Final Exam

1) Clear your desk top of all **handwritten** papers and personal notes. You may keep only your textbook, the test paper, a calculator and a pencil.

2) Read through the test completely and work the problems you can, leave the difficult ones till last.

3) Keep your eyes on your own paper. Cheating will not be tolerated

4) Work problems on the back of the previous page if necessary.

5) **Show your work!**

NAME: _____________________________________________ _________________________
Question 1

The hand frame $T_H$ of a robot is given below. The corresponding inverse Jacobian of the robot at this location relative to this frame is also shown. The robot makes a differential motion relative to this frame described as:

$$\tau_n D = [0.05, 0, -0.1, 0, 0.1, 0.1]^T$$

(a) Find which joints must make a differential motion, and by how much, in order to create the indicated differential motions.

(b) Find the change in the frame.

(c) Find the new location of the frame after the differential motion.

(d) Find how much the differential motions (given above) should have been, if measured relative to the World/Universe frame, to move the robot to the same new location as in part (c).

$$T_H = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_HJ^{-1} = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 0 & -0.2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$
Question 2

Consider the two-link manipulator with rotary joints as shown below. The manipulator is moving its end-effector along the $x_0$-axis at 1 meter/sec.

a) Find the 2x2 Jacobian matrix with respect to the base coordinate frame.

b) Indicate the condition(s) when a singularity will occur. (Hint, a singularity occurs when we cannot find the necessary changes $d\theta/dt$ given the desired changes in, e.g., $dx/dt$.)

c) Calculate the joint rates when the manipulator is at a singular position.
Question 3

Using the Lagrangian method (directly or through D-H) derive the equations of motion (torque/forces) for the 2-DOF polar robot arm below. The center of mass for each link is at the center of the link.

Hints: The total length of the arm is \( r + \left( \frac{l_2}{2} \right) \), and the moments of inertia (direct method) and pseudo inertia matrices (D-H method) are:

\[
I_1 = \frac{1}{3} m_1 l_1^2 \quad I_2 = \frac{1}{12} m_2 l_2^2
\]

\[
J_i = \begin{bmatrix}
\frac{1}{3} m_i l_i^2 & 0 & 0 & \frac{1}{2} m_i l_i \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{1}{2} m_i l_i & 0 & 0 & m_i
\end{bmatrix}
\]
Question 4

Describe the algorithm used by your team in the robot competition – i.e. flowchart, block diagram, etc. Make sure you give details on both the image processing part and the game strategy part.