In two lines called epipolar lines of M, the plane \( \mathbf{W'O} \) intersects the image planes.

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Projectively, the epipolar geometry is all there is to know about a stereo rig. It establishes the correspondence and allows 3D reconstruction of the scene to be carried out up to an overall 3D projective deformation (which is all that can be done with any number of completely uncalibrated cameras, without further constraints). An important practical application of epipolar geometry is to aid the search for corresponding points, reducing it from the entire second image to a single epipolar line. The epipolar geometry is sometimes obtained by calibrating each of the cameras with respect to the same 3D frame, but it can easily be found from a few point correspondences without previous camera calibration.
The line \( \ell \) is \( \perp \) to \( m \) if and only if \( \ell \cdot m = 0 \).

By definition:

\[ \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \perp \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} \]

Let \( \ell = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \) be a line on the same plane.

Linearly independent.

On the plane, such that \( m \neq 0 \) and \( n \neq 0 \):

\[ \begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix} \] be two points

\[ \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} \]

Let \( m = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \)

\[ \text{(Note:)} \]

Lines \& Planes
Let $L = \mathbb{W} \times \mathbb{U}$. 

The kernel of the mapping to $\mathbb{W}$ is:

\[
\begin{bmatrix}
\mathbf{w} \\
\mathbf{u} \\
\mathbf{n}
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{w} \\
\mathbf{u} \\
\mathbf{n}
\end{bmatrix}
\]

For each $\mathbf{n}$, we have:

\[
\begin{bmatrix}
\mathbf{n} \\
\mathbf{w} \\
\mathbf{u}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{n} \\
\mathbf{w} - \mathbf{w} \times \mathbf{u} \\
\mathbf{u} - \mathbf{w} \times \mathbf{n}
\end{bmatrix}
\]

Summary (to remember):
\[ (L = 5 \text{m}) \]

\[
\begin{bmatrix}
7 & 0 & -1 \\
0 & 7 & -1 \\
1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
= 
\begin{bmatrix}
-1 \\
0 \\
0
\end{bmatrix}
\]

\[ e \times n = e \times n \]

\[
\begin{bmatrix}
0 \\
8 \\
0
\end{bmatrix}
\]

The Epipolar Line \[ L = \begin{bmatrix}
c \\
0 \\
9
\end{bmatrix} \]

\[ \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} \]

Let \[ m = \begin{bmatrix}
n \\
0 \\
0
\end{bmatrix} \]

\[ e \times m = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} \]

Fundamental Matrix
To the line \( L \) in image 2

Given point \( M \) is constant.

That is, given a point in image 1, the line

\[ L = AC = FM \]

Then we can determine \( F \) - \( AC \) (Fundamental Matrix)

If we find 3 distinct epipolar lines, we can extract the projective function. So,

A has eight degrees of freedom. So,

\[ A = \begin{bmatrix} 1 \end{bmatrix} \]

How about mapping line \( L \) to \( L' \).
(Eigenvector Constraint)

\[ \text{null } w = 0 \]

\[ \text{im } f = \text{ker } f = \text{im } w = 0 \]

If \( w \) is an eigenvector, then \( \text{null } w = 0 \). Thus \( \text{null } f \) is the corresponding null space.

Also, if \( w \) is an eigenvector at \( P \), then the left kernel of \( f \) at \( P \) is the set of all eigenvectors at \( P \) of \( f \). Since all eigenvectors of \( f \) are in the image, \( \text{ker } f \) is the set of all \( w \) such that \( f(w) = 0 \). Hence, \( f(w) = 0 \) if and only if \( w \) is an eigenvector at \( P \) of \( f \).

Also, the right kernel of \( f \) at \( P \) is the set of all vectors \( w \) such that \( w f = 0 \). Since \( \text{rank}(A) = 3 \) and \( \text{null}(C) = 2 \), we have:

\[ \text{rank}(f) = 2 \]
The determinant is zero:

$$\det(f) = 0$$

We can estimate

$$\left[ \begin{array}{c} m_1^2, m_1^2, \ldots, m_1^2 \end{array} \right]$$

plus

So, with matrices

But rank \( r = 2 \) then \( \text{det}(f) = 0 \)

\( \mathbf{f} = \mathbf{AC} \), \( \mathbf{f} = \mathbf{AC} \)

Estimating the fundamental matrix

\( \mathbf{f} = \mathbf{AC} \)
\[
\begin{align*}
\|f\|_1 &= 1 \\
\|f\|_2 &= 1 \\
\min_{f + A \neq 0} f &= 0 \\
\text{Similiar to unit vector.}
\end{align*}
\]

\[
\begin{bmatrix}
\mathbf{w}_1 \\
\vdots \\
\mathbf{w}_m
\end{bmatrix} = A
\]

where 

\[
A = 0
\]

\[
\mathbf{f} = \mathbf{w}_1 + \mathbf{w}_2 + \mathbf{w}_3 + \mathbf{w}_4 + \mathbf{w}_5 + \mathbf{w}_6 + \mathbf{w}_7 + \mathbf{w}_8 + \mathbf{w}_9 + \mathbf{w}_{10} + \mathbf{w}_{11} + \mathbf{w}_{12} + \mathbf{w}_{13} + \mathbf{w}_{14} + \mathbf{w}_{15} + \mathbf{w}_{16} + \mathbf{w}_{17} + \mathbf{w}_{18} + \mathbf{w}_{19} + \mathbf{w}_{20}
\]

\[
\begin{bmatrix}
1 \\
1 \\
\vdots \\
1 \\
1
\end{bmatrix} = \mathbf{w}_1 + \mathbf{w}_2 + \mathbf{w}_3 + \mathbf{w}_4 + \mathbf{w}_5 + \mathbf{w}_6 + \mathbf{w}_7 + \mathbf{w}_8 + \mathbf{w}_9 + \mathbf{w}_{10} + \mathbf{w}_{11} + \mathbf{w}_{12} + \mathbf{w}_{13} + \mathbf{w}_{14} + \mathbf{w}_{15} + \mathbf{w}_{16} + \mathbf{w}_{17} + \mathbf{w}_{18} + \mathbf{w}_{19} + \mathbf{w}_{20}
\]

So, from the equivalence constraint,

\[
\text{However, } \det(f) = 0 \text{ provides a cubic equation (8)}
\]
Let $F$ be the least upper bound of \((\text{smallest})\) \(\lambda \geq 0\) that solves

\[
F = \inf \left\{ \lambda \geq 0 \mid \forall x \in D \right\}
\]

However, this does not guarantee that \(\lambda = 2\).

Smallest eigenvalue of \(A + \lambda I\) is the eigenvector associated with the solution for \(\min \left\{ \| x \| \mid x^T (A + \lambda I)x = 1 \right\} \)
See [27] & [12]

Normalized pixel coordinates [-1, 1]

Solution is numerically unstable

A is externally ill-conditioned

At ζ = 0

\[
\frac{1}{k^2} \approx 200
\]

Thus eliminate row 1 from \( A \) and the last entry in \( A \) is 1

Image is 200, so \( x, x' \approx 200 \)

Problems: A vertical pixel coordinate in A 512x512
\[ A \cdot m = N \cdot m \]

Cross product \( \times \) \( A \times m \)

\[ m' = A \cdot (m + N) \]

\[ N = A_1 \cdot m \]

\[ A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

Translation method:

Given point \( (x, y, z) \):

\[ m' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = m + N \]
\[ F = (AT)^\top A^\top R A_{\hat{r}} \]

**Fundamental Matrix**

\[ m_e^T (AT) A_{\hat{r}} e_{m_e} = 0 \]

\[ m_e^T (AT) e_{m_e} = 0 \]

\[ m_e\perp (AT) m_e = 0 \]

\[ m_e\perp (AT) m_e = 0 \]


In defence of the 8-point algorithm.


R. Hartley. Understanding positioning from multiple images.
R. Mohr, B. Boufama, and P. Brand.

P. Meer, S. Ramabhadran, and R. Lenz. Correspondence of coplanar features through P2-invariant representations.

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