Change of Coor. Sys. (Polar) & Wrench. Kinematics Constraints

Lecture 12
\[
\begin{bmatrix}
\theta \\
\phi \\
\alpha \\
\beta
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\sin \alpha & 0 & \cos \alpha & 0 \\
-\cos \alpha & 0 & \sin \alpha & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\theta \\
\phi \\
\alpha \\
\beta
\end{bmatrix}
\]

Controller

\[
\begin{bmatrix}
\theta \\
\phi \\
\alpha \\
\beta
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\sin \alpha & 0 & \cos \alpha & 0 \\
-\cos \alpha & 0 & \sin \alpha & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\theta \\
\phi \\
\alpha \\
\beta
\end{bmatrix}
\]

Feedback
\[ \sum \alpha = \phi \]
Connectivity

\[
\begin{bmatrix}
\pi / 2 & 0 \\
\sin \alpha & \cos \alpha \\
-\sin \alpha & \cos \alpha
\end{bmatrix}
\]

We use

\[
\left(\begin{array}{c}
\pi / 2 \\
0 \\
-\pi / 2
\end{array}\right)
\]

If \( \alpha \) is in the 2nd or 3rd quadrant, i.e., \( (-\pi, -\pi / 2) \) or \( (\pi / 2, \pi) \), \( \alpha \) is in the 1st or 4th quadrant, i.e., \( (\pi, 0) \) or \( (-\pi / 2, 0) \).

Notes: The expression in (*) is correct.
For that to be true, the eigenvalues must have only negative real parts (i.e., not purely negative).

A system is stable if all its eigenvalues have negative real parts. A system is said to be bounded-input, bounded-output (BIBO) stable if its transfer function is a rational function with all poles in the open right half of the complex plane. For linear time-invariant systems, the state equation is:

\[ y(t) = Cx + Du \]

\[ x(t) = Ax + Bu \]

Stability of Lumped Systems
But if we look at the plane A  doesn't consist of ... show that there are no such poses (x, s, p).

Along poses where the eigenvalues are of

All poses of the object and entities

If we would have to calculate A is eigenvalues

\[
\begin{bmatrix}
    K_{p \text{bias}} & 0 & 0 & -K_{p \text{bias}} \\
    0 & K_{p \text{bias}} & 0 & -K_{p \text{bias}} \\
    0 & 0 & K_{p \text{bias}} & -K_{p \text{bias}} \\
    0 & 0 & 0 & K_{p \text{bias}}
\end{bmatrix} = A
\]

\[
\begin{bmatrix}
    K_{r \text{bias}} & 0 & 0 & -K_{r \text{bias}} \\
    0 & K_{r \text{bias}} & 0 & -K_{r \text{bias}} \\
    0 & 0 & K_{r \text{bias}} & -K_{r \text{bias}} \\
    0 & 0 & 0 & K_{r \text{bias}}
\end{bmatrix} = A
\]

In our case
Polynomial:

Characteristic:

\[
\begin{align*}
\Delta &= 0 = \begin{bmatrix}
-\lambda & -k_2 & 0 \\
-k_4 & -\lambda & -k_3 \\
-k & -k & -\lambda
\end{bmatrix} \\
&= \det
\end{align*}
\]

In our case, \( \Delta = A \), then

\[
0 = \begin{bmatrix}
\lambda^2 - \lambda - \beta^2 \\
\lambda^2 - \lambda - \beta^2 \\
\lambda^2 - \lambda - \beta^2
\end{bmatrix}
\]

That is

\[
0 = \begin{bmatrix}
\lambda - \beta \\
\lambda - \beta \\
\lambda - \beta
\end{bmatrix}
\]

Recall: Eigenvalues/vectors of a matrix \( \beta \) is:

A value \( \lambda \); a vector \( \phi \); such that

\[
\phi^T A \phi = \lambda \phi^T \phi
\]
Now if we set other goals (after)

Can we add kinetic constraints?

\[ 0 < \theta < k^d \]

In other words: