Remarks: Lecture 11
Note: Focus on constant forces?

Focus:

Two steering wheels

1. \( \theta \), \( \xi \), \( \alpha \)
2. \( x(t) \), \( y(t) \)

Path:

Assuming linear forces & moments

Both nodes move as some node moves.
**Trajectory Following**

- Not easy to compute all feasible trajectories based on all kinematic, dynamic, and workspace constraints.

- Dynamic environments affect planning (adaptation).

- Smooth trajectories (IROS paper on rubber band model vs prediction).
Open vs. Closed Loop Control
Graph shown below.
Assuming linear variation, and that 0° throttle angle results in zero speed, you can get the speed vs. throttle angle:

Solution:

Given these experimental observations, develop an automobile model for control purposes.

1. The measured steady-state characteristics of the vehicle on level road around 55 mph and found that a 1 degree change in

   - throttle angle (control variable) caused a 10 mph change in speed
   - the speed of the vehicle at 55 mph

   This is the input/output relation of the dynamic responses of an automobile. For

Part A: Modeling

Problem: An engineer performed the following tests to determine the dynamic response of an automobile for...
With this controller check for error with disturbance

\[ \frac{1}{\alpha} \text{ controller } \]

So ignore it. Let's set \( w = 0 \).

for –ve disturbance

Note: Have no idea what \( w \), it will be. It can be \( \pm \)ve

How to decide on the controller? Let's what number

or

\( \frac{1}{\alpha} \) closed loop

Add a controller (decision-maker)

Solved

Problem: Get \( \alpha \) as close to \( \beta \) as possible

Part B: Open Loop vs. Closed Loop Control

where \( \lambda \) is the speed (mph), \( n \) is the throttle angle (degrees), and \( w \) is the road grade (\%).

\[ \lambda = 10n - 5w \]

So, the model developed from experimental testing is as follows:
The issue of how to get the gain large enough to reduce the errors without making the system become unstable and squeal is what much of feedback control design all about.

How large can the controller gain be?

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<thead>
<tr>
<th>Error (%)</th>
<th>0.0%</th>
<th>5.5%</th>
<th>10%</th>
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Effect of Parameter Variation

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Effect of Disturbance

\[ m = 0.996^p - 0.05w \]

Closed Loop Diagram
\[
\begin{bmatrix}
0 \\
e^y \\
e^x
\end{bmatrix} = K e
\]

Feedback Loop:

\[
\lim_{t \to \infty} e = 0
\]

Without using kinematic constraints, feedback control.
Assume GOAL: \[
\begin{bmatrix}
X \\
Y \\
\Theta \\
\beta
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{\Theta} \\
\dot{\beta}
\end{bmatrix} =
\begin{bmatrix}
\cos \Theta & 0 \\
\sin \Theta & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
V \\
W
\end{bmatrix}
\]

\[p = \sqrt{\Delta x^2 + \Delta y^2}
\]
\[\alpha = -\Theta + \arctan(\Delta y, \Delta x)
\]
\[\beta = -(\Theta + \alpha)
\]

\[
\begin{bmatrix}
\dot{\rho} \\
\dot{\theta} \\
\dot{\beta}
\end{bmatrix} =
\begin{bmatrix}
-\cos \alpha & 0 \\
\frac{\sin \alpha}{\rho} & -1 \\
-\frac{\sin \alpha}{\rho} & 0
\end{bmatrix}
\begin{bmatrix}
\dot{V} \\
\dot{W}
\end{bmatrix}
\]
Choices of $K_p, K_d$:

$$\begin{align*}
\dot{x} &= -\frac{\sin x}{\sin \theta} x - \cos x + 1 \\
\theta &= \begin{bmatrix} 0 & 0 & 1 \\ -K_p & 0 & K_d \\ 0 & K_p & 0 \end{bmatrix} x
\end{align*}$$

Controller