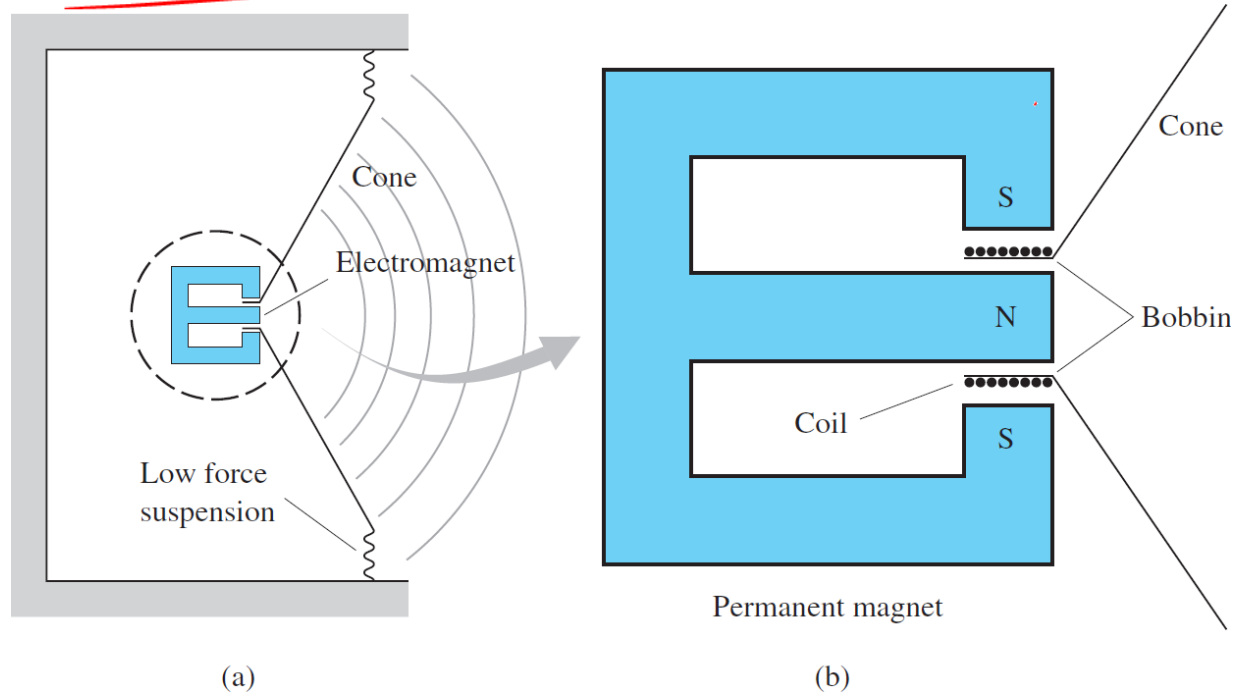


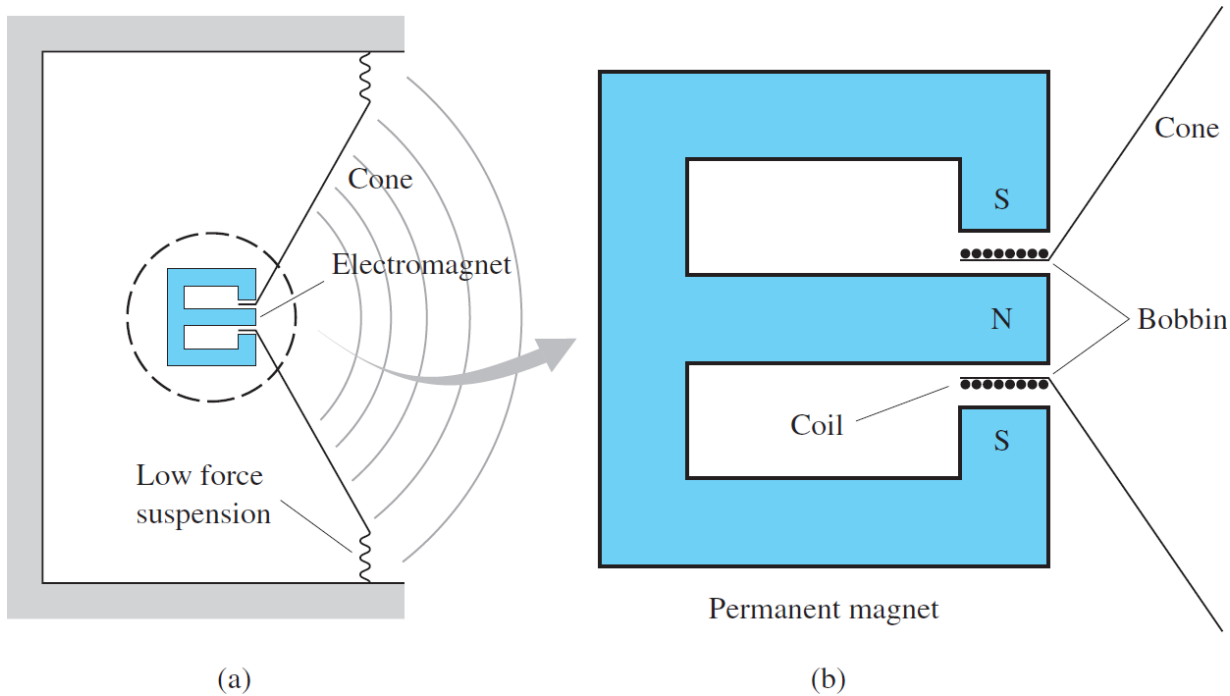
# LECTURE 4

## Modeling a loudspeaker



**Write the dynamic equations for the loudspeaker circuit below**

# Modeling a loudspeaker



**Write the dynamic equations for the loudspeaker circuit below**

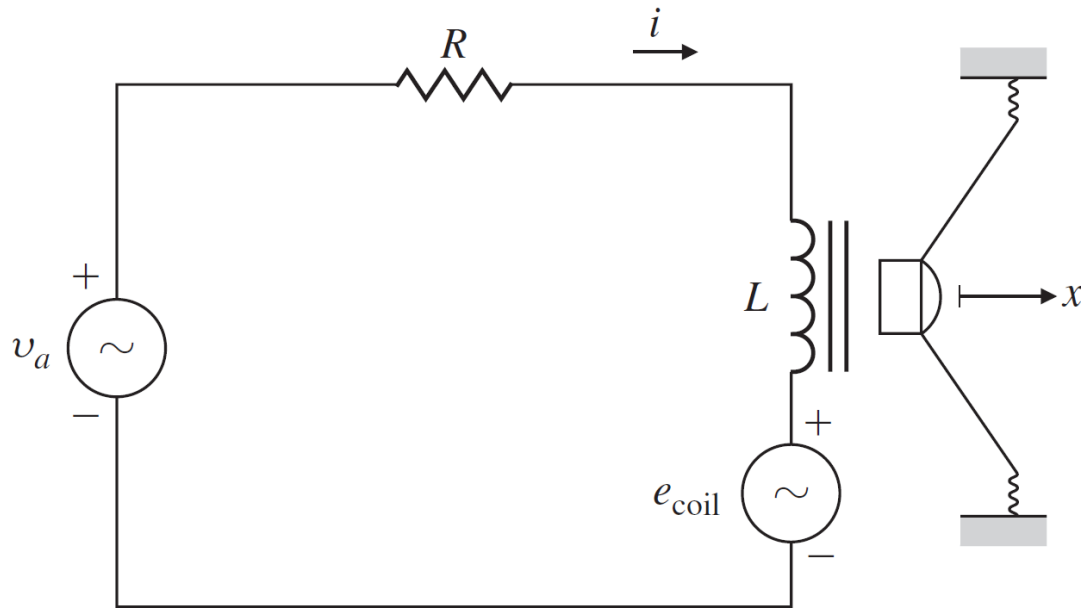
*Applying the law of motors*

$$M\ddot{x} + b\dot{x} = Bli$$

$$M\ddot{x} + b\dot{x} = 0.5 * ( 20 * 2 * \pi / 100 ) * i = 0.63i$$

$$\frac{X}{I}(s) = \frac{0.63 / M}{s( s + b / M )}$$

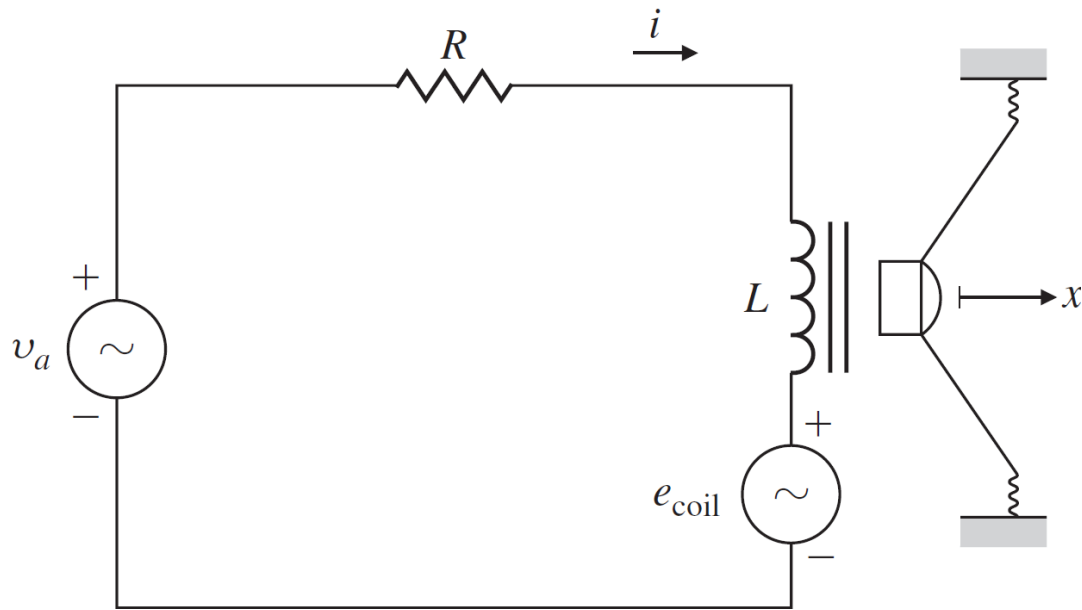
# Loudspeaker with circuit



Write the dynamic equations for the loudspeaker with circuit – **the circuit shown alongside generates the current “ $i$ ” now!**

**What is the equation for  $i$ ?**

# Loudspeaker with circuit



Write the dynamic equations for the loudspeaker with circuit – **the circuit shown alongside generates the current “i” now!**

**What is the equation for i?**

*KVL gives*

$$v_a - i * R - L \frac{di}{dt} - e_{coil} = 0$$

*Note that  $e_{coil} = Bl\dot{x}$  (law of ?)*

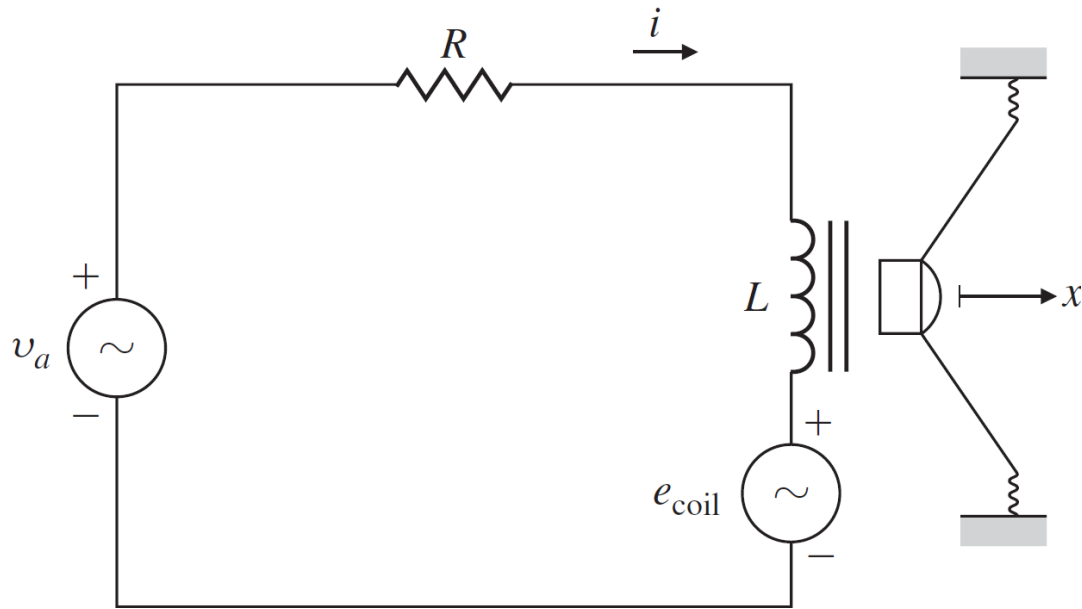
$$\text{So, } L \frac{di}{dt} + Ri - e_{coil} = v_a - 0.63\dot{x} \quad (\text{Eqn. 1})$$

$$\text{Recall, } M\ddot{x} + b\dot{x} = 0.63i \quad (\text{Eqn. 2})$$

**Eqns. 1 and 2 form the dynamic model for the loudspeaker**

**The TF from this loudspeaker circuit is then**

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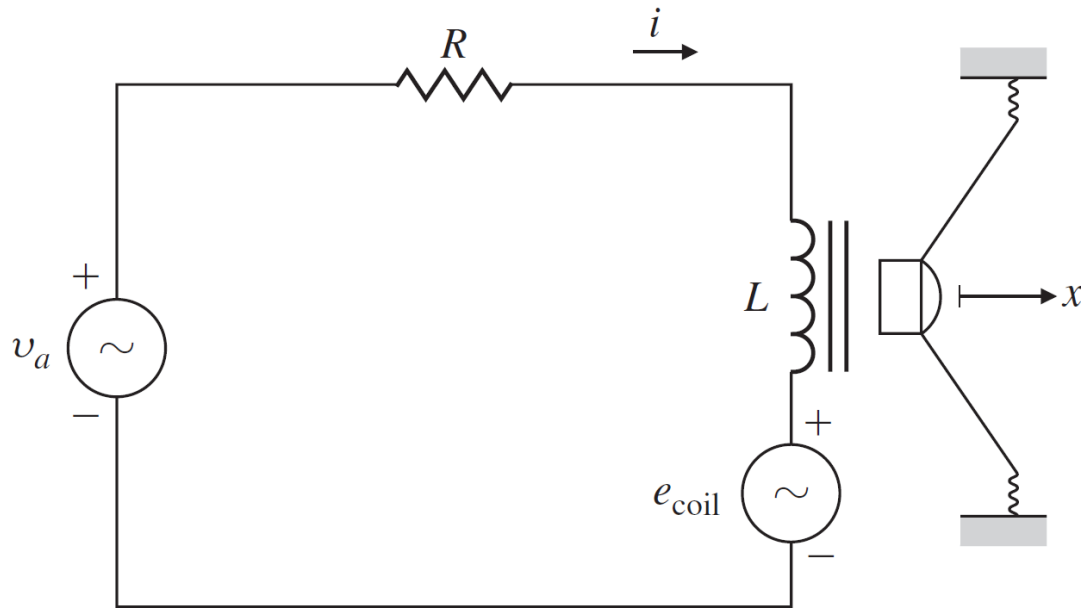
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The TF from this loudspeaker circuit is then

$$\frac{X}{I}(s) = \frac{0.63}{s[(Ms + b)(Ls + R) + (0.63)^2]}$$

**Eqns. 1 and 2 form the dynamic model for the loudspeaker**

## Application of Faraday's Law

In order to make something useful of this generated force, more is required than simply a single, current-carrying

conductor in a magnetic field. Figure 3 illustrates a coil carrying a current  $I$  in a static magnetic field of flux density  $B$ . Assume that the coil is mounted on a rotational axis  $O - O'$ . The interaction of the current  $I$  in coil segment  $AB$  with the magnetic field  $B$  will create a force  $F$ , shown downward in the drawing.

The interaction of the current  $I$  in coil segment  $CD$  in the same magnetic field  $B$  will also generate a force  $F$ . Since the direction of the current in  $CD$  is opposite to the direction of the current in  $AB$ , the resulting force is in the opposite direction (upward in Figure 3). Note that the current in coil segment  $AC$  is parallel to the magnetic field, and the net force in this coil segment must be zero.

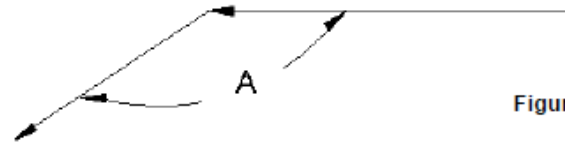
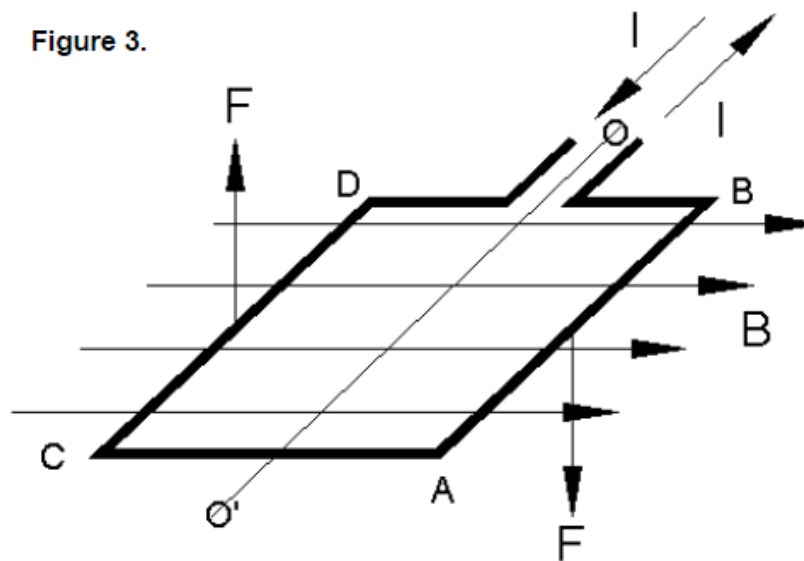


Figure 2.

Figure 3.



The coil in Figure 3 will rotate about axis  $O - O'$  ninety degrees and stop with the coil oriented vertically (segment  $AC$  perpendicular to the magnetic field). If the direction of the current flow is then reversed, the coil will rotate  $180^\circ$  and, again, come to a stop. Thus, the arrangement shown in Figure 3 represents a crude rotary solenoid.

One could, theoretically, continue to switch the direction of current at the appropriate angular position and generate continuous rotational motion. In order to make such a device practical, however, it is necessary to add a means by which current can be conducted to the coil and switched at the proper time to create continuous motion. These functions are performed (in brush-type motors) by the brushes and commutator of the motor.

## A Single Coil DC Motor

Figure 4 illustrates one method by which the commutation function might be accomplished. Rather than hard wiring the current source to the coil, the current is conducted through sliding contacts (brushes) connected to the current source. The brushes ride on the ends of the coil wires, thus conducting current through the coil. In this simplified motor, the brushes switch coil connections about once every 180° of rotation. Therefore, the direction of current flow remains fixed with respect to the magnetic field.

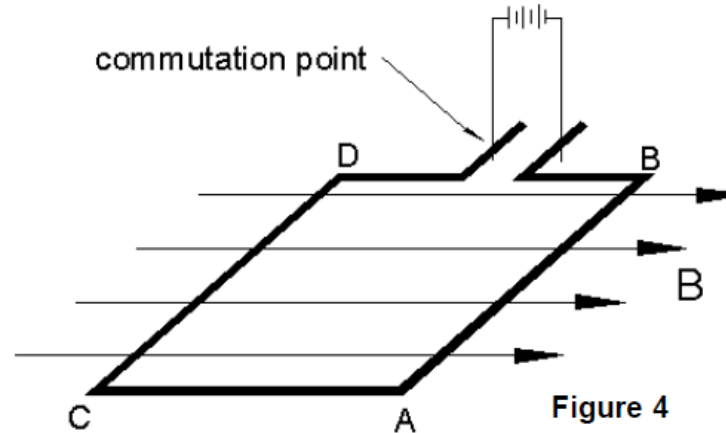
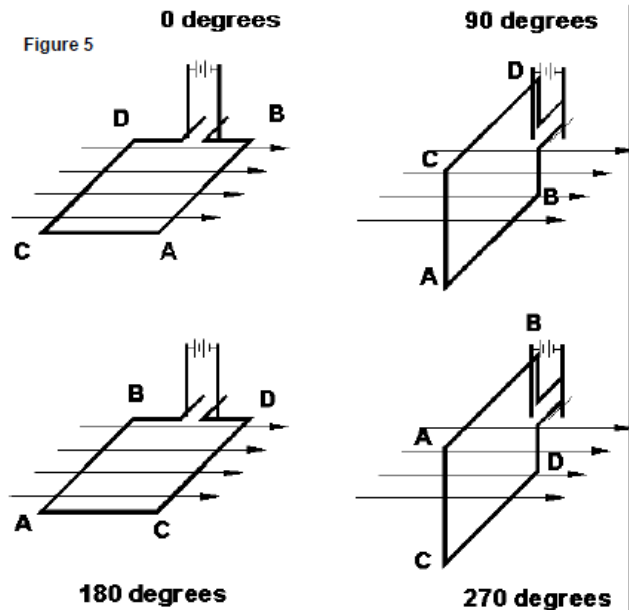


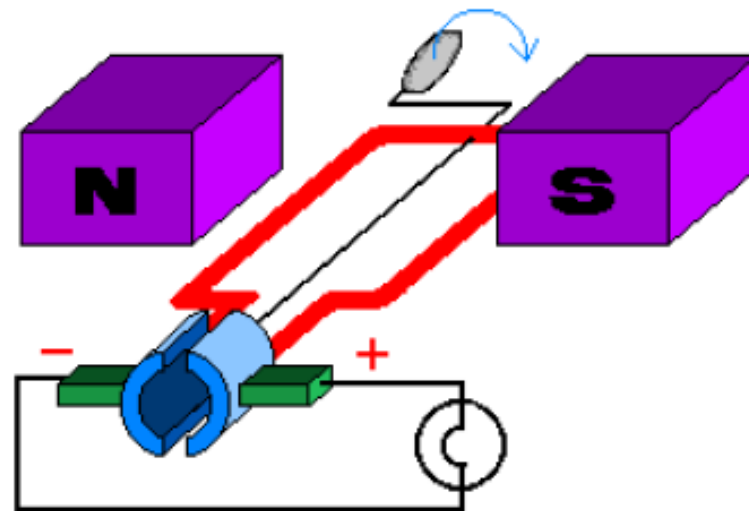
Figure 5 illustrates the coil of the single-coil motor at angular positions of 0 degrees, 90 degrees, 180 degrees, and 270 degrees. At 0 degrees, the ends of the coils are contacting the "brushes", and current flows through the coil. The interaction of the current flow in coil segment CD with the magnetic field produces a force upward on segment CD. The current flowing through segment AB interacts with the magnetic field to produce a force downward.



The two forces are identical in magnitude but opposite in direction since the direction of the current flow in the two segments is reversed with respect to the magnetic field. As mentioned earlier, there is no net force produced on coil segment AC. As a result of the forces on segments AB and CD, there is a net rotational force (torque) clockwise on the coil. As the coil reaches the 90-degree position, the coil ends lose contact with the "brushes", and there is no current flowing in the coil. Therefore, there is no force produced on any of the coil segments, and the crude motor depends upon the rotational kinetic energy of the coil to rotate it past the 90-degree position. As the coil approaches the 180 degree position, the coil ends contact the "brushes" again, and current flow is reestablished. The current flow in segment AB interacts with the magnetic field to produce a force upward in segment AB. The current flow in segment CD produces a downward force on that coil segment. This pair of forces produces an angular acceleration of the coil in the clockwise direction. At the 270-degree position, there is, once again, no current flow in the coil, and the coil continues to rotate only due to its own inertia.

## SIMPLIFIED DC MOTOR.

(adapted from <http://library.thinkquest.org/15433/unit6/6-5.htm>)



As we turn the handle in the picture, the wire loop rotates. This moves the wires through a field. Because of the split-ring commutator, the side of the loop on the left always produces current flowing out of the loop, and the side on the right always has current flowing in. This means that a D.C. current is produced. A.C. current can also be produced, using two slip rings instead of the split ring commutator. Each side of the wire loop is connected to a separate ring, which is in turn connected to a brush contact. As a side of the loop rotates, it moves first in one direction through the field, then in the opposite. This produces an A.C. output at each ring, and therefore for the entire circuit.

An E.M.F. (voltage), as well as a current, is produced by this simple **generator**, by a process called **electromagnetic induction**. This voltage, also called an **induced E.M.F.**, is then used in the circuit to light the light bulb. The voltage is produced because there is mechanical power going into the handle of the generator.

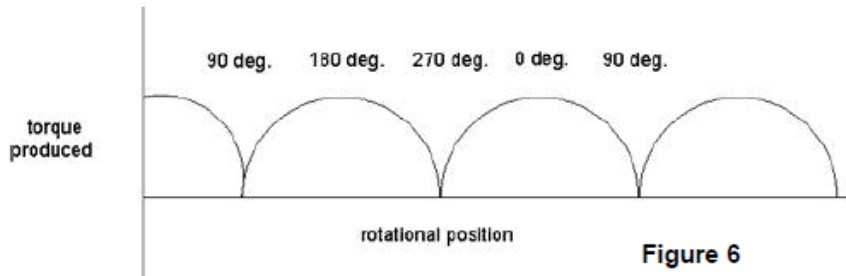


Figure 6

to the structure of the first so that the two coils are 90 degrees apart, the torque generated from the two coils would be represented by the curves in Figure 7. A simplified curve for the output of the two segment motor is shown in Figure 8. With this construction, the output torque never reaches zero but is smoothed out at some nominal value.

The number of coil segments can be (and usually is) increased well beyond two coil segments. The resulting motor has very little torque ripple. A second problem with the motor previously developed is that it is modeled using a single conductor in each coil. In practice, the motor coils contain multiple conductors, each adding to the torque production as the single conductor model has been shown to do. The use of multiple conductors in the coils is discussed further in the next section.

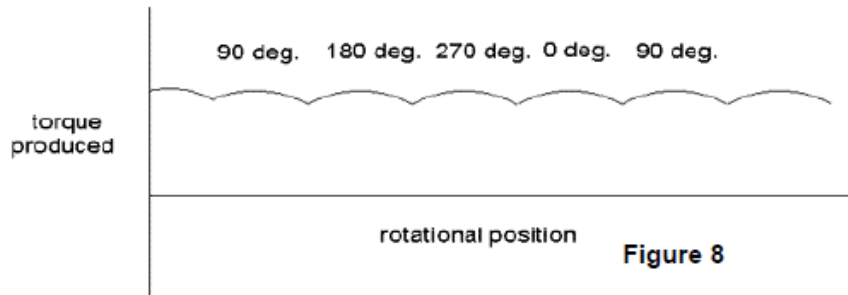


Figure 8

The crude motor developed in the preceding paragraphs has several design flaws which prevent its use in most practical applications. Perhaps its most limiting feature is the large amount of torque ripple produced during operation. Figure 6 is an illustration of the motor torque output as a function of angular position.

At each 90 degree and 270 degree position, the generated torque drops momentarily to zero. If a second coil is added

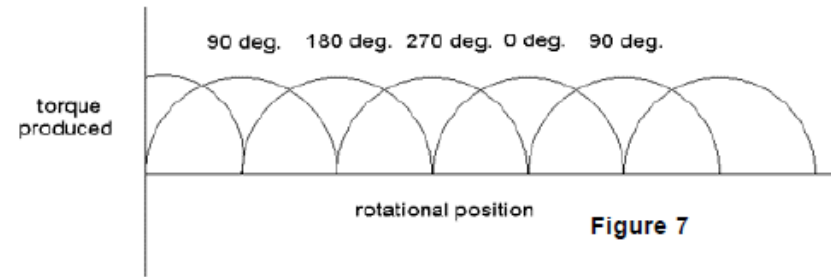


Figure 7

### Motor Constants

The nominal value of the torque generated by the single coil motor can be calculated. In Figure 9, the single coil model is shown again. As previously discussed, the coil rotates about axis O - O'. The force on conductor AB is given by:

$$F = \mathbf{B}L\mathbf{I} \sin A$$

where A is the included angle between **B** and **I**, and L is the length of coil segments AB and CD. Assume for the sake of discussion that A is 90°, then  $\sin A = 1$ , and the  $\sin A$  factor can be dropped. Further, assume that multiple

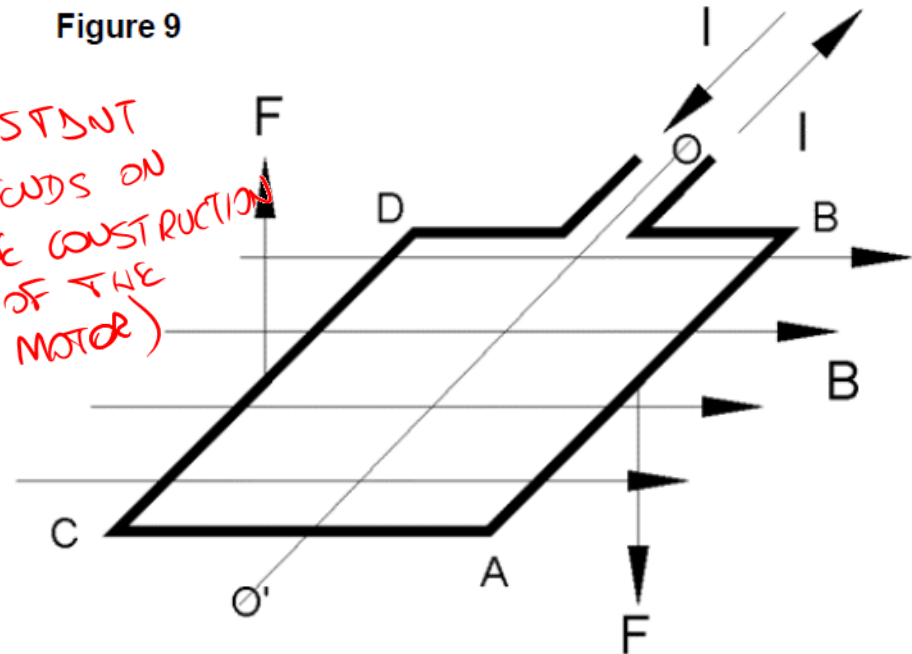
conductors are used in the coil. The force on the conductor AB is then given by:

$$F = NBLI$$

where N is the number of conductors in the coil. The torque generated in the coil as a result of this force F is the product of the force F and the distance from segment AB to the rotational axis O - O'. If this distance is called R (radius), then the resulting torque is given by:

$$T = FR = RNBLI$$

Figure 9



CONSTANT  
(DEPENDS ON  
THE CONSTRUCTION  
OF THE  
MOTOR)

Coil segment CD also contributes to the generated torque, however, and has a contribution equal to that of segment AB. Therefore, the total torque generated is simply twice that generated by segment AB and is given by:

$$T = 2FR = 2RNBLI$$

Note that the generated torque is dependent upon the current I, the radius of the coil R, the number of conductors (turns) N, the magnetic field flux density B, and the length of the conductors L. With

the exception of the motor current, all other factors are determined by the geometry of the motor and the materials from which it is made. Since it is generally safe to assume that construction features of a finished motor will not change, a constant of proportionality between the motor current and the materials/geometry dependent factors can be assigned to the motor. In the case at hand, there is a constant that describes the torque generated by the motor for a specific motor current, the torque constant.

$$K_t = T/I = 2RNBL$$

depends on the  
core used

In the early part of this section, it was shown that an EMF,  $E$ , is developed in a conductor moving through a magnetic field of flux density,  $B$ , at velocity  $V$ . If one were to use the same assumptions adopted to develop the crude motor using a coil, brushes, and commutator, it can be shown that an EMF will be developed across the brushes when an external torque physically rotates the coil. Like the principles involved in the development of the torque constant, the magnitude of the EMF is dependent upon materials/geometry factors. It is also dependent upon the velocity at which the coil is rotated. Once again, there is a constant of proportionality that describes the relationship between coil rotational velocity and materials/geometry factors, commonly known as the back EMF constant ( $K_e$ ). The back EMF constant is typically given in volts per unit of rotational velocity.

If one takes the reciprocal of the back EMF constant, the result is a proportionality constant that relates the voltage applied to the motor terminals to the rotational velocity of the coil. This version of the motor constant is commonly known as the velocity constant. The velocity constant is given in units of rotational velocity per volt. There is, of course, one constant for a motor. The differences between the torque constant and the back EMF constant are simply a matter of the units used to make routine calculations while the velocity constant is simply a useful form of the back EMF constant. In fact, it can be shown that if the torque constant is specified in Nm/A and the back EMF constant in V-sec/rad, then:

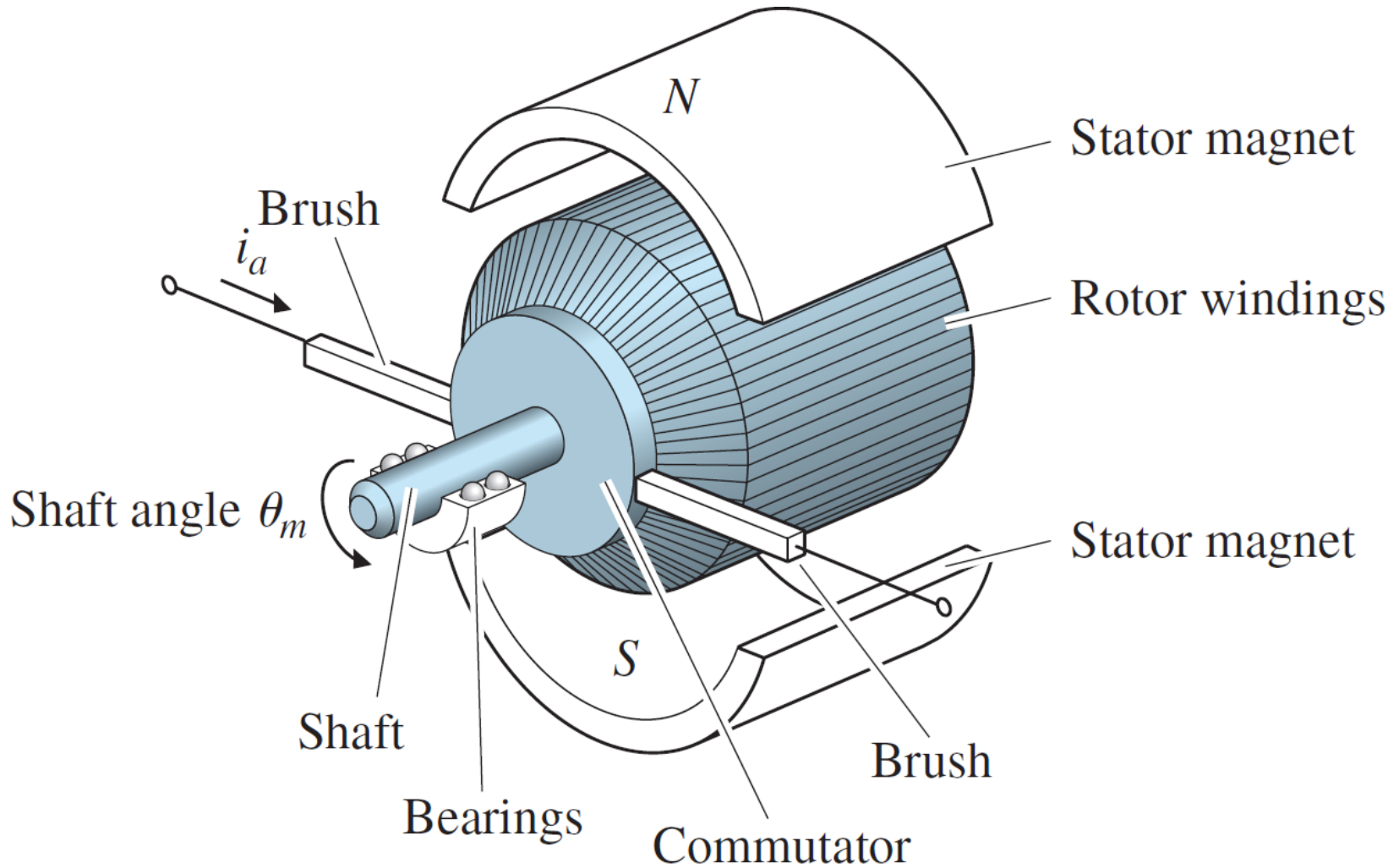
$$K_t = K_e = 1/K_v$$



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Phone: (727) 572-0131 - Fax: (727) 573-5918 - Toll-Free: (800) 807-9166  
Email: [info@micromo.com](mailto:info@micromo.com) - Web Address: [www.micromo.com](http://www.micromo.com)



## 2.8 BASIC OPERATIONAL PRINCIPLE OF DC MOTORS



- Typical electromechanical system: DC motor

The underlying principles of a DC motor are given by the motor and generator laws. However, rather than express these relationships in terms of the field strength and winding geometry, it is standard to relate the torque  $T$  developed in the rotor in terms of the armature current  $i_a$  and torque constant  $K_t$ , and to express the voltage generated as a result of rotation in terms of the shaft's rotational velocity  $\dot{\theta}_m$  and an electric or "emf" (electromotive force) constant  $K_e$ . Thus, we get the two laws as

$$T = K_t i_a, \quad \text{where } K_t \equiv \text{torque constant (Nm/A)}, i_a \equiv \text{armature current (A)}$$

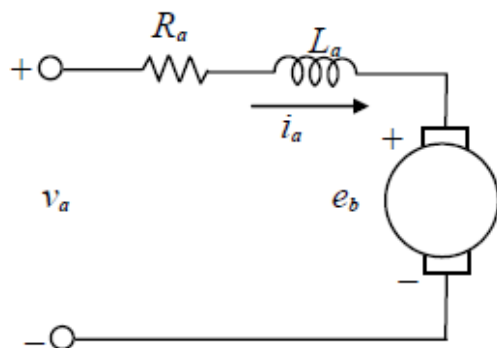
$$e = K_e \dot{\theta}_m, \quad \text{where } K_e \equiv \text{emf constant (Nm/A)}, \dot{\theta}_m \equiv \text{shaft's rotational velocity (rad/sec)}$$

Note that these relationships connect the mechanical variable  $T$  and  $\dot{\theta}_m$  to the electrical variable  $i$  and  $e$ .

**Exercise 1:** Derive the two equations above by using law of the generator and law of motors.

**Exercise 2:** Using the two 'laws', come up with a dynamic model for the DC motor system.

- Modeling a DC motor



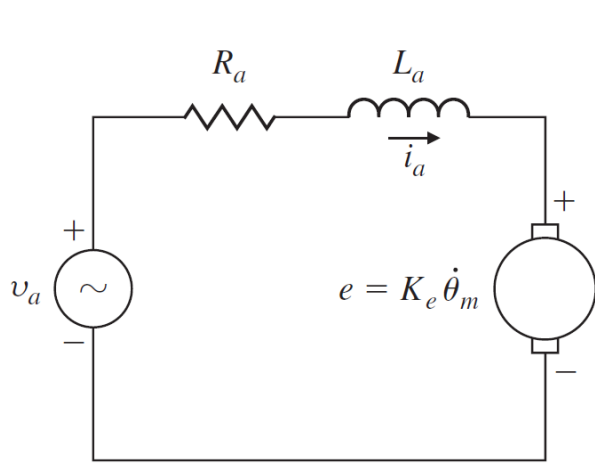
Assume the rotor has inertia  $J_m$  and friction coefficient  $b$ ; and the motor armature resistance is  $R_a$  and inductance is  $L_a$ . How many equations will the dynamic model have and why?

Two equations are needed in this case since there are two energy storage elements (inductor and inertia) in this system.

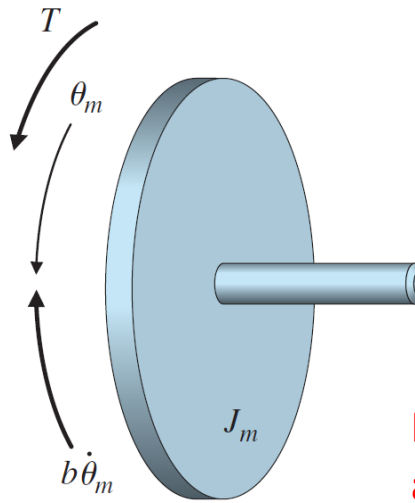
**Rule:** *Number of Equations = Number of Energy storage elements*

Assuming the effects of inductance is negligible, the model is 2<sup>nd</sup> order:

$$J_m \ddot{\theta}_m + (b + \frac{K_t K_e}{R_a}) \dot{\theta}_m = \frac{K_t}{R_a} V_a$$



(a)



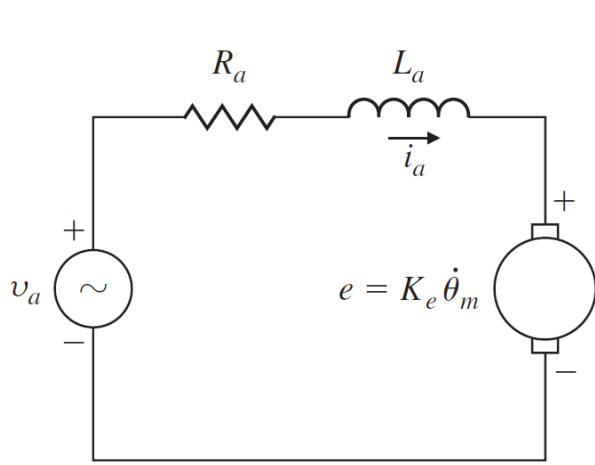
(b)

**Write the dynamic equations for the DC motor**

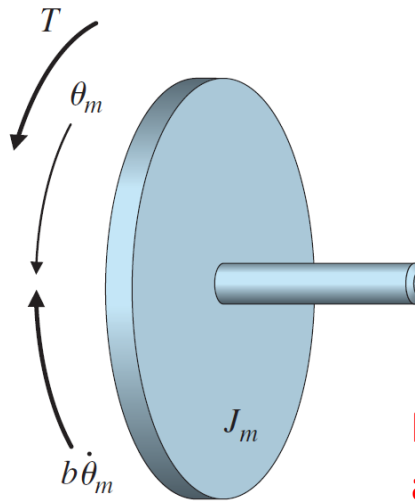
**Need to apply Newton's law, and circuit laws. How?**

**Applying Newton's law to the rotor →**

**Similarly, applying KVL for the electric circuit →**



(a)



(b)

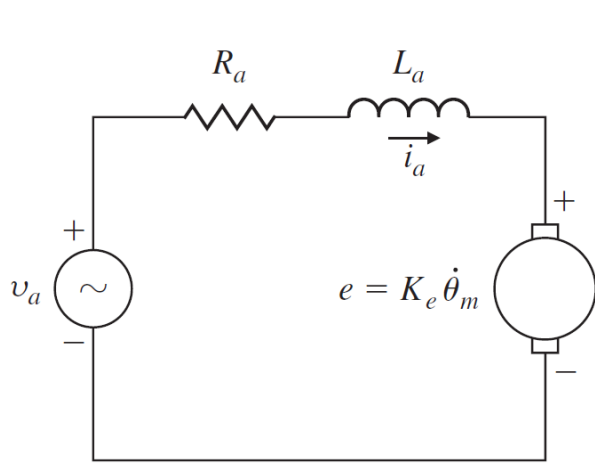
Write the dynamic equations for the DC motor

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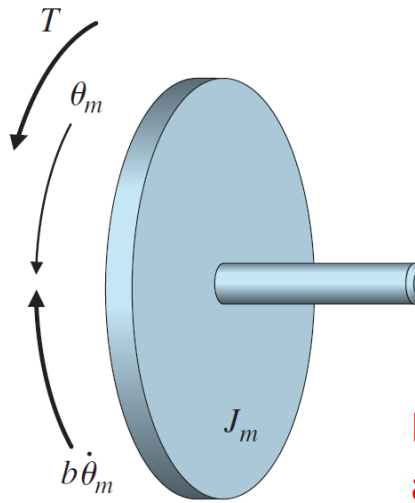
Applying Newton's law to the rotor  $\rightarrow J_m \ddot{\theta}_m + b \dot{\theta}_m = K_t i_a$  (Eqn.1)

Similarly, applying KVL for the electric circuit  $\rightarrow v_a - i_a * R_a - L_a \frac{di_a}{dt} - K_e \dot{\theta}_m = 0$  (Eqn.2)

The TF connecting input  $v_a$  to output  $\theta_m$  can then be shown to be ?



(a)



(b)

Write the dynamic equations for the DC motor

Need to apply Newton's law, and circuit laws. How?

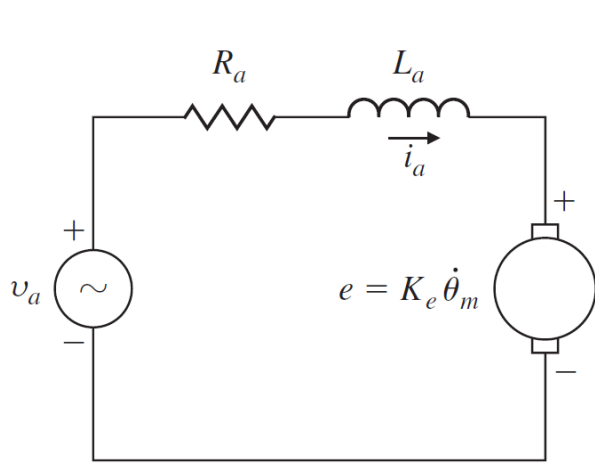
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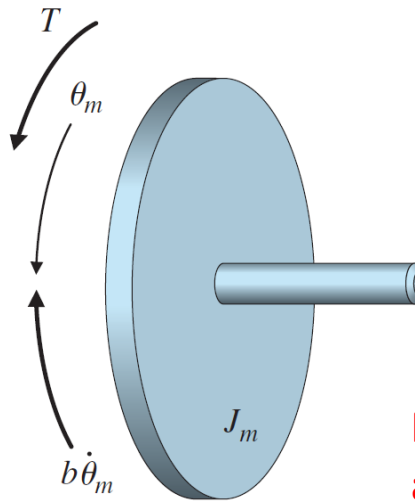
The TF connecting input  $v_a$  to output  $\theta_m$  can then be shown to be

$$\frac{\theta_m(s)}{v_a(s)} = \frac{K_t}{s[(J_m s + b)(L_a s + R_a) + K_t K_e]}$$

Inductance effects negligible  $\rightarrow$ ?



(a)



(b)

Write the dynamic equations for the DC motor

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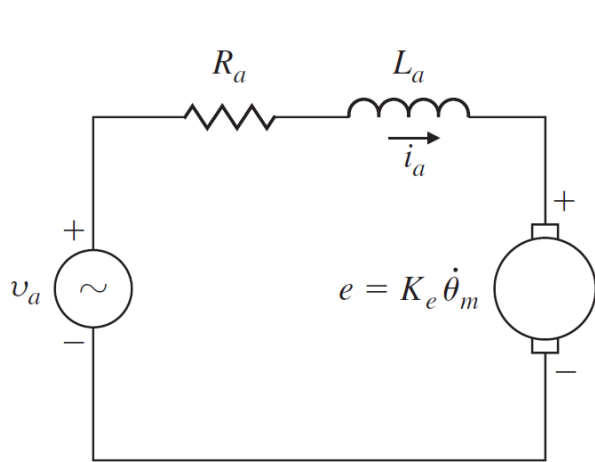
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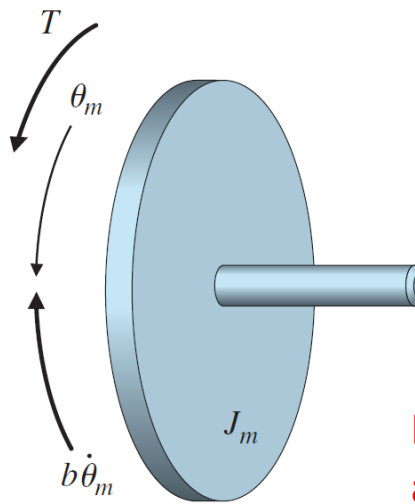
Inductance effects negligible  $\rightarrow \frac{\theta_m(s)}{v_a(s)} = \frac{K_t / R_a}{J_m s^2 + (b + K_t K_e) / R_a} = \frac{K}{s(\tau s + 1)}$

This gives the gain  $K = ?$

and time constant  $\tau = ?$



(a)



(b)

Write the dynamic equations for the DC motor

Need to apply Newton's law, and circuit laws. How?

Applying Newton's law to the rotor  $\rightarrow J_m \ddot{\theta}_m + b \dot{\theta}_m = K_t i_a$  (Eqn.1)

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This gives the gain  $K = \frac{K_t}{bR_a + K_t K_e}$  and time constant  $\tau = \frac{R_a J_m}{bR_a + K_t K_e}$