\[ x_t = f(x_{t-1}, v_{t-1}) \]

AND NOW DON'T MEASUREMENTS NLA TO STATES

We may know how the system evolves:

But we can observe \( y_t \). We cannot "see" \( x_t \).

Goal: Estimate \( x_t \) - the current state of the system.

Dynamic State Estimation

Bayesian Approaches

Lecture 23
The system model \( p(x_t | z_{1:t-1}) \) to predict the next state \( x_t \) given prior knowledge \( p(x_t | z_{1:t-1}) \).

We assume that if we know \( p(x_t | z_{1:t-1}) \), we might be able to use...

**Prediction:**

\[
p(x_t | z_{1:t-1}) = \mathbb{E} \]

**Update:**

\[
p(x_t | z_{1:t}) \rightarrow p(x_t | z_{1:t})
\]

Bayesian approach involves trying to estimate all \( x_t \) given \( z_{1:t} \) and the prior. But the problem is — if \( x_t \) are usually non-Gaussian.
\[
\frac{p(y \mid x_1, x_2, \ldots, x_k)}{p(x_1, x_2, \ldots, x_k)} = \frac{p(x_1, x_2, \ldots, x_k)}{p(x_1, x_2, \ldots, x_k)}
\]
This PDF can be generated by calculating analytically. The problem is simple, if \( k < \infty \), and the fact that

\[
p(x^k | z_{1:k-1}) = \int p(x^k | z_{1:k-1}) \, dx \frac{p(z_{1:k-1})}{p(z_{1:k-1})}
\]

this model's previous model. The previous model's measurement.

\[
\frac{p(x^k | z_{1:k-1})}{p(x^k | z_{1:k-1})} \frac{p(z_{1:k-1})}{p(z_{1:k-1})} = \frac{p(z_{1:k-1})}{p(z_{1:k-1})}
\]

\[
\frac{p(x^k | z_{1:k-1})}{p(x^k | z_{1:k-1})} \frac{p(z_{1:k-1})}{p(z_{1:k-1})} = \frac{p(z_{1:k-1})}{p(z_{1:k-1})}
\]

\[ P(x_{1:t} | x_{1:t-1}, \epsilon_{1:t}) = \mathcal{N}(m_{x_{1:t-1} | x_{1:t-1}, \epsilon_{1:t}}, \sigma_{x_{1:t-1} | x_{1:t-1}, \epsilon_{1:t}}) \]

Given \( \sigma \) is the covariance matrix of the process noise, we assume

\[ m_{x_{1:t-1} | x_{1:t-1}, \epsilon_{1:t}} = \mathcal{N}(m_{x_{1:t-1}}, \sigma_{x_{1:t-1}}) \]

That is:

\[ z \sim \mathcal{N}(0, \sigma) \]

\[ u = w + x \]

\[ f = f(x, 0, \sigma) \]

\[ v = f(x, 0, \sigma) \]

And the PDP's are Gaussian?

What if \( f(x, 0, \sigma) \) are linear?
\[ S = \frac{1}{k} p \cdot \frac{k}{k-1} f + r \]

(CaunCy)

\[ K = \frac{p}{P-1} k \cdot S \]

where

\[ P = P - \frac{K}{k} - \frac{1}{c} \cdot \frac{P}{k} \]

\[ \frac{m_{k-1}}{k-1} + K \cdot (x - k \cdot m_{k-1}) \]

(\text{Vest}: \text{Pkl} = 0 + \frac{1}{k} \cdot \frac{P}{k} \cdot f \]

\[
\text{Explanation:} \quad m_{k-1} = \frac{f}{m_{k-1}} \text{ (From the slide)}
\]

Note: The notation \( m_{k-1} \) and \( f \) are not defined in the image.
(1st or 2nd)\text{order}\text{wise}\nonumber
\begin{align*}
\frac{d}{dx} f(x) &= \frac{d}{dx} (x) \\
&= 1
\end{align*}
\text{So same as before, except that}
\text{now linear?}
\text{Must } f(0) \neq \lambda_0(0) \text{ and indeed}
\text{untraced Kalman filter
Unsupervised Localization