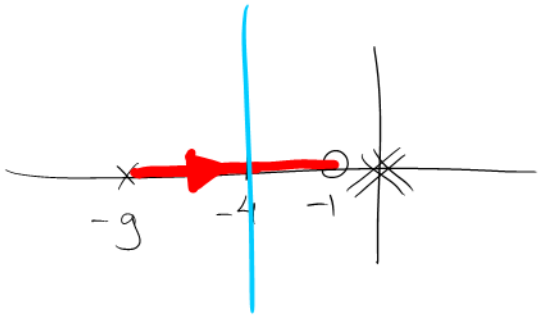
RULE 1

POLES ⟶ ZEROS



RULE 2

REAL AXIS ✓

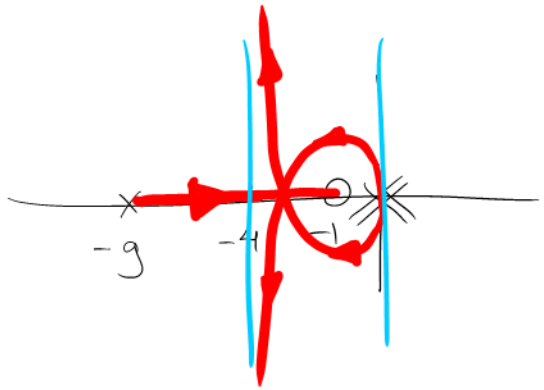


RULE 3

$\Phi_{Asym} = \dfrac{180 + 360(l-1)}{2} = \pm 90°$

$\alpha_{Asym} = \dfrac{-8-0-0+1}{2} = -4$

(ADAPTED FROM SATISH NAIR)



Rule 4

$$\phi_{DEP\ @0} = \frac{0+0-0-180-360(\ell-1)}{2}$$

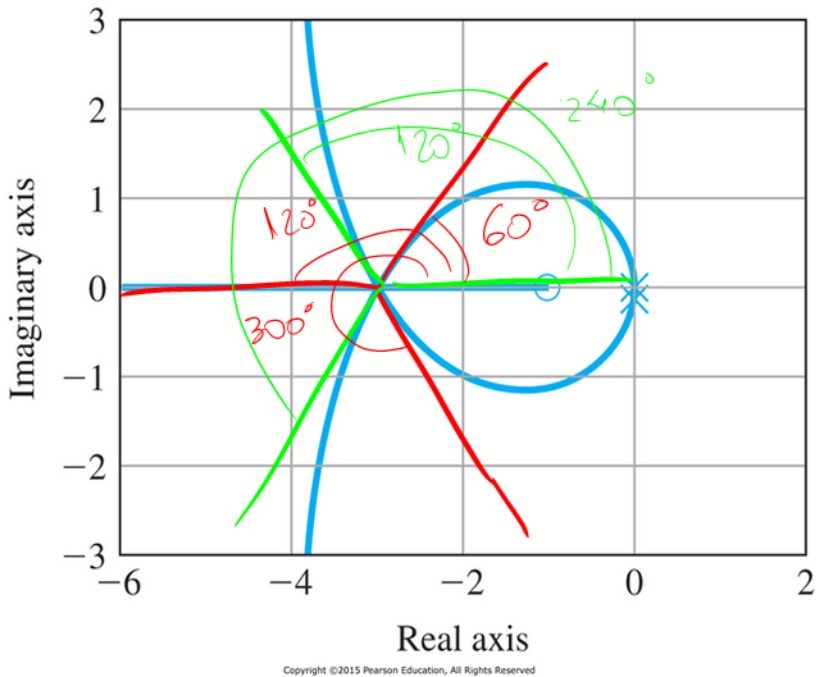
$$= \pm 90^\circ$$

$$\phi_{ARR\ @-1} = 180+180+0+180+360(\ell-1)$$

$$= 180^\circ$$

$$\phi_{DEP\ @-9} = \frac{180+180-180-180-360(\ell-1)}{1}$$

$$= 0^\circ$$



Rule 5

Arriving
60°/180°/300°

Departing
0°/120°/240°

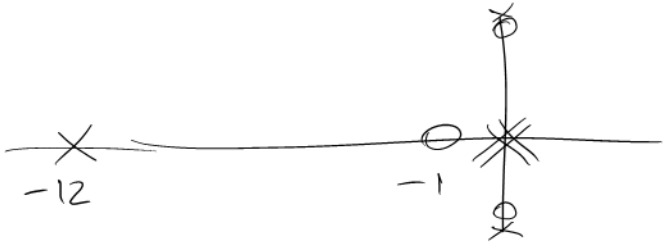
$$\frac{180+360(\ell-1)}{3}$$

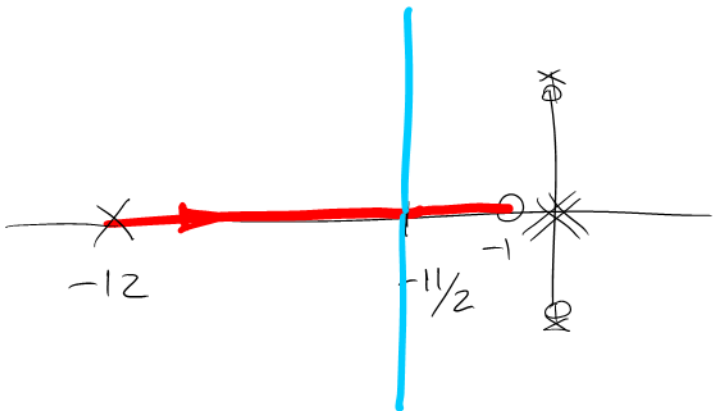
Ex 6

$$D(s) = K\frac{(s+1)}{(s+12)} \quad \& \quad G(s) = \frac{(s+0.1)^2 + 6^2}{[(s+0.1)^2 + 6.6^2]}$$

$$L(s) = \frac{a(s)}{b(s)} \Rightarrow m = 3 \text{ zeros } @ -1, -0.1 \pm 6j$$

$$b(s) \Rightarrow n = 5 \text{ poles } @ 0, -12, -0.1 \pm 6.6j$$





RULE 1

POLES → ZEROS

RULE 2

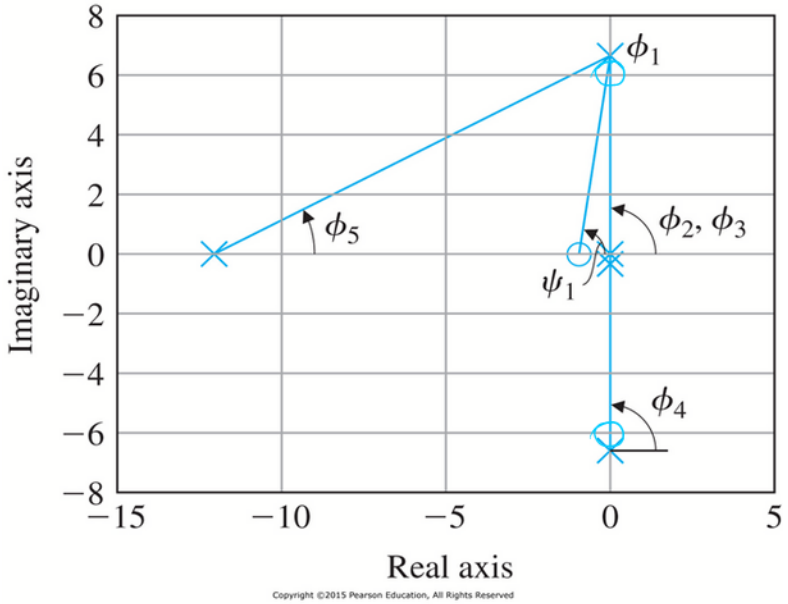
REAL AXIS ✓

RULE 3

$$\phi_{Asym} = \frac{180 + 360(l-1)}{2} = \pm 90^\circ$$

$$\alpha_{Asym} = \frac{-12 - 0.1 - 0.1 + 0.1 + 0.1 + 1}{5 - 3}$$

$$= -11/2$$



RULE 4

$$\phi_{DEP}\Big|_{@0} = \frac{-90 - 0 + 90 - 0 - (0 - 90 + 90) - 180 - 360(k-1)}{2}$$

$$= \pm 90^\circ$$

Similarly, $\phi_{DEP}\Big|_{@-12} = 180^\circ$

$$\phi_{DEP}\Big|_{@-0.1+6.6j} = 90^\circ + 90^\circ + \arctan\left(\frac{6.6}{1}\right) - \left[90^\circ + 90^\circ + 90^\circ + \arctan\left(\frac{6.6}{12}\right)\right] - 180^\circ$$

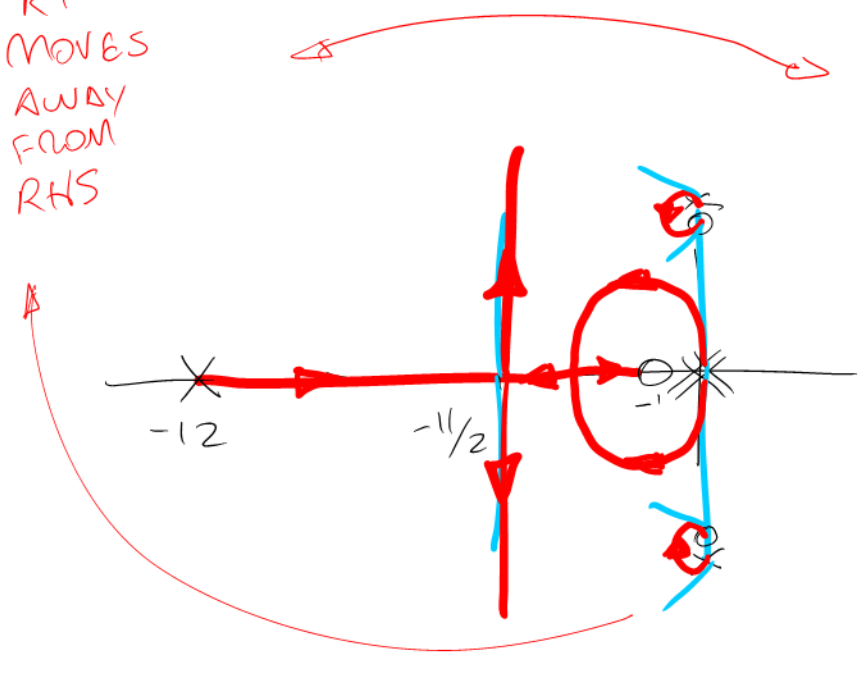
$$\phi_{DEP}\Big|_{@-0.1+6.6j} = 81.4^\circ - 90^\circ - 28.8^\circ - 180^\circ = 142.6^\circ$$

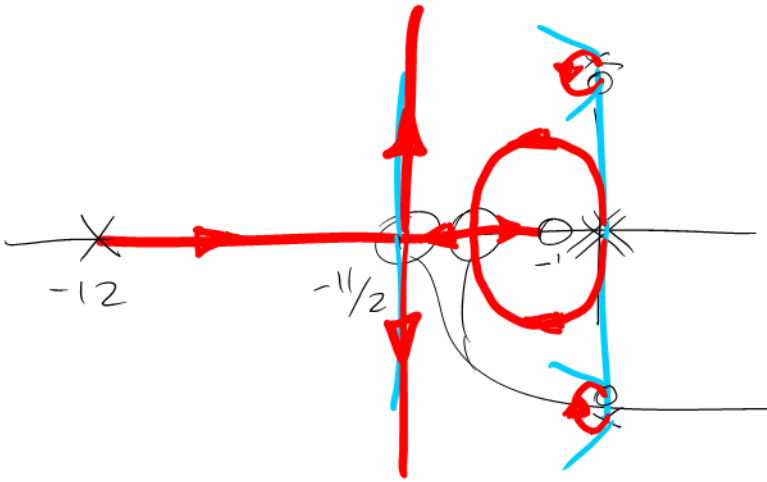
Similarly, $\phi_{DEP}\Big|_{@-0.1-6.6j} = -142.6^\circ$

$$\phi_{ARR}\Big|_{@-0.1+6j} = 90^\circ + 90^\circ + 90^\circ + \arctan(6/12) - [90^\circ + 90^\circ + \arctan(6/1)] + 180^\circ$$

$$\phi_{ARR}\Big|_{@-0.1+6j} = -143.9 \qquad \phi_{ARR}\Big|_{@-0.1-6j} = 143.9$$

RT MOVES AWAY FROM RHS

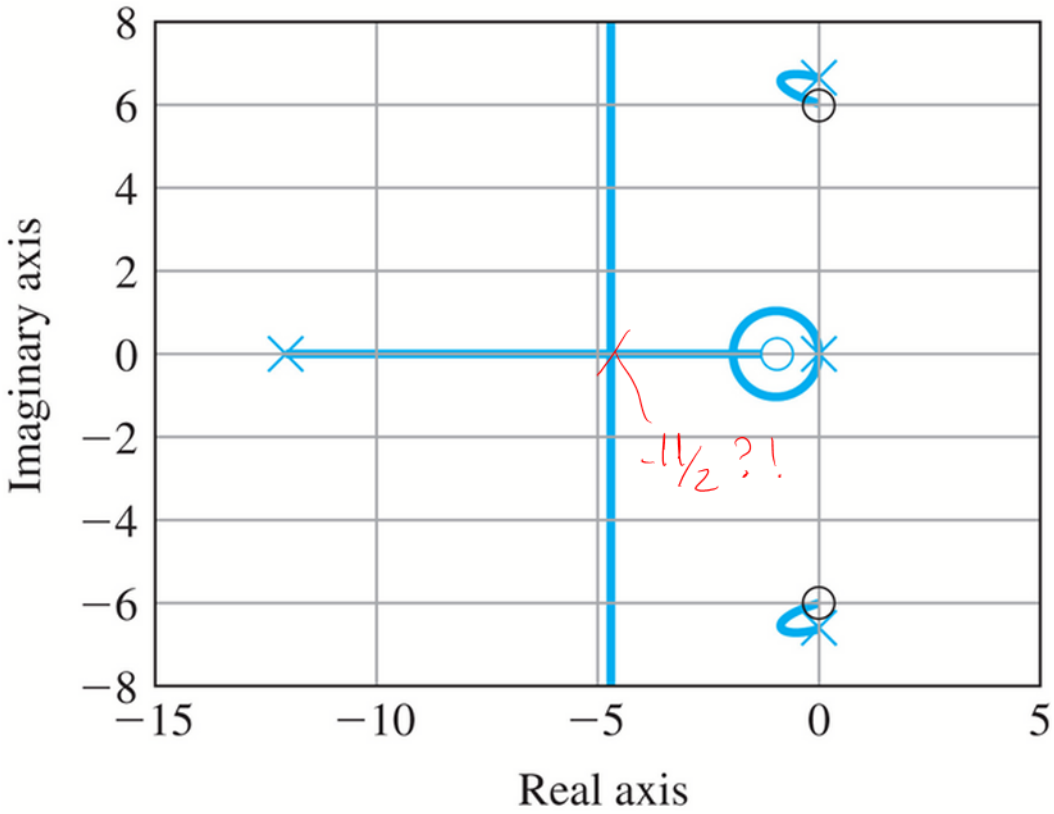




Rule 5

$$\frac{180^\circ + 360(\ell - 1)}{2} = \pm 90^\circ$$

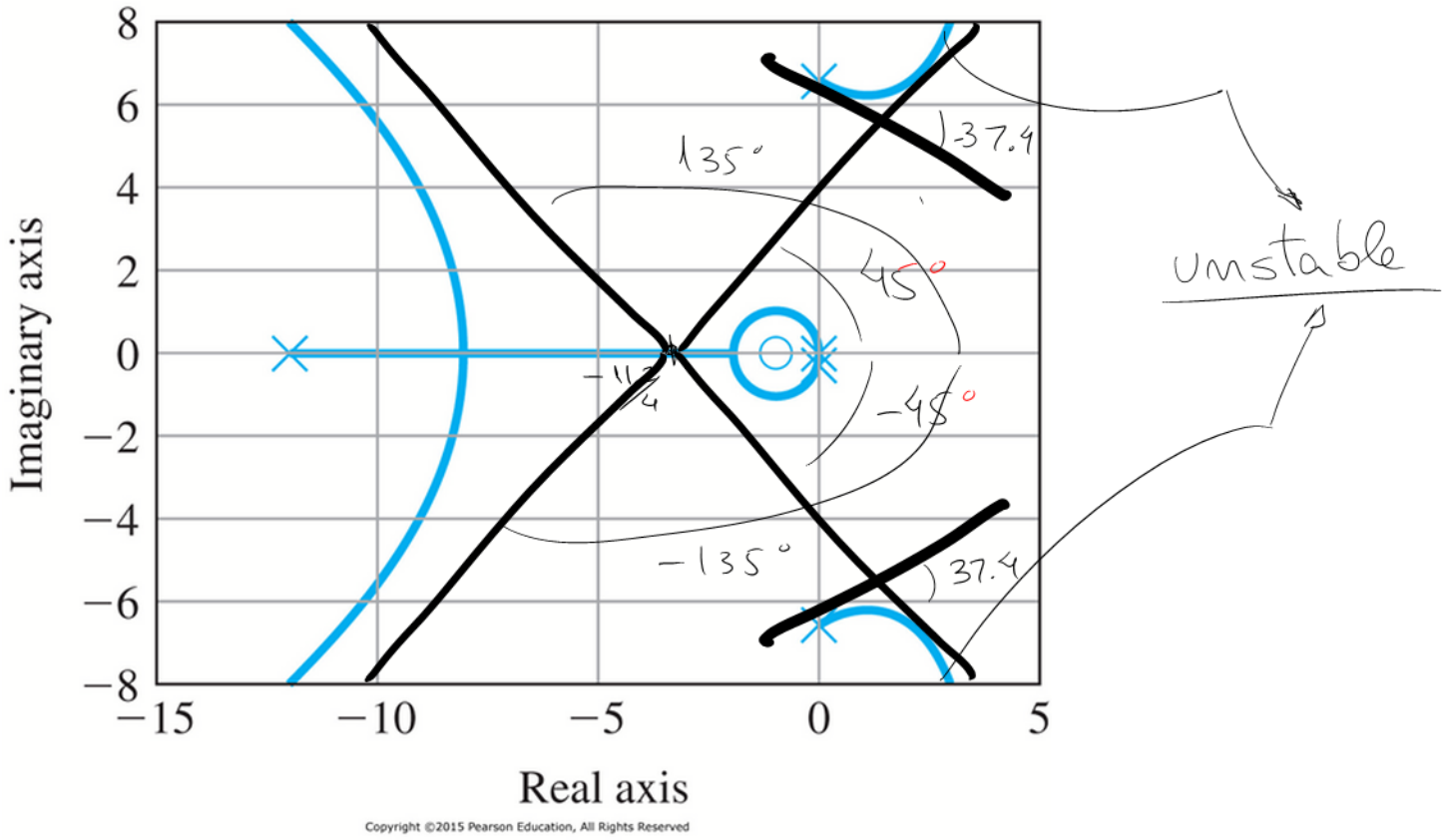
$q = 2$



Ex 7

$$D(s) = K\,\frac{s+1}{s+12} \quad \& \quad G(s) = \frac{1}{[(s+0.1)^2 + 6.6^2]}$$

$$L(s) = \frac{a(s)}{b(s)} \Rightarrow \begin{array}{l} m = 1 \text{ zero } @ -1 \\ n = 5 \text{ poles } @ 0, -12, -0.1 \pm 6.6j \end{array}$$



Ex $L(s) = \dfrac{1}{s(s+2)\left[(s+1)^2+4\right]} \Longrightarrow$ $m = 0$ no zeros

$n = 4$ poles @ $0, -2, -1 \pm 2j$

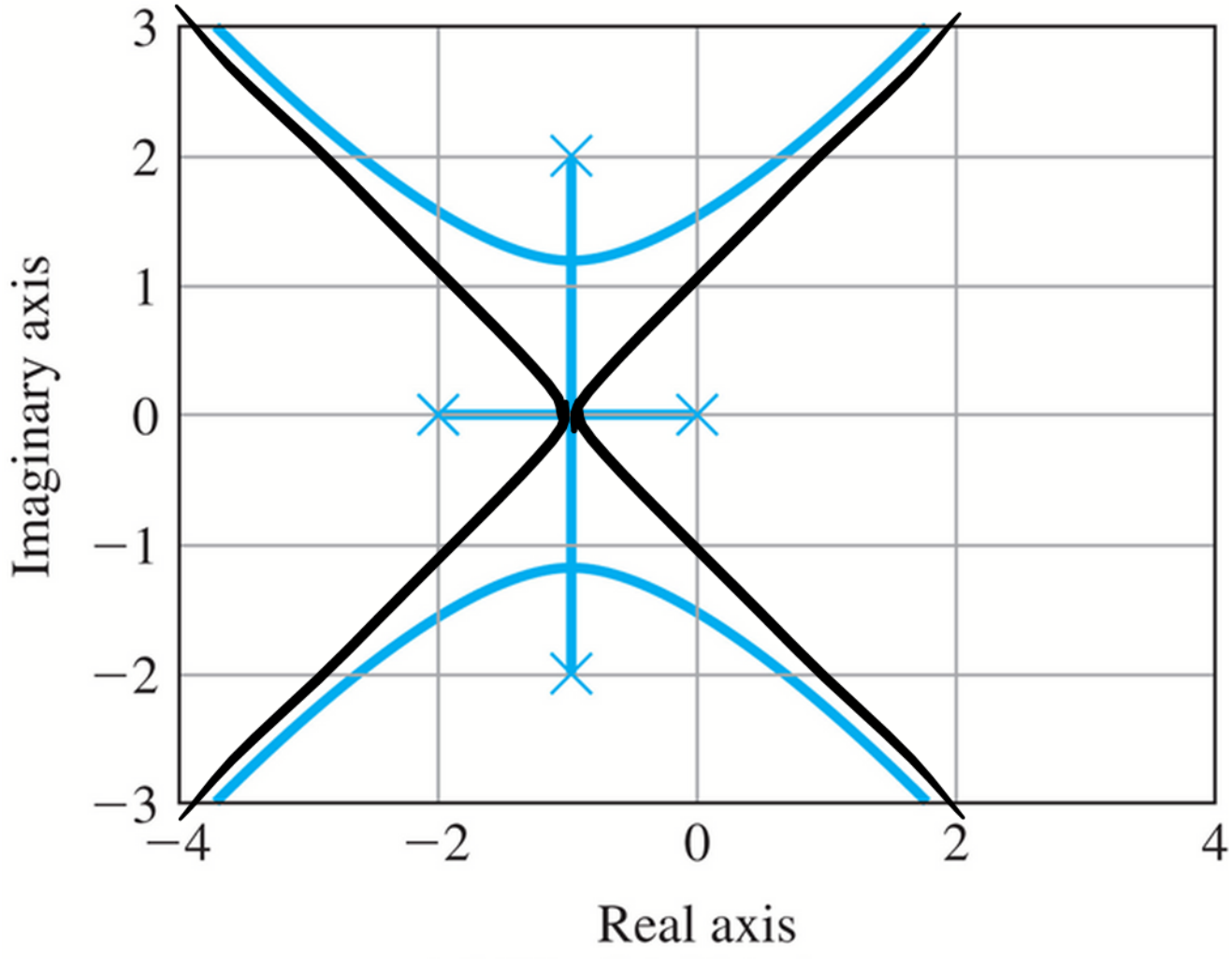
RULE 1: 4 BRANCHES, ALL APPROACHING ASYMPTOTES

RULE 2: REAL AXIS

RULE 3: $\alpha = -1$ $\phi = 45°, 135°, -45°, -135°$

RULE 4: ϕ_{DEP} $= \mp 90°$

$-1 \pm 2j$



Breakaway & Break-in Points

Those are the $s \in \mathbb{R}$ for which

$$\frac{dK}{ds} = 0, \quad \text{where} \quad K = \frac{-1}{L(s)} = -\frac{a(s)}{b(s)}$$

That is:

$$\frac{dK}{ds} = -\frac{(a'b - ab')}{b^2} = 0$$

$$\Rightarrow \boxed{a'b - ab' = 0}$$