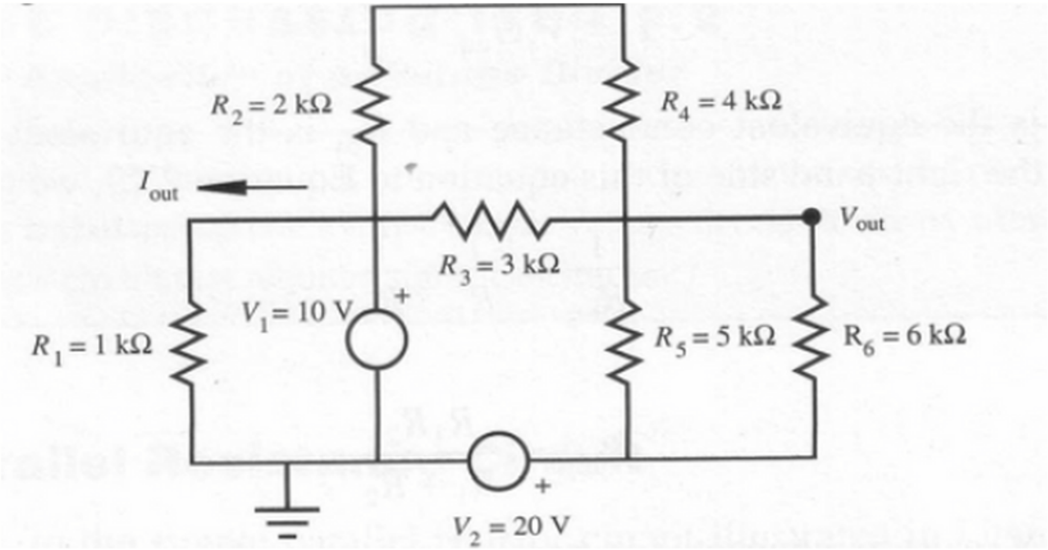
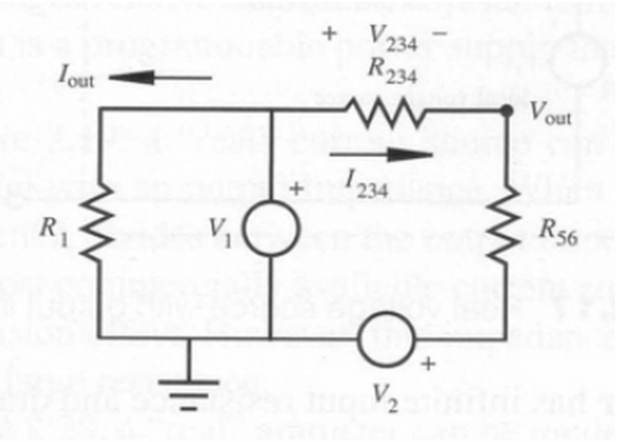
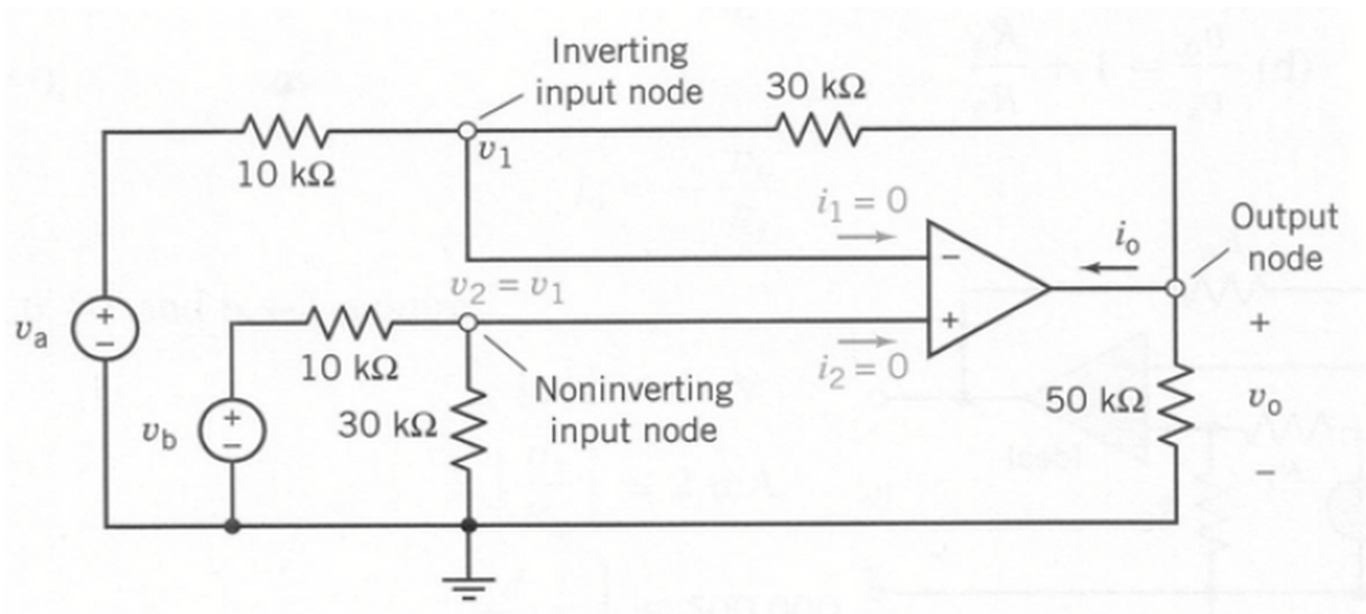
LECTURE 5

Ex 1



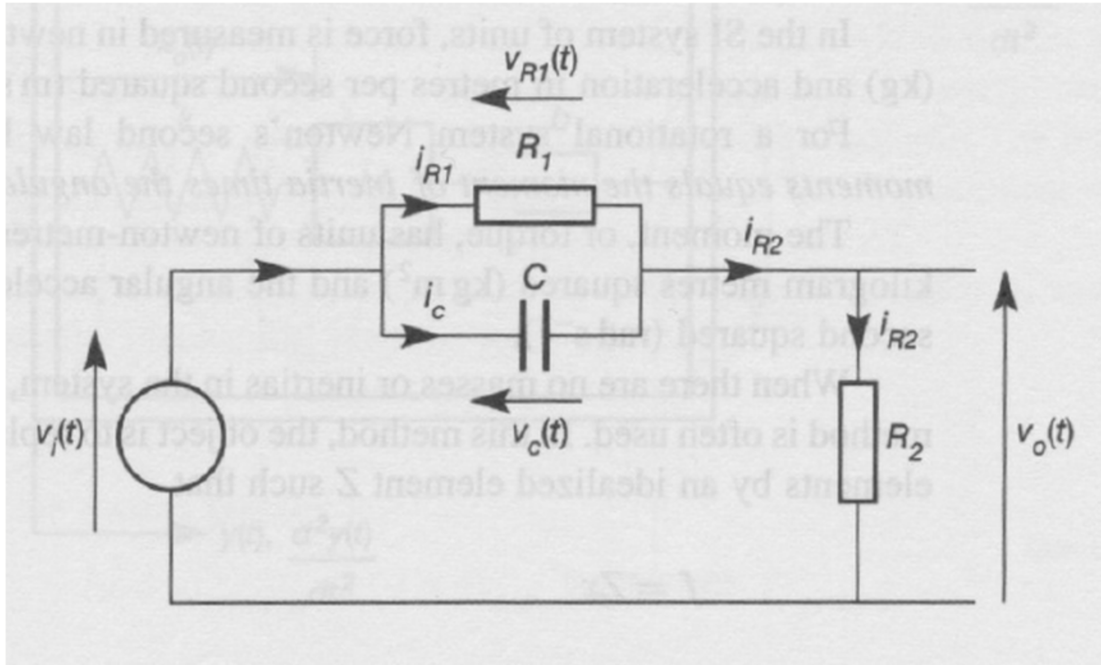


Ex 2



Inverting
input node
30 kΩ
10 kΩ
v_1
$i_1 = 0$
i_o
Output
node
$v_2 = v_1$
v_a
10 kΩ
+
$i_2 = 0$
Noninverting
input node
50 kΩ
v_o
v_b
30 kΩ
−

Ex 3



2.9 Linearization

Linearization is the process of finding a linear model that approximates a nonlinear one.

Case I One and Two Variable Versions

1. Taylor series expansion for one variable

$$f(x) = f(x_0) + \frac{f'(x_0)(x-x_0)}{1!} + \frac{f''(x_0)(x-x_0)^2}{2!} + \frac{f'''(x_0)(x-x_0)^3}{3!} + \cdots \text{ Higher Order Terms}$$

$$\text{Linearized Version}: \; f(x) \approx f(x_0) + \frac{f'(x_0)(x-x_0)}{1!}$$

2. Taylor's Series for Two Variables

$$f(x,u) = f(x_0,u_0) + f'(x,u)\big|_{x_0,u_0}(x-x_0) + f'(x,u)\big|_{x_0,u_0}(u-u_0) + \cdots \text{ Higher Order Terms}$$

$$\text{Linearized Version}: \; f(x,u) \approx f(x_0,u_0) + f'(x,u)\big|_{x_0,u_0}(x-x_0) + f'(x,u)\big|_{x_0,u_0}(u-u_0)$$

Case II General vector version

Determine equilibrium values of $\mathbf{x}_0$, u_0, that is, values where $\dot{\mathbf{x}}_0 = 0 = \mathrm{f}(\mathbf{x}_0, u_0)$. We then expand the nonlinear equations in terms of perturbations from these equilibrium values: that is, we let $\mathbf{x} = \mathbf{x}_0 + \delta\mathbf{x}$ and
$u = u_0 + \delta u$, so that

$$\dot{\mathbf{x}}_0 + \delta\dot{\mathbf{x}} \cong \mathrm{f}(\mathbf{x}_0, u_0) + \mathbf{F}\,\delta\mathbf{x} + \mathbf{G}\,\delta u$$

where F and G are the linearized matrices for the function f($\mathbf{x}$, u) at $\mathbf{x}_0$ and u_0. (ADAPTED FROM SATISH NAIR)

Example

1. Sinusoidal Function

The motion of the simple pendulum system is described by the differential equation

$$(l + R\theta)\ddot{\theta} + g \sin\theta = 0$$

l = length of the cord in the vertical (down) position,

R = radius of the cylinder.

Linearize the equation around the point $\theta = 0$.

2. Polynomial Function

Find the linearization $L(x)$ of $f(x)$ at $x = 2$.

$$f(x) = x^3 - 2x + 3$$

SOLUTION

Example

1. Sinusoidal Function

The motion of the simple pendulum system is described by the differential equation

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Linearize the equation around the point $\theta = 0$.

SOLUTION

Nonlinear term $\sin\theta$ term should be linearized. By using Equation (1.1), $\sin\theta$ can be approximated as

$$\sin\theta = \sin\theta_0 + \cos\theta\big|_{\theta=\theta_0}(\theta - \theta_0) \text{ where } \theta_0 = 0$$

$$= \sin(0) + \cos(0)\,(\theta - 0)$$

$$= \theta$$

For small values of θ, the system equation reduces to an equation for a simple pendulum,

$$\ddot{\theta} + \frac{g}{l}\theta = 0.$$

2. Polynomial Function

Find the linearization $L(x)$ of $f(x)$ at $x = 2$.

$$f(x) = x^3 - 2x + 3$$

SOLUTION

Since the function $f(x)$ has only one variable x, we can use

$$f(x) \approx f(x_0) + \frac{f'(x_0)(x - x_0)}{1!}$$

where $f'(x) = 3x^2 - 2, x_0 = 2$

Therefore,

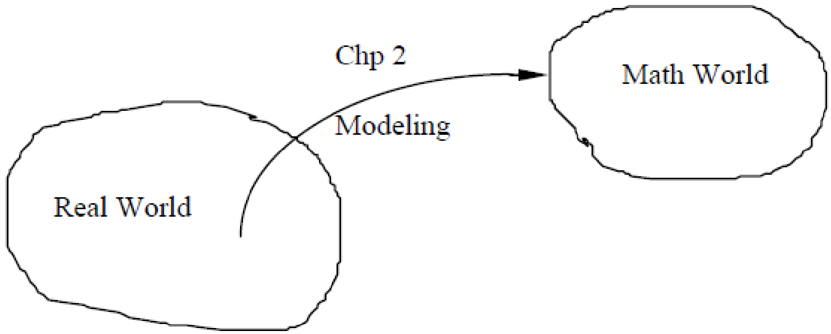
$$f(x) \approx f(2) + \frac{f'(2)(x-2)}{1!}$$

$$\approx 10x - 13$$

Linearize the equation $f(x) = 2x^5 - 2x^2 + 3x + 4$

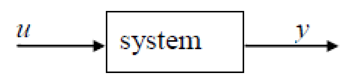
Linearize the equation $\ddot{x} + 3x^2\dot{x} + x^2 + 3 * \sin(x) = 0$

2.10 MODELING – SUMMARY



- For linear time-invariant (LTI) systems the 'model' consists of a set of ordinary differential eqns. with constant coefficients.
- Examples:

1). Standard form of first order system



$$\tau\dot{y} + y = Ku$$

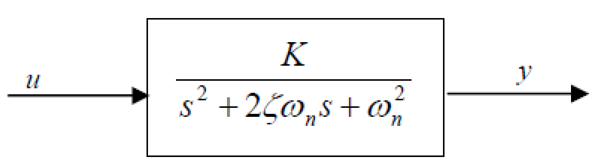
Laplace transforming (with zero ICs)

$$\tau s Y(s) + Y(s) = KU(s)$$

$$\Rightarrow (\tau s + 1)Y(s) = KU(s) \Rightarrow \boxed{\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}}$$

$K \equiv$ Gain
$\tau \equiv$ time constant

2). Standard form of a second order system



$K \equiv$ Gain
$\omega_n \equiv$ natural freq., rad/s
ζ = damping ratio